ADAPTIVE COMPENSATION FOR NONLINEAR AND TURBULENT DISTORTIONS OF LIGHT BEAMS IN THE ATMOSPHERE

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The paper presents an overview of recent publications devoted to application of the simplex method to problems of atmospheric adaptive optics. A comparison of this method with the gradient procedure of hill climbing is carried out. The regimes of stationary and nonstationary wind–induced refraction in a stable medium, wind velocity fluctuations along a path, and large–scale fluctuations of the refractive index are considered in this paper.

1. INTRODUCTION

The problem of light power transfer through the fixed distance is of a great importance for present–day atmospheric optics. The transferred power is limited by such factors as turbulent and nonlinear spreading of a beam as well as its random wandering caused by large–scale fluctuations in the refractive index and pulsations in the wind velocity being available along the path. Adaptive systems of control over the phase of a light beam in the real time are used to compensate for these effects. A control in the problems of atmospheric optics is usually aimed at maximization of the light power received by the prescribed aperture. The principle of aperture sounding is widely used for controlling in the multivibrator adaptive systems. However, the gradient procedure of "hill climbing" being originally as the basis of the above principle results frequently in a search of only a local extremum of quality criterion. It also depends strongly on the initial conditions and turns out inefficient in the case of fluctuations in the parameters of a beam and medium. Therefore the development of such methods of control over the phase of light beam which does not require calculation of a goal function gradient is of interest. In particular, the simplex search can refer to such methods.

This paper presents an overview of the original recent publications devoted to application of the simplex method to the problems of atmospheric adaptive optics.

2. MATHEMATICAL MODEL OF THE SYSTEM OF CONTROL OVER THE PHASE OF A LIGHT BEAM

A statement of the problem on beam propagation through the atmosphere is mathematically described by the system of differential equations[1] for the complex amplitude of light field $E(x, y, z, t)$ and temperature of a medium $T(x, y, z, t)$

$$2ik\frac{\partial E}{\partial z} = \Delta_\perp E + 2\frac{k^2}{n_0}\left(\frac{\partial n}{\partial t}T + \tilde{n}\right)E ; \quad (1)$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + (\mathbf{v}\nabla)T\right) = \alpha I , \quad (2)$$

where $k = 2\pi/\lambda$ is the wave number, n is the refractive index of a medium, $\tilde{n}$ is the random field describing turbulent fluctuations of refractive index, ρ is the density, C_p is the heat capacity, $\mathbf{v}$ is the motion velocity of a medium, α is the absorption coefficient, and $I = cn|E|^2/8\pi$ is the light intensity. The nonlinearity parameter R proportional to the radiation power and to the convection time $\tau_v = a/v$, where a is the beam radius, is the main likeness criterion for systems (1) and (2).

The complex amplitude of the light field $E(x, y, z, t)$ at the point of entry into a medium (at $z = 0$) is prescribed in the form

$$E(x, y, 0, t) = E_0(x, y)\, f(t) \exp(i\, U(x, y, t)) , \quad (3)$$

where E_0 is the amplitude profile, $f(t)$ is the light pulse envelope characterized by the duration τ_p, and U is the controllable wave front. In the problem of beam focusing it is convenient to use one of the following quality criteria for control (goal functions of control):

– normalized peak intensity in the observation plane

$$J_m = (1/I_0) \max_{x,y} (I(x, y, z_0, t)) ; \quad (4)$$

– focusing criterion in the observation plane

$$J_f = \frac{1}{P_0}\int\int_{-\infty}^{\infty} \sigma(x, y)\, I(x, y, z_0, t)\, \mathrm{d}x\, \mathrm{d}y , \quad (5)$$

where P_0 is the total power in the beam, I_0 is the peak intensity at the point of entry into a medium, and σ the aperture function describing a localization spot of light in the observation plane. Under nonstationary conditions a control efficiency is best estimated using the parameter

$$\eta(T) = W(T)/W_0(T) , \quad (6)$$

where $W(T) = \int_0^T J_f(t)\, \mathrm{d}t$ is the total light energy coming onto the receiving aperture for the fixed time T, and $W_0(T)$ is the same energy in the lack of control.

