INVESTIGATION OF THE OPTICAL RADIATION PROPAGATION THROUGH STRONGLY ABSORBING INHOMOGENEOUS MEDIA BY THE DIFFRACTION RAYS METHOD

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The problem of the optical radiation propagation through a medium with parabolic distribution of the imaginary part of dielectric constant has been solved analytically by the diffraction rays method in the paraxial approximation. It is shown that the diffraction rays method allows energy flow lines to be traced for the known arbitrary distribution of the dielectric constant of the medium taking into account the diffraction and refraction on an inhomogeneous absorption profile. The geometrical optics approximation for the given media is considered. It is found that the optical ray trajectory in the medium with inhomogeneous absorptance depends not only on the complex dielectric constant distribution, but also on the wavefront phase distribution. This means that the rays outgoing from one point of space in the same directions but belonging to wavefronts with different curvature radii in the vicinity of the given point will propagate along different trajectories. It is the fundamental difference between the geometric optics of homogeneously and inhomogeneously absorbing media.

Traditionally, investigators use the methods based on the introduction of complex ray trajectories when solving the problems of the radiation propagation through absorbing inhomogeneous media.[1–4] Phenomenological tracing of rays in real space is described in Ref. 5 where the radiation propagation through a strongly absorbing inhomogeneous media is related to the amplitude trajectories with minimum extinction. However, the concept of the amplitude trajectory is based primarily on physical considerations rather than on rigorous derivations and therefore is of heuristic nature.

This paper describes the rigorous solution of the problem in the paraxial approximation based on tracing of diffraction rays in real space. The diffraction rays mean the lines perpendicular at each point to the wavefront phase, i.e., the tangent to the given line coincides with the direction of the Poynting vector.

1. The optical radiation propagation through strongly absorbing inhomogeneous media is considered on the basis of the parabolic wave equation

$$2ik\,\frac{\partial E}{\partial z} + \nabla_\perp^2 E + k^2\,\Delta\tilde{\varepsilon}(z, \mathbf{R})\,E(z, \mathbf{R}) = 0, \qquad (1)$$

where k is the wave number and $\Delta\tilde{\varepsilon}(z, \mathbf{R})$ is the perturbation of the complex dielectric constant of the medium: $\Delta\tilde{\varepsilon}(z, \mathbf{R}) = \varepsilon(z, \mathbf{R}) + i\sigma(z, \mathbf{R})$. The field is defined as $E(z, \mathbf{R}) = A(z, \mathbf{R})\,e^{ik\varphi(z,\mathbf{R})}$, where $A(z, \mathbf{R})$ and $\varphi(z, \mathbf{R})$ are the real amplitude and the eikonal of the field. It can be demonstrated[6] that Eq. (1) is identical to the system of equations

$$\frac{d^2\mathbf{R}}{dz^2} = \frac{1}{2}\,\nabla_\perp\left(\varepsilon(z, \mathbf{R}) + \frac{1}{k^2}\,A^{-1}\,\Delta_\perp\,A(z, \mathbf{R})\right),$$
$$\frac{dA^2}{dz} + \nabla_\perp\,(\nabla_\perp\,\varphi\,A^2) = -\,k\sigma(z, \mathbf{R})\,A^2. \qquad (2)$$

From this system of equations in the geometrical optics approximation $(k \to \infty)$ the following equation can be derived[6]:

$$\frac{d^2\mathbf{R}}{dz^2} = \frac{1}{2}\nabla_\perp\left(\varepsilon(z, \mathbf{R}(z)) + \frac{1}{4}\left[\int\limits_0^z \nabla_\perp\sigma(z', \mathbf{R}(z'))\,dz'\right]^2\right). \qquad (3)$$

An analysis of this equation demonstrates that a ray trajectory in the medium with the inhomogeneous absorption coefficient depends not only on the distribution of the complex dielectric constant, but also on the wavefront phase distribution. That is, the rays, emerged from one spatial point in the same directions

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but belonging to the wavefronts with different curvature radii in the vicinity of the given point will propagate along different trajectories. This is the fundamental difference of geometrical optics of inhomogeneously absorbing media from geometrical optics of the media with homogeneous absorption.

This fact can be illustrated by an example of the exact analytical solution for the parabolic profile of perturbation of the complex dielectric constant of the medium.

