THE EFFICIENCY OF MODAL CONTROL OF THE LASER BEAM PHASE

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The results of experimental and theoretical studies of compensation for nonlinear distortions by means of the adaptive control of light beam in the basis of the lowest–order wave aberrations are presented. It is demonstrated that for a wide range of variation of physical parameters the modal control of the beam phase significantly improves the spatial localization of radiation energy. Applicability of the similarity theory, developed in nonlinear optics, is experimentally proved.

INTRODUCTION

The principle of modal formation of the beam phase is being increasingly employed in adaptive optics. Such an approach is based on the results of theoretical studies of mode composition of phase distortions of optical beam propagating through both nonlinear and randomly inhomogeneous media as well as by advances in designing the wavefront sensors and correctors and in construction of the algorithms for adaptive control. For example, the lowest–order aberrations produce the main contribution to the phase distortions of the beam under conditions of stationary wind–induced refraction being the lowest threshold nonlinear atmospheric effect.[1,2] Perturbations of the beam phase in the turbulent atmosphere may be represented in the form of an expansion in a system of the Karhunen–Loev functions or Zernike polynomials.[3] Starting from the third power term, the coefficients of the expansion in a system of Zernike polynomials rapidly decrease becoming negligible. For this reason forming the beam phase as a superposition of the base modes is undisputedly advantageous, because the needed number of control channels is significantly reduced thereby improving the stability of the adaptive system. Simultaneously modal approach makes it possible to monitor the accuracy of correction of phase distortions.

At present various types of wavefront correctors have already been designed intended to modal formation of the phase of the light beam. The most promising among them are those with nonlocalized response functions. For example, for the bimorph mirrors with 13 drives the deflections were obtained which coincide with the first Zernike polynomials to an accuracy of 10%. The sensitivity then was about 0.03 μm/V (see Ref. 4). A flexible mirror with six electromechanical drives clamped at its contour reproduces the first five Zernike polynomials to an accuracy of 20% for the range of displacement of the reflecting surface up to 300 μm (see Ref. 5). Apparently, modal formation of the beam phase can be also implemented by a corrector with a local response function if the number of its drives is sufficiently large (up to several tens).

In systems for modal control it is preferable to use the wavefront sensors, which yield the information on the mode composition of the distorted wave.[6] To organize operation of such adaptive systems, algorithms have been constructed which operate in the space of the base modes. One of such algorithms uses the multicriterional approach, which permits one to decouple the channels of modal control thereby improving the operational stability of the system affected by natural noises.[7]

Despite such significant advances in developing the components of the system of adaptive optics, we still have comparatively few experimental studies performed with the help of the working set up.

The present paper is devoted to studying an adaptive system for modal control intended to compensate for nonlinear phase distortions of a laser beam propagating through an absorbing medium. The principal goal of the study is obtaining a quantitative estimate of the efficiency of modal control of the lowest–order in lower aberrations for the spatial location of the beam under conditions of wind–induced refraction. This paper continues and generalized the cycle of studies presented elsewhere.[7,9,10]

THE ANALYSIS OF MULTICRITERIONAL REFRACTION ALGORITHM FOR CORRECTION OF THE WIND–DRIVEN

The principal idea of the multicriterion algorithm consists in adaptive control based on optimizing against the set of scalar criteria describing the quality of compensation for the corresponding base mode of the corrector. Such an approach results in eliminating the cross talk between the different control channels there by speeding up the convergence and improving the stability of the iterative control. Let the beam phase be controlled in the basis of the lowest–order optical aberrations, which are described by the first five Zernice polynomials (tilts, axisymmetric focusing, and astigmatisms). As the analysis of the case of the linear and homogeneous medium showed,[9] the following functionals of distribution of beam intensity $I(x, y, z_0)$ in the focusing plane z_0 may be used for such quality criteria:

$$\hat{F}_1[I] = M\{x\} = x_z \; ; \; \hat{F}_2[I] = M\{y\} = y_z \; ;$$

$$\hat{F}_{3,4}[I] = M\{(x - x_z)^2 \pm (y - y_z)^2\} = a_x^2 \pm a_y^2 \; ;$$

$$\hat{F}_5[I] = M\{(x - x_z)\,(y - y_z)\} = a_{xy} \; , \quad (1)$$

where

$$M\{f(x, y)\} = \int\int f(x, y)\, I(x, y, z_0)\, \mathrm{d}x\, \mathrm{d}y \Big/ \int\int I(x, y, z_0) \mathrm{d}x\, \mathrm{d}y.$$