According to the principle of modal control, the beam wave front formed by a corrector can be represented as superposition of the selected basis modes $S_i(x, y)$, i.e.,

$$U(x, y, t) = k \sum_{i=1}^{N} a_i(t)\, S_i(x, y) , \quad (7)$$

where a_i are the controllable coordinates (control signals). The temporal dependences of the signals a_i can be characterized by a

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pause duration between the subsequent processes of wave front correction τ_c. The algorithms developed in theory of control and automatic adjustment are increasingly applied to a search for optimal values of a_i in the systems of aperture sounding. The simplex method for finding an extremum of the goal function (quality criterion) of control[2] refers to them.

3. ALGORITHMS FOR CONTROL

It is naturally to begin a subsequent description of algorithms with the simplest regime at which an extremum of the goal function exists in the system of controllable coordinates and its position is independent of a search path. In the problems of atmospheric optics such a regime takes place when quasicontinuous radiation propagates through a medium with time–independent parameters. If the pauses between phase corrections are quite long in order that the stable temperature field could be established in the beam, then the maximum of object illumination can be achieved at an identically determined set of controllable coordinates. Thus in the N–dimensional space the motion to an optimum (hill climbing) can be implemented by multiple reflection of a certain object with $N + 1$ top (so–called simplex).

The direction of motion to an extremum is based on the values of goal function at all the tops. For example, motion to the maximum is carried out from the top with the least value of a goal function to the opposite face of simplex. A step in the search is determined by passing from "old" simplex to "new" one by means of elimination of the worst top after plotting its specular reflection relative to the face being common for both simplexes. Multiple reflection of the worst tops results in step–by–step motion of the simplex center to the given aim along a certain curve. Essentially that the goal function is required to be calculated once for every step in a search excepting the start when its $N + 1$ value should be calculated.

The search with the variable step is frequently used under the above–indicated conditions when position of the goal function maximum is independent of the values of controllable coordinates. That makes it possible to combine the high velocity of motion at the beginning of optimization with an accuracy in a search of extremum at the stage of finishing. The power or exponential laws are usually used for a change in the simplex size. In so doing the accuracy for optimum achievement, initial size of simplex L_0, and the number of steps in the search turn out to be related by the simple relation.[2] The value of L_0 empirically estimated can be refined in the test problems for optimization.

In the real problems of atmospheric optics which are known to be nonstationary, the search of the object illumination maximum is accompanied by the transient processes in the "beam –– medium" system. Such processes occur when the parameters as well as controllable wave front vary along the path. In the regime of a long pulse that propagates through a regular medium, a monotonic shift of the beam towards the wind direction in the controllable space produces the effect, which is called "target drift" in optimization theory.[2] In addition, the search for the optimal phase can be represented in terms of two processes: climbing to the "movable hill" and following its motion. Essentially that "hill" motion, i.e., position of the goal function maximum depends on the path of its search.

In other words, since the processes of the thermal lens formation along the path and of the beam phase optimization occur at the same time the unsuccessful initial steps can cause such decrease in the goal function which cannot be eliminated by subsequent, even successful, corrections. Therefore, it is important to carry out the first steps in the correct direction which is, in its turn, determined by the initial configuration of simplex.

The *a priori* choice of the latter, as the experience shows, can be complicated if the dimensionality of the control space N is greater than three. In this connection, the problem arises for reasonable limitation of the number of independent controllable coordinates that is closely related to the problem of increasing in the speed and stability of an adaptive system. In addition, a choice of any regular method for changing in the face length of simplex as approaching to the extremum involves difficulties under nonstationary conditions. Therefore it stands to reason to remain the size of simplex unchanged in the case of available transient processes accompanying the search of the optimal phase along the path.

The size of simplex can be optimized, for example, using the criterion of maximum light energy coming to the fixed aperture of the object in the fixed time of control (6). Let us note also that "cycling" of simplex is one of the effects appearing in the course of the target drift, i.e., no reflection of any tops for a long time, as a result, simplex stops a translational motion to the target. To eliminate this effect the algorithms with forced reflection of tops remaining unmovable during the certain number of steps are used in optimization theory.[2]

In the regime of wind velocity pulsations available along the path the beam defocusing is close to axisymmetric one that should be accounted for choosing the basis and strategy of control. In particular, it is reasonable to apply two tilt angles and axisymmetric focusing to the basis as well as to use the search with variable strategy after its dividing into two stages. The first stage is the control at the initial stage of heating of the medium which allows one to follow the drifting target using the algorithm with forced reflection of tops and to avoid the cycling of simplex. Then, at the second stage, when random wandering of a beam and transient processes resulting from variation in the states of a medium predominate, the algorithm with unforced reflection of tops should be applied.[2] Its basic rule is the reflection of the worst top of simplex without any additional conditions.