Now we consider the case in which the medium is strongly absorbing, i.e., $\varepsilon(z, \mathbf{R}) \ll \sigma(z, \mathbf{R})$ and the real part of perturbation of the dielectric constant can be neglected. In this case, for parabolic absorption distribution $(\sigma(z, \mathbf{R}) = \sigma_2\mathbf{R}^2)$ the derived system of equations (2) has the automodel solution

$$\begin{cases} \mathbf{R}(z) = \mathbf{R}_0\, g(z), \quad g(z=0) = 1, \\ A(z,R) = \dfrac{A_0}{\sqrt{g}} \exp\left(-\mathbf{R}_0^2/2(a_0 g)^2\right) \exp\left(-\dfrac{k}{2}\sigma_2 \displaystyle\int_0^z \mathbf{R}^2(z')\,\mathrm{d}z'\right), \end{cases} \tag{4}$$

where $g(z)$ is the beam broadening, for which from Eq. (2) it follows that

$$\mathrm{d}^2 g/\mathrm{d}z^2 = g^{-3}\,(1/ka_0^2 + \sigma_2\,\lambda(z))^2, \tag{5}$$

where $\lambda(z) = \int\limits_0^z g^2(z')\,\mathrm{d}z'$.

The obtained equation is nonlinear even for the limit of geometrical optics, although when considering the refraction effects only for the real part of perturbation of the dielectric constant of the medium, the equation for the function $g(z)$ is linear.[7] However, for the function $f(z) = g^2(z)$ from Eq. (5) the linear equation of the fourth order follows

$$\frac{\mathrm{d}^4 f}{\mathrm{d}z^4} = 4\,\sigma_2^2\, f(z). \tag{6}$$

Because the four independent initial conditions are required for obtaining the general solution of the fourth-order equation, the knowledge of only initial point and the angle of ray tilt is insufficient. The two additional initial conditions may be taken from the form of the second and third derivatives of function $f(z)$, which follow from Eq. (5):

$$\frac{\mathrm{d}^2 f}{\mathrm{d}z^2} = \frac{1}{2f}\left(\frac{\mathrm{d}f}{\mathrm{d}z}\right)^2 + \frac{2}{f}\,[L_D^{-1} + \sigma_2\,\lambda(z)]^2,$$

$$\frac{\mathrm{d}^3 f}{\mathrm{d}z^3} = 4\,\sigma_2\,[L_D^{-1} + \sigma_2\,\lambda(z)],$$

where, in its turn, $\dfrac{\mathrm{d}f}{\mathrm{d}z} = 2g\,\dfrac{\mathrm{d}g}{\mathrm{d}z}$ and $f(z) = g^2(z)$. Moreover, all the derivatives of function $f(z)$ are taken in the initial plane $z = 0$ and have the forms

$$f_0 = 1,\ f_0' = 2F^{-1},\ f_0'' = 2\,(F^{-2} + L_D^{-2}),\ f_0''' = 4\,\sigma_2\,L_D^{-1}.$$

The solution of Eq. (6) can be represented as

$$f(z) = f_1(\bar{z}) + L_\sigma F_0^{-1} f_2(\bar{z}) + \frac{L_\sigma^2}{2}\,(F_0^{-2} + L_D^{-2})\, f_3(\bar{z}) + L_\sigma L_D^{-1} f_4(\bar{z}), \tag{7}$$

where F_0 is the focal length of the initial wavefront phase, $L_\sigma = (2/\sigma_2)^{1/2}$, $L_D = ka_0^2$, $\bar{z} = z/L_\sigma$,

$$f_1(z) = \frac{1}{2}(\cosh 2\bar{z} + \cos 2\bar{z}),\quad f_2(z) = \frac{1}{2}(\sinh 2\bar{z} + \sin 2\bar{z}),$$

$$f_3(z) = \frac{1}{2}(\cosh 2\bar{z} - \cos 2\bar{z}),\quad f_4(z) = \frac{1}{2}(\sinh 2\bar{z} - \sin 2\bar{z}).$$

It should be noted that for the parabolic absorption distribution the derived solution is exact for the system of equations (2) and hence it is exact for the initial parabolic wave equation (1) as well.

The solution (7) confirms the above-indicated fact that even in the approximation of geometrical optics the two initial conditions (the starting point and the initial tilt) do not determine uniquely the ray trajectory, because in the third term of solution (7) after the limit $(L_D \to \infty)$ is taken, the dependence on the initial wavefront curvature F_0^{-2} remains.