The values x_c and y_c denote the coordinates of the energy center of gravity, and a_x and a_y give the effective beam size over its cross section. The functionals $\hat{F}_1$ and $\hat{F}_2$ specify the position of the energy center of gravity of the beam, $\hat{F}_3$ determines the focusing, $\hat{F}_4$ and $\hat{F}_5$ describe the

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roundness of the beam image associated with the astigmatisms of the wavefront.

Since these functionals cannot completely decouple the channels, step–by–step compensation of aberrations was used in the experiment to improve the stability of the iterative process of control.[9]

When such an approach is expanded to cover the problems of control of the beam phase in the nonlinear medium, one should account for arising the additional cross talk between the various control channels.[10] The principle of superposition breaks down in the nonlinear media, so that in the case of control for some channel all intensity functionals changes.[1] This may result in a retarded convergence of iterative focusing, and even in its breakdown for high nonlinearities. The problem of theoretical investigation of adaptive control of the beam phase in a nonlinear medium, based on multicriterional algorithms became very important.

The principal regularities of multicriterional control of the phase in the basis of the lowest–order optical aberrations (up to the second order, inclusively) may be studied within the framework of the abberation–free approximation. The radiation field under conditions of stationary wind–induced refraction is then represented as a Gaussian–like beam whose phase can be described by the second order polynomial:

$$E(x, y, z) = A(z) \exp\left\{-\frac{(x - r(z))^2}{2a^2(z)} - \frac{y^2}{2b^2(z)}\right\} \exp\{i\varphi(x, y, z)\} ;$$

$$\varphi(x, y, z) = \kappa\left\{\theta(z)\,(x - r(z)) + \frac{S_x}{2}(x - r(z))^2 + \frac{S_y}{2}y^2\right\}, \quad (2)$$

where $\theta(z)$, $S_x(z)$, and $S_y(z)$ are the tilt and the curvatures of the wavefront; $a(z)$, $b(z)$, and $r(z)$ are the effective size and the shift of the beam's energy center of gravity. Wind direction is assumed to coincide with the *OX* axis.

The beam phase $\varphi(x, y, 0)$ at the transmitting aperture is formed as a superposition of base modes reproduced by the wavefront corrector. For a flexible controllable mirror, whose deflection is described by the modes $\omega_j(x, y)$, the phase $\varphi(x, y, 0)$ is equal to

$$\varphi(x, y, z) = 2\kappa \sum_{j=1}^{5} U_j\,\omega_j\,(x, y) , \quad (3)$$

where U_j are the control coordinates. In the case in which Zernike polynomials are selected chosen for the modal basis, the control coordinates are related to the coefficients of phase expansion (2) in the plane $z = 0$ by the formulas

$$U_1 = \theta(0) ; \quad U_3 = \frac{S_x(0) + S_y(0)}{2} ;$$

$$U_4 = \frac{S_x(0) - S_y(0)}{2} . \quad (4)$$

In the case of stationary wind–induced refraction, the following similarity criteria are chosen for the problem of beam phase control:[11] path length $z/\kappa a_0^2$ expressed in diffraction lengths; optical depth at diffraction distance $\alpha n_0\,\kappa a_0^2$; parameter of nonlinearity $R_\upsilon = \dfrac{2\kappa^2\alpha\, n_0 a_0}{\pi\, C_p\, \rho}\dfrac{\partial n}{\partial T}\dfrac{P_0}{V}$; dimensionless coordinates of control of tilt, focusing, and astigmatisms of the wavefront $U_1\,\kappa a_0$, $U_3\,\kappa a_0^2$, $U_4\,\kappa a_0^2$.