4. DISCUSSION OF NUMERICAL RESULTS

Below we discuss the results obtained for the Gaussian beams along the path $z_0 = 0.5ka_0^2$ long, where a_0 is the initial radius of a beam. The relation $\sigma = \exp(-(x^2 + y^2)/S_t^2)$ is used as the aperture function σ. Here S_t is the effective radius of the region of light beam localization which is equal to the double radius of the focal spot bounded by diffraction.

4.1. *Stationary wind–induced refraction in a regular medium* ($f(t) \equiv 1$, $\tau_c \gg \tau_v$, $\mathbf{v} = \text{const}$, $\tilde{n} = 0$). Based on the salient features of thermal blooming in the moving medium (non–axisymmetric defocusing and shift of a beam due to wind), as the controllable coordinates it is naturally to choose the tilt angle of wave front in the wind plane ϑ_x and two parameters of focusing S_x and S_y in the planes longitudinal and transverse to the wind direction. According to that

$$U = \vartheta_x x + S_x \frac{x^2}{2} + S_y \frac{y^2}{2} . \qquad (8)$$

A comparison of the convergence rates and the accuracies in a search of the extremum of a goal function obtained by both the simplex method and ordinary gradient procedure is of most interest for the considered model problem. As calculations of the paths in a search of the maximum of focusing criterion show,[3] the obtained optimal values of J_f and number of optimization steps appear to be quite close for the comparative methods. However the number of measurements (calculations) of the goal function for the simplex method is 2–2.5 times less than for the gradient one since each gradient step is accompanied by the test measurements of a quality criterion response to the small variations in all the coordinates. The summary data are listed in Table I. In the stationary problems the simplex method, as is seen from the table, actually provides a higher rate of the search of the extremum comparing to that of the gradient procedure.

TABLE I. Correction of stationary wind–induced refraction in a regular medium

Control method	Nonlinearity parameter R	Number of goal function measurements	Radiation parameters on the object	
			J_f	J_m
Simplex	– 15	15	0.49	1.11
	– 30	16	0.30	0.56
Gradient	– 15	36	0.49	1.14
	– 30	36	0.30	0.60

4.2. *Nonstationary wind–induced refraction in a regular medium* ($f = 0$ at $t < 0$, $f = 1$ at $t \geq 0$, $\tau_c \leq \tau_v$, $\mathbf{v}$ = const, $\tilde{n} = 0$). As was above–mentioned, the main problem arising during compensation for the nonstationary variations in the beam in the real time by means of the simplex method is a choice of the optimal size of simplex L and the control basis. Following to Ref. 4 we can consider the available potentialities of optimization of L using three–dimensional basis (8). Table II represents the data calculated for the duration of compensation $T = 3\tau_v$ within the wide range of the nonlinearity parameter R and the values of the integral criterion of quality of correction η at the different values of L. As can clearly be seen from the table, the optimal size of simplex exists actually at each R, in addition, L_{opt} increases with increase in R. The optimal size of simplex can be also defined by the control duration as an analysis of dependences of L_{opt} on T shows (Table III). The decrease in L_{opt} with increase in T is associated with a need for compensation for the focusing criterion oscillations that can appear at long duration of control ($T \gg \tau_v$) in the case when the amplitude of phase variations is excessively large.