From the solution of Eq. (7) it also follows that in the approximation of geometrical optics $(L_D \to \infty)$ for a plane wave $(F_0 \to \infty)$ the radiation intensity along the axial ray decreases significantly, although the absorption along this ray is absent. This decrease of the radiation intensity is explained by the beam broadening on the inhomogeneous absorption profile. Thus, the rays are bent not only on the real part, but also on the imaginary part of the dielectric constant of the medium.[8]

2. Now we consider the radiation propagation through the medium with the absorption distribution different from the parabolic one. The real part of dielectric constant of the medium is neglected as in the derivation of the automodel solution. In addition, we

restrict ourselves to the case of a two-dimensional (slit) beam. The system of equations (2) is transformed into a form more suitable for numerical realization. Taking into account the absorption

$$\tau(z,\, R(R_0,\, z)) = k \int\limits_0^z \sigma(z',\, R(R_0,\, z'))\, dz' ,$$

the expression for variation of the field amplitude along a given ray has the form

$$A(z,\, R(R_0,\, z)) = \frac{A_0(R_0)}{\sqrt{g(z)}}\, e^{-\tau/2}.$$

In what follows that using the expressions $A = e^{\chi}$ and $\nabla_{\perp}\varphi = \frac{dR}{dz}$ and going to the normalized variables

$$\bar{R} = \frac{R}{a_0},\ \bar{z} = \frac{z}{L_\sigma},\ \bar{\chi} = \frac{L_\sigma}{L_D}\chi,\ \bar{\tau} = \frac{L_\sigma}{L_D}\tau,\ \bar{\sigma} = \frac{L_\sigma^2}{a_0^2}\sigma,$$

we derive the system of equations

$$\begin{cases} \dfrac{d^2\bar{R}}{d\bar{z}^2} = \dfrac{1}{2}\dfrac{d}{d\bar{R}}\left(\left(\dfrac{d\bar{\chi}}{d\bar{R}}\right)^2 + \beta\,\dfrac{d^2\bar{\chi}}{d\bar{R}^2}\right), \\ \dfrac{d\bar{\tau}}{d\bar{z}} = \bar{\sigma}(\bar{z},\, \bar{R}), \\ \bar{\chi} = \beta\ \ln\left(\dfrac{A_0\ (R_0)}{\sqrt{g}}\right) - \dfrac{\bar{\tau}}{2}, \end{cases} \qquad (8)$$

where $\beta = L_\sigma/L_D$.

Based on the given system of equations, numerical analysis was made of the peculiarities of the optical radiation propagation for the Gaussian $[\sigma(z, R) = \sigma_2\,(1 - e^{-R^2})]$ and the power-law $[\sigma(z, R) = \sigma_2\,(R/a_0)^{20}]$ distributions of absorption.

When the Gaussian beam (with the Gaussian initial distribution of the field amplitude $A_0(z, R) = A_0\, e^{-R^2/2a_0^2)}$ propagates through the medium with the Gaussian absorption distribution over the beam cross section and the diffraction is small ($\beta = 0.01$), the ray focusing is observed at the point $\bar{R} = 1/\sqrt{2}$ (Fig. 1). This point is the point of inflection of the field amplitude distribution. In the right-hand side of the first equation of the system, the derivatives enter the function containing the field amplitude distribution. So we are able to verify that at this point the focusing condition is fulfilled. However, although we observe strong focusing on the inhomogeneous absorption profile, at this distance the extinction $e^{-\tau}$ dominates; therefore, the intensity at the focusing point is close to zero and consequently the intensity distribution is smooth (Fig. 2).

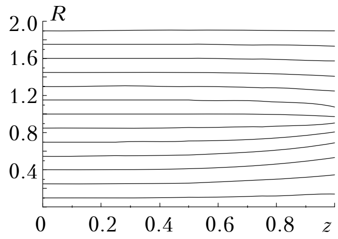

FIG. 1. Ray trajectories of the Gaussian beam propagating through the medium with the Gaussian distribution of inhomogeneous absorption for $\beta = 0.15$.

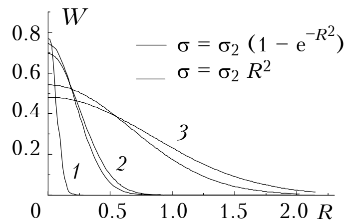

FIG. 2. Radial intensity distribution of the Gaussian beam propagating through the medium with the Gaussian and parabolic absorption distributions at $z = 1$ *for* $\beta = 0.01$ *(1),* 0.15 *(2), and* 1 *(3).*

One can readily see that the intensity distribution remains close to Gaussian. With the increase of the diffraction (with increasing β), focusing gradually disappears, because the diffraction becomes a suppressing factor.