To determine the parameters of the beam (2) along the path, a system of ordinary differential equations was derived using the variational approach.[12] Solving that system for several control coordinates (U_3, U_4) the values of the functionals $\hat{F}_3$, $\hat{F}_4$ were found which characterize the spatial localization of the beam for various conditions of propagation.

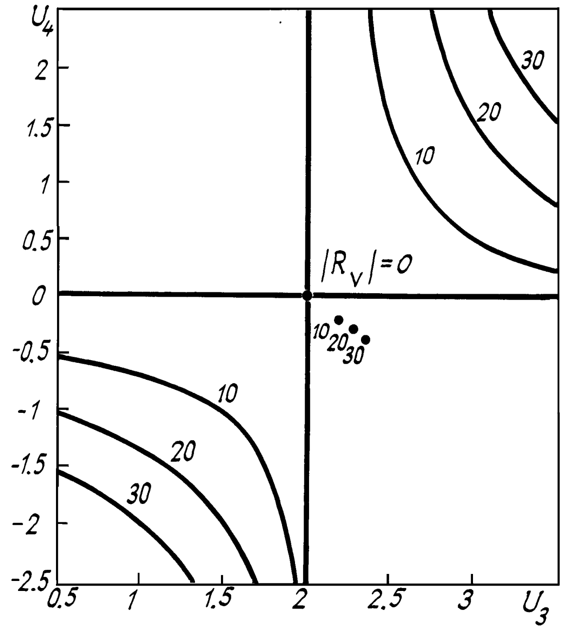

FIG. 1. Coordinates of the minimum of functional of focusing $\hat{F}_3$ and curves of zero functional of astigmatism $\hat{F}_4$ in the plane of modal control (U_3, U_4) for different values of the nonlinearity parameter R_υ. Path length $z = 0.5\kappa a_0^2$.

The degree and character of the effect of nonlinear inreaction between the radiation and the medium on the process of multicriterional control is vividly seen from the topology of the functionals $\hat{F}_3$ (focusing) and $\hat{F}_4$ (astigmatism). Fig. 1 presents the coordinates of the minimum of the functional $\hat{F}_3$ and the zero level curves for the functional $\hat{F}_4$ in the plane of modal control (U_3, U_4) for a number of values of the nonlinearity parameter R_υ. In a linear case the positions of that minimum of the functional $\hat{F}_3$ and of the zero value of the functional $\hat{F}_4$ coincide with each other (see points $U_3 = 2.0$ and $U_4 = 0.0$ in Fig. 1). This fact results in a maximally focused axisymmetric beam.

With the increase of nonlinearity the points of extremum split in the space of control coordinates (U_3, U_4). Namely, if the initial phase profile at the transmitting aperture minimizes the functional of focusing, the axial symmetry in the beam image is absent ($\hat{F}_4 \neq 0$). Conversely, if the beam is axisymmetric in coordinates (U_3, U_4), belonging to the curve of zero value of the functional $\hat{F}_4$, the effective radius of the beam exceeds extremely small. It then follows that the adaptive control of the beam phase in the nonlinear medium based on multicriterional algorithm with step–by–step correction for astigmatism U_4 first, and focusing U_3 second, may result in temporal oscillations of the parameters of the field. That statement is illustrated by Fig. 2 which shows the change of the peak intensity of the beam in the focusing plane in the process of multicriterional control and control based on gradient optimization against the single scalar criterion $\hat{F}_3$. Note that irrespective of the gain in the feedback loop (the gradient step size), multicriterional, control results in speeding up of convergence to peak intensity in as comparison with a single criterion control.

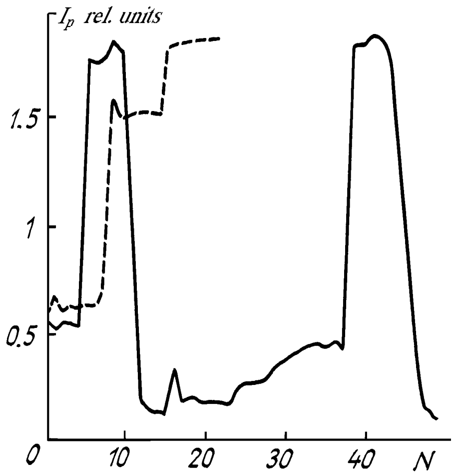

FIG. 2. Variation of the peak intensity of the beam I_p in the process of control by multicriterional algorithm (solid line) and by method of gradient optimization against a single scalar criterion $\hat{F}_3$ (dashed line). $|R_\upsilon| = 14$, $z = 0.5ka_0^2$, and N is the iteration number.