TABLE II. Efficiency parameter of control $\eta(3\tau_v)$ in compensating for nonstationary wind–induced refraction in a regular medium

R	Length of simplex face L										
	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
– 10	1.16	1.29	1.38	1.56	1.54	1.50	–	–	–	–	–
– 20	1.16	1.26	1.40	1.52	1.60	1.64	1.54	1.46	1.42	1.40	–
– 30	–	–	–	–	1.16	1.26	1.32	1.29	1.20	1.19	–
– 40	–	–	–	–	1.18	1.25	1.30	1.34	1.36	1.30	1.24

TABLE III. The optimal size of simplex in compensating for nonstationary wind–induced refraction in a regular medium

R	Duration of control T/τ_v										
	2	3	4	5	6	7	8	9	10	11	12
– 20	0.65	0.64	0.60	0.50	0.45	0.41	0.40	0.37	0.35	0.35	0.34
– 30	0.71	0.70	0.68	0.66	0.64	0.60	0.50	0.45	0.44	0.41	0.40

To optimize the control basis let us draw on the temporal dependences of coordinates $\vartheta_x(t)$, $S_x(t)$, and $S_y(t)$ in the course of dynamic correction.[5] The variables S_x and S_y, as their analysis shows, appear to be proportional each other at every instant of time in the wide range of R. Therefore it is natural to decrease the number of independent coordinates of control by introducing the combination mode ($x^2/4 + y^2/2$), i.e., to assign the wave front in the form

$$U = \vartheta_x x + S\left(\frac{x^2}{4} + \frac{y^2}{2}\right). \tag{9}$$

The summary data on simulation of the simplex search of the object illumination maximum in basises (8) and (9) are listed in Table IV, where the correction efficiency is estimated as earlier according to the parameter $\eta(3\tau_v)$. As is seen from the table, the control using the different basises turns out to be more efficient depending on nonlinearity of a medium (radiation power). In particular, three–dimensional basis (8) is preferable to use at $|R| \leq 20$ while two–dimensional basis (9) – at large nonlinearities.

TABLE IV. Efficiency parameter of control $\eta(3\tau_v)$ in compensating for nonstationary wind–induced refraction in a regular medium

Control basis	Nonlinearity parameter $\|R\|$			
	10	20	30	40
(8)	1.36	1.60	1.38	1.37
(9)	1.40	1.56	1.52	1.50

4.3. *Nonstationary wind–induced refraction at velocity pulsations* ($f = 0$ at $t < 0$, $f = 1$ at $t \geq 0$, $\tau_c \leq \tau_v$, $\mathbf{v}(z, t) = \langle v \rangle + \delta\tilde{\mathbf{v}}(z, t)$, $\tilde{n} = 0$). Going to the stochastic problems of control let us consider first the beam propagation through the medium with random pulsations of the wind velocity along the path, neglecting the natural fluctuations in the refractive index. The regime of the sufficiently frequent pulsations in the velocity $\delta\tilde{\mathbf{v}}$ can be available along the near–ground horizontal paths, the transient processes in the beam–medium system are quite essential at such pulsations.[6] According to Refs. 6 and 7, we can assume for definiteness the average time, during which the pulsations are "frozen", to be $T_v = 2\tau_v$ at the numerical experiments, while the standard deviation for the fluctuation component of the velocity σ_v to vary within $0 \leq \sigma_v \leq 0.5\langle v \rangle$ range.

As the calculations[7] show for the above–considered regime the distortions of the beam can successfully be compensated in the three–dimensional basis

$$U = \vartheta_x x + \vartheta_y y + S\left(\frac{x^2}{2} + \frac{y^2}{2}\right). \tag{10}$$

In addition, the optimal size of simplex L_{opt} is determined by only the average parameter of nonlinearity $\langle R \rangle$ and estimated on the above–considered grounds.

It was found in Ref. 7 that the variable strategy of a simplex search (see above) developed under conditions of sufficiently strong pulsations of the wind velocity ($\sigma_v \geq 0.3\langle v \rangle$) allows one actually to compensate the random wandering of the beam and to avoid unstable regimes in the course of control over the phase for a long time (T/τ_v = 10 ... 12). In so doing at $\langle |R| \rangle$ = 10 ... 30 the phase compensation increases the energy $W(T)$, on the average, by a factor of 1.5 comparing to the case of propagation of the uncontrollable (both collimated and focused) beam.