Let us consider a medium with a sharp boundary of absorption variation. For example, $\sigma(z, R) = \sigma_2(R/a_0)^{20}$ can be taken, where a_0 is the Gaussian beam width. When diffraction is small ($\beta = 0.001$), the refraction is manifested on the inhomogeneous absorption profile. The rays gradually move aside and attenuate rapidly (Fig. 3). In this case, the intensity inside the weak absorption region remains unchanged. As the value of β increases up to 0.01, the rays in the vicinity of the sharp boundary of absorption variation undergo the diffraction distortions (Fig. 3) thereby initiating small spikes of radiation intensity at the points of their concentration (Fig. 4).

As β increases up to 0.15, the diffraction is manifested almost instantly. The rays deviate from the boundary with the sharp change of absorption and start to focus (Fig. 3). Gradually, with the increase of the propagation distance, strong focusing on the axis is observed (Fig. 4), resulting in the intensity increase more than twice.[9] This effect is unexpected, because the diffraction effects on the aperture with such a profile are less pronounced. This is confirmed by the numerical solution for the beam diffraction behind an infinitely thin screen, whose optical thickness is equal to the propagation channel optical thickness at the distance being studied. Figures 5 and 6 show the results indicating that at equal distances the diffraction effects behind the screen are less pronounced than in the propagation channel.

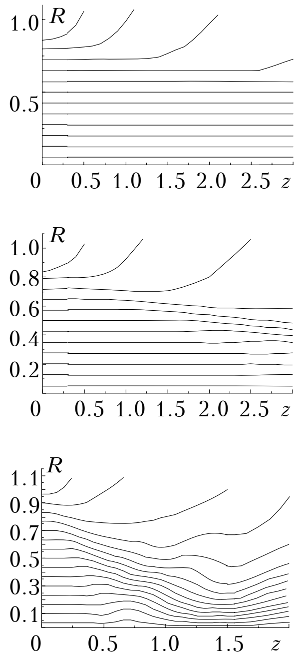

FIG. 3. Ray trajectories of the Gaussian beam propagating through the medium with the power-law distribution of inhomogeneous absorption $\sigma(z, R) = \sigma_2(R/a_0)^{20}$ *for* $\beta = 0.001$, 0.01, and 0.15.

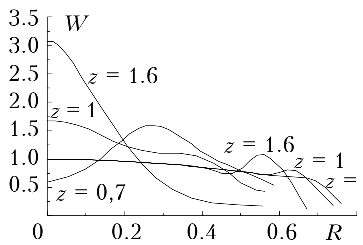

FIG. 4. Radial intensity distribution of the Gaussian beam propagating through the medium with the absorption distribution by the law $\sigma(z, R) = \sigma_2(R/a_0)^{20}$ *at distances up to* $z = 1.6$.

This behavior of rays propagated through the medium with the sharp boundary of absorption change as, for example, $\sigma(z, R) = \sigma_2\,(R/a_0)^{20}$ can be explained by a joint effect of diffraction and refraction on the inhomogeneous absorption profile. The refraction leads to the deflection of rays from the axis toward the region of strong absorption. The absorption results in the appearance of large gradients in the intensity distribution, that in their turn strengthen the diffraction ray bending. The bent rays fall within the region of strong refraction and the process is repeated.

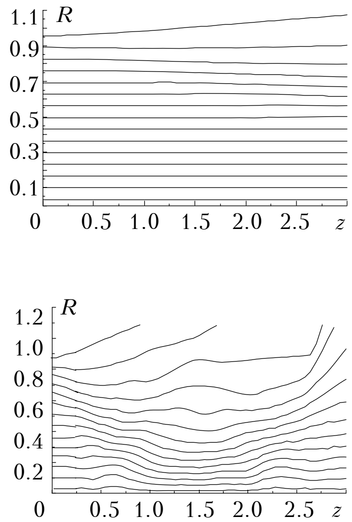

FIG. 5. Ray trajectories of the Gaussian beam propagating through the field stop of radius a_0 *for* $\beta = 0.01$ *and* 0.15.

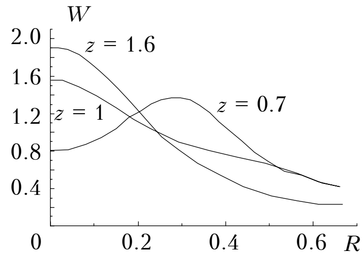

FIG. 6. Radial intensity distribution of the Gaussian beam propagating through the field stop of radius a_0 *at distances up to* $z = 1.6$.

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