However laboratory tests of Ref. 10 do not reveal any breakdown of adaptive control after the range of optimal compensation for nonlinear distortions was attained. It may be explained by the fact that during the experiments in the range of optimal compensation the gradient of the astigmatism functional $|\hat{F}_4|$ can not be distinguished from the background noise level due to its small value.

EXPERIMENTAL STUDIES OF THE EFFICIENCY OF MODAL CONTROL

Experimental studies were performed using the laboratory model of the adaptive optical system described in Ref. 9. The principal component of the set up was the flexible controllable mirror capable of reproducing the mode basis consisting of the first five Zernike polynomials with satisfactory accuracy.[5]

Propagation of laser beam under conditions of wind–induced refraction was modeled using a rotating vertical cell filled with alcohol solution of fuchsine (the cell length was 0.7 m and the absorption coefficient of the solution was 1.0 m^{-1}).

Beam parameters were measured in the process of control by a specially designed recording system, which contained a standard TV camera interfaced with a computer.[13] Beam image was recorded in a plane located at a distance of 12 m from the exit window of the cell, which corresponded to the far zone of diffraction of the beam. From that image, the computer calculated the integral characteristics of the beam needed for the to adaptive control. The process of such control was implemented by a multicriterional algorithm of correction of aberrations.[10]

Experiments were performed with the us of the TEM_{00} mode of an Ar–laser at $\lambda = 0.488$ μm, the beam radius upon entering the medium was $a_0 = 1.2$ mm.

Below we present the results of three series of experiments on the adaptive compensation for the distortions caused by wind refraction at different speeds of flow (υ = 1.1, 2.0, and 3.0 mm/s). Measurements were performed with different input radiative power ($P = 0 - 200$ mW) during each series. Since one and the same value of the dimensionless parameter R_υ is attained different speeds t and powers P, results from each series are only given as functions of R_υ.

a) *Effective beam size.* The two effective beam radii in the directions parallel a_x and perpendicular a_y to the flow were calculated from the beam image from formula (1). For measurements in the far zone of diffraction effective radii of the beam determine its angular divergence.

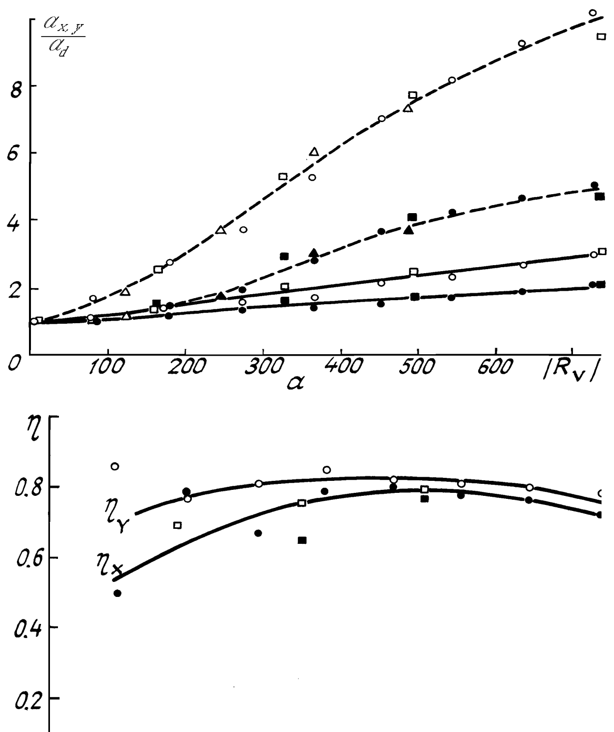

FIG. 3. Effective beam size a_x (filled squares and circles) and a_y (empty squares and circles); squares: υ = 1.1 mm/s; circles: υ = 2 mm/s; triangles: υ = 3 mm/s. Dashed lines: in the "off" mode of control; Solid lines: in the "on" mode of control. a) effective size vs. nonlinearity parameter R_υ. b) control efficiency η vs. nonlinearity parameter R_υ.