A comparison made with the gradient procedure shows the identical and average over time values of the focusing criterion $\langle J_f \rangle$ (see Table V) can be attained by using both the methods. The algorithm of the simplex search is stable for pulsations of the wind velocity within a range of $\sigma_v \le 0.5\langle v\rangle$, in addition the standard deviation of the focusing criterion is virtually unchanged with increase in σ_v. This can apparently be explained by the fact that the used algorithm provides an uniform scanning by a beam over the mutually perpendicular planes. As a result, the average displacement of the center of gravity of the beam $\langle r_c\rangle$ is less than $a/2$. We can note also that to provide the stability, the control procedure is required to be more complicated when using of the gradient method under conditions of $\sigma_v \le 0.3\langle v\rangle$, for example, we should apply the sounding isolated over both focusing and tilt.[6]

TABLE V. Average values of focusing criterion in compensation for nonstationary wind–induced refraction in a medium with velocity pulsations ($\sigma_v = 0.3\langle v\rangle$)

Control method	Nonlinearity parameter $\langle\|R\|\rangle$ 10	20	30	40
Simplex	0.40	0.27	0.24	0.23
Gradient	0.42	0.29	0.26	0.25

4.4. *The turbulent atmosphere characterized by the random wind along the path* ($f = 0$ at $t < 0$, $f = 1$ at $t \ge 0$, $\tau_c \le \tau_v$, $\mathbf{v}(z, t) = \langle\mathbf{v}\rangle + \delta\tilde{\mathbf{v}}(z, t)$, $\tilde{n} \ne 0$). For the regime under consideration, we assume that the average time during which the wind velocity pulsations become frozen and the time of each realization of the field of random fluctuations in the refractive index $\tilde{n}$ are identical and equal to $2\tau_v$. In addition, both the wind velocity and refractive index change at the same instant of time.

The control quality is studied in Ref. 7 depending on the parameter $D_s(2a)$ (Ref. 8) which characterizes the turbulence of the atmosphere along the path and makes sense of the structure function of phase fluctuations of the spherical wave in the beam diameter. To form the random fields $\tilde{n}$ the method of modal representation of the atmospheric inhomogeneities was used.[9] The efficiency of correction is estimated by the normalized total light energy η arrived to the receiving aperture in the control time $T = 12\,\tau_v$.

The calculational results averaged over 30 samplings following one by one in time $T = 12\,\tau_v$ are listed in Table VI. As the analysis of the table shows the control based on the simplex method is stable and rather efficient within the wide range of the parameter $D_s(2a_0)$.

TABLE VI. Average values of efficiency parameter of control $\langle\eta(12\tau_v)\rangle$ *in compensation for nonstationary wind–induced refraction in a medium with velocity pulsations* ($\sigma_v = 0.3\langle v\rangle$, $\langle|R|\rangle = 20$)

Atmospheric turbulence parameter $D_s(2a_0)$							
0.0	0.6	1.2	1.8	2.4	3.0	3.6	4.2
1.27	1.32	1.33	1.34	1.36	1.35	1.34	1.33

5. CONCLUSION

Investigations carried out show that the simplex method can be successfully used in the systems of atmospheric adaptive optics based on the aperture–sounding principle. Comparing to the gradient procedure of hill climbing the simplex method has the advantages such as stability and high speed.

The control based on the simplex method provides the reliable maximum of the goal function in the regime of the stationary wind–induced refraction. In addition, the number of its measurements is as low as 1.5–2 times comparing to the gradient procedure.

In the nonstationary regime of wind–induced refraction for the fixed parameters of a medium and beam there is the optimal size of simplex determined by the control duration and speed of the adaptive system. Optimization of the simplex size allows us to increase the total energy coming to the receiving aperture by a factor of 1.3–1.5 as comparing to the gradient method.

The simplex method and analysis of *a priori* data on the behavior of a goal function in the regime of nonstationary wind–induced refraction makes it possible to decrease the number of controllable variables. That permits us to reduce the technical requirements for the system of control over the beam.

As applied to the regime of random pulsations of the wind velocity along the path we developed the variable strategy of control. Such a strategy is capable of compensating the random wandering of a beam and to eliminate the losses in stability. We elucidate that the available large–scale fluctuations in the refractive index do not virtually diminish the quality of correction within the wide range of the parameter $D_s(2a)$.

In conclusion we would like to note that the simplex method extends strongly the possibility of the adaptive control over the beams in the real time as comparing to the gradient procedures.

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