Stability of the fractal properties of laser radiation in turbulent media

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Using a model of the turbulent atmosphere, we have experimentally investigated the fractal characteristics of fluctuations in the position of laser beam center of gravity. It was established that the fractal radiation properties manifest themselves most obviously when the beam diameter is smaller or comparable with the characteristic size of the inhomogeneities. It was shown that in a turbulent medium with invariable statistical characteristics, the Hurst parameter of fluctuations and, correspondingly, the fractal dimensionality are characterized by high stability.

Numerous publications can be found in literature[1–4] on the study of fluctuations of laser radiation structure in a turbulent atmosphere. Most often, such studies use traditional methods of probability theory and mathematical statistics for describing the fluctuations. As a rule, the role of scaling characterizing the fractal and multifractal properties of structure fluctuations is ignored.

In Refs. 5 and 6 it was reported on the detection and investigation of the fractality in fluctuations of laser radiation propagated along the near-ground atmospheric paths. However, the above-mentioned studies pointed out instability of the fractality characteristics recorded. In the experiments carried out in the ground atmosphere, it was impossible to fully control the turbulence along the entire path. As a result this, in its turn, didn't allow one to uniquely relate the changes in fractal properties of radiation fluctuations with the variations of the propagation medium parameters. Therefore, there was a necessity in supplementary investigations of the fluctuation structure under laboratory conditions when the turbulence parameters could be reliably controlled.

In this study, investigation of this kind has been carried out using a model of turbulent atmosphere. The modeling has been performed using a multipass cylinder cell where a beam of laser radiation with the wavelength $\lambda = 0.532$ μm was propagated (Fig. 1).

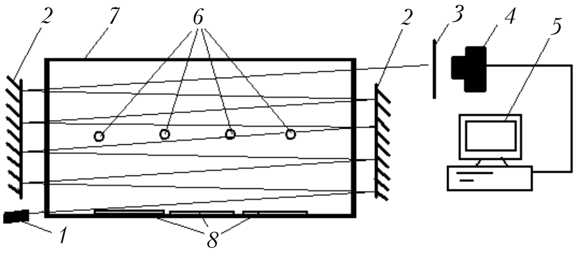

Fig. 1. Optical arrangement of the setup for simulating the atmospheric turbulence: *1* is the laser; *2* are the driving mirrors; *3* is the semitransparent screen; *4* is the camera; *5* is the personal computer; *6* are the nozzles; *7* is the cylinder cell; *8* are the heating elements.

The turbulent medium was formed by means of mixing the heated and cold air injected into the cylinder through the nozzles. The fluctuations of the beam gravity center were recorded with a video recorder and computer processing of the images of the laser beam passed through the cylinder. The image recording rate was up to 60 frames/s.

It was revealed in the experiments, that the fractal properties of radiation manifest themselves when the beam size was smaller or compared with the characteristic size of the medium inhomogeneities. Usually, this condition corresponded to the temperature difference between hot and cold air about 60°C. Typical sizes of the inhomogeneities were determined by means of the Young interferometer, which was additionally illuminated with an expanded light beam that had once passed through the turbulent medium. It was assessed that value of the structure characteristics of fluctuations of the air refractive index C_n^2 optimal for recording the fractal properties of radiation in a turbulent medium, is to be about 10^{-8} cm$^{-2/3}$.

Figure 2 presents the fluctuations of the collimated beam gravity center with diameter of 5 mm along the horizontal X_c for different number of the beam passages along the cylinder.

As follows from these data, the variance of fluctuations decreases with the decrease in the number of beam passages through the cell.

Scaling in fluctuations of the beam gravity center was determined using the method based on studying the behavior of the structure function S_n of the first order[7,8]:

$$S_n = E\left[\left|X_{k+n} - X_k\right|\right] = \frac{1}{K-n}\sum_{k=1}^{K-n}\left|X_{k+n} - X_k\right|.$$

Here X is the signal studied, k and $k+n$ are the order numbers of significant points, K is the total number of significant points; $0 < n < K$. If the structure function plotted on a double logarithmic scale has a portion close to a straight line, in a certain range of n, one can conclude that the signal

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possesses fractal properties. The Hurst parameter H connected with the fractal dimensionality D by the relation $D = 2 - H$ is estimated by the slope of the straight line.

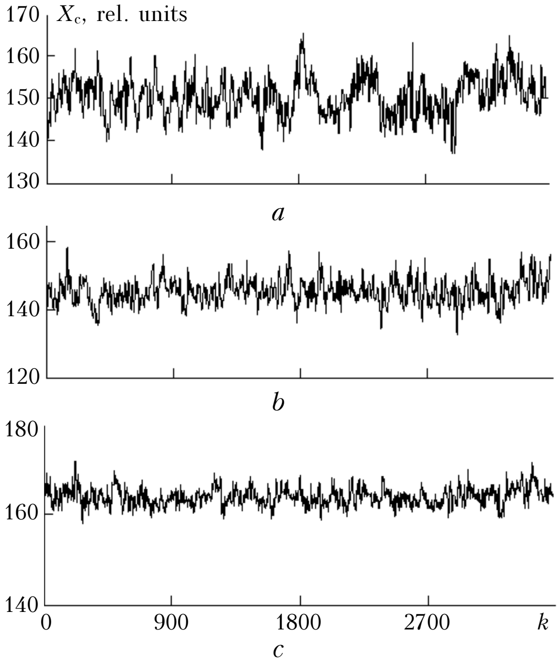

Fig. 2. Fluctuations of the laser beam center of gravity for different numbers of beam passages along the cylinder: 8 (*a*), 6 (*b*), and 4 passages (