Fig. 3*a* shows the dependence of the effective size of the beam on the nonlinearity parameter R_υ, obtained in both "off" and "on" modes of the system of adaptive control.

First of all the satisfactory agreement between the experimental points obtained at various values of the physical parameters of speed υ and power P, which, however, correspond to one and the same value of the dimensionless nonlinearity parameter R_υ, should be noted That testifies to the applicability of the similarity theory to interpretation of the results of laboratory modeling of propagation of intense laser beams through the atmosphere and of the adaptive control of the beam phase.

When control is "off", the effective size of the beam along and across the flow typically differ quite strongly. This the characteristic feature of manifesting the anisotropy of the thermal lens induced in the moving medium by the beam. As can be seen from the figure, in the process of control of the base modes of focusing and astigmatism we obtain a significant reduction of the effective beam size in the far zone of diffraction. The control brings the angular divergence of the beam in a plane perpendicular to the flow, close to that in the plane parallel to that flow.

To give the numerical estimate of control efficiency, we may introduce the value η given by the formula

$$\eta = \frac{a^0_{x,\,y} - a^A_{x,\,y}}{a^0_{x,\,y} - a_\mathrm{d}}\,, \tag{5}$$

where $a^0_{x,\,y}$ are the beam radii prior to the control, and $a^A_{x,\,y}$ are these radii after control; a_d is the diffraction limited radius of the beam in the absence of nonlinearity. The introduced parameter η characterizes the ratio of the decrease in the beam size due to control to the deviation of that size from the diffraction limited value in the "off" mode of control. Thus it defines the relative improvement of the beam quality. For $\eta = 0$ the parameters of the beam do not change, while for $\eta = 1$ a complete compensation is achieved and the beam size $a^A_{x,\,y}$ coincides with diffraction limited a_d. The value $\eta < 0$ indicates that control actually gets worse the beam quality.

The dependence of η on the parameter of nonlinearity, shown in Fig. 3*b*, demonstrates that control is always somewhat more efficient in the direction across rather than along the flow. The beam formed as a result of such correction acquires a more symmetric shape, and the values a^A_x and a^A_y are brought close in value to each other.

With $|R_\upsilon|$ growing, the efficiency of control decreases because of intensifying the amplitude perturbations and because the principle of additivity of the nonlinear and correcting phase aberrations breaks down. As for the case of low nonlinearities, the reason of such a decrease in control quality has different nature there and results from the specific technical parameters of the design of the mirror drives. The nonlinear run–on of the phase, estimated in the fixed–field approximation, amounts to $\sim|R_\upsilon|z \sim 10$ rad for the parameter of nonlinearity $|R_\upsilon| \sim 10^2$. For the accuracy of positioning the surface of phase corrector being about 0.7λ, the relative local error in obtaining the needed phase profile may reach 50%.

Thus the parameter η, introduced above, characterizes both the principal possibility of compensating for the nonlinearities and the quality of operation of a given adaptive system.

b) *Spatial energy localization.* In connection with the problem of laser energy transfer through the atmosphere, there arises the question of change in the spatial localization of the light field in the process of the adaptive control. To obtain a quantitative estimate of spatial localization of radiation, we consider its normalized power, $W(a_W)$ in the observation plane z_0 incident upon the circular aperture of radius a_W. Since wind–induced refraction results in several extremuma in the distributions of beam intensity, the choice of the center of that circular aperture is not unique (Fig. 4*a*). During our measurements the center of that aperture coincided with the energy center of the beam (x_c, y_c). The normalized power $W(a_W)$ is given by the relation

$$W(a_W) = \iint_S I(x,\,y,\,z_0)\,\mathrm{d}x\,\mathrm{d}y/P\,, \tag{6}$$

where S is the aperture of the receiver and P is the beam power.

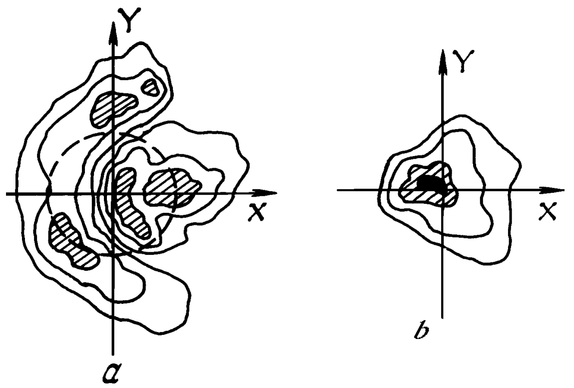

FIG. 4. Intensity isophotes in the beam crosssection (far zone of diffraction) for $|R_\upsilon| = 640$. a) in the "off" mode of control; b) in the "on" mode of control. The origin of the coordinate system coincides with the beam energy center (z_c, y_c). Dashed curve shows the receiving aperture.

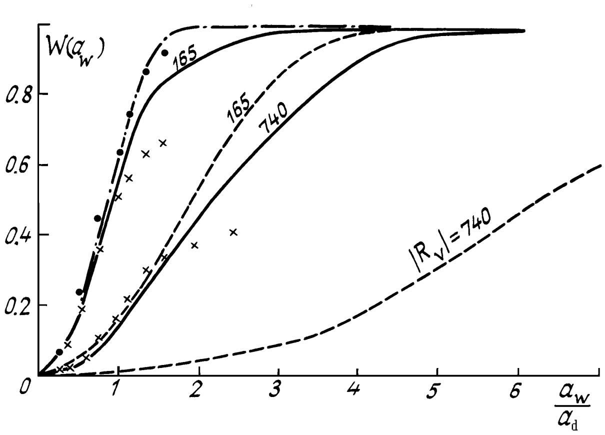

FIG. 5. Normalized power $W(a_W)$ vs. receiving aperture radius a_W. a_d is the diffraction radius of a Gaussian beam. Data are obtained from the digitized image: dash–dot line shows the linear case $R_\upsilon = 0$, dashed line shows the "off" mode of control; solid line shows the "on" mode of control. The PM data: dots indicate the linear case and crosses show the corresponding parameter after the control.

The normalized power $W(a_W)$ was calculated from the digitized beam images recorded with the control both "on" and "off". The accuracy of recording of that image was limited by the dynamic range of the TV and was 30–40 dB. That range makes any complete reproduction of the intensity change across the beam impossible. The camera was tuned to reproduce the maximum of the beam power density, so that the low intensity base was cut off during the measurements. Thus the spatial localization of the beam energy was overestimated. To eliminate such errors, simultaneous measurements of the beam power were performed using the power meter (PM), equipped with a tunable diaphragm. The limited aperture of the PM makes it impossible to measure the beam power $W(a_W)$, when control was "off" and the spatial spreading of the beam was high.

Figure 5 shows the dependence of the normalized power $W(a_W)$ on the radius of the receiving aperture a_W for two different values of the nonlinearity parameter. The curves are plotted from the digitized images (dots and crosses correspond to the PM readings).

When control is "off" wind–induced refraction distorts the directivity pattern of the beam. Local maxima appear in the intensity distribution, which deteriorate the spatial localization of the beam energy. As a result, the power entering the receiving aperture decreases for higher parameters of nonlinearity $|R_\upsilon|$, provided that the aperture radius remains fixed (Fig. 5).

Phase correction of the lowest–order optical aberrations can be used to significantly improve the spatial localization of energy, practically over the entire range of nonlinearity parameter R_υ. The fraction of power entering the prescribed aperture drastically increases. This effect is explained by the formation of a single distinctly pronounced maximum containing most of the beam power (Fig. 4*b*).

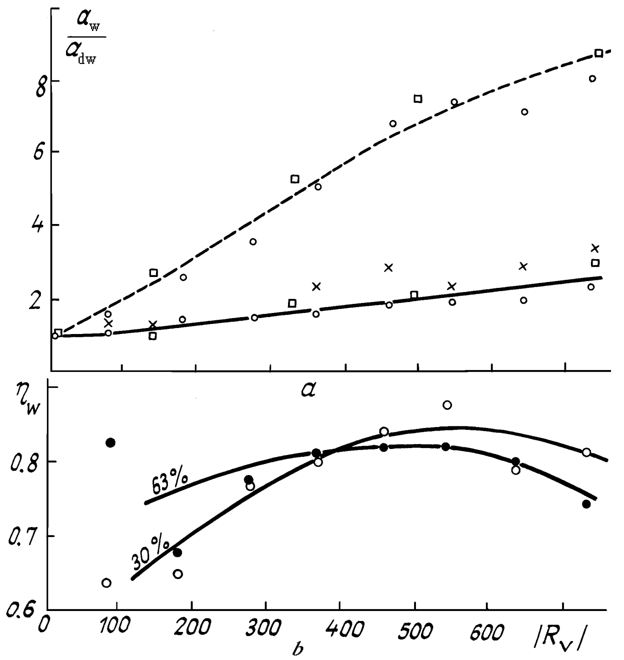

FIG. 6. Energetic radius of the beam a_W. a) Energetic radius a_W at 63% level of the total power vs R_υ. Empty circles and crosses: flow speed υ = 2 mm/s; squares; υ = 1.1 mm/s; dashed lines: without control; solid lines: after control. Clear circles and squares: digitized image; crosses; PM data. b) Energetic efficiency of control η_W vs the parameter of nonlinearity R_t at 63% and 30% levels of the total power and for the flow speed t = 2 mm/s.

To have a numerical measure of spatial localization of the beam, its energetic radius a_W may be introduced. It is equal to the radius of aperture covering a prescribed fraction of the total beam power. If that fraction constitutes 63% of the power, the values a_W and a_g for a Gaussian beam coincide with each other.

Figure 6*a* shows the energetic radius of the beam a_W vs the nonlinearity parameter R_υ, at the level of 63%. Control results in a reduction of the beam energetic radius by several times.

To estimate the efficiency of spatial localization of energy, we may introduce a variable η_W similar to η. The dependence of η_W on R_υ is given in Fig. 6*b* at two levels (63% and 30%) of the receiving power. It can be seen that the efficiency of energy control is quite high, amounting to about 0.8 in the range $|R_\upsilon| \sim$ 300–600.

It follows from the results shown above that when nonlinearities are large ($|R_\upsilon| \gtrsim 500$), the efficiency of such control, estimated at the level of 0.63, is somewhat lower than that at the level of 0.30. That effect is explained by the formation of a narrow kern of high power density against the background of a wide low–intensity pedestal in the process of control. Observations showed that the spatial scale of that low intensity pedestal remains practically un changed when control is "on".

CONCLUSIONS

High efficiency of modal control of aberrations of phase of the first and second order aimed to compensate wind–induced refraction of the beam is experimentally established. The beam size is reduced by more than a factor of two in the direction along the flow and by more than a factor of three across it. The beam image becomes almost axisymmetric.

With the parameter of nonlinearity varying in the range within $|R_\upsilon| \sim$ 200–700, the efficiency of control, treated as the relative reduction of the beam cross size, reaches 60–70% in the far zone of diffraction. The efficiency of spatial localization of radiation energy reaches 80% at the level of 63% of the total power.

The criteria from similarity theory are experimentally confirmed to be applicable to nonlinear optics. They may serve to interpreted the results of experimental studies of the adaptive systems.

In the case of wind–induced refraction, the nonlinearity parameter R_υ uniquely determines the distortions along the path of a prescribed length.

When the medium is nonlinear, the control channels in the adaptive optical system are cross talked, and the use of multicriterional algorithm should result in a breakdown of the correction process. However, that algorithm of modal control results in a stable convergence of iterative process of adaptive correction of phase distortions under the actual conditions of interference.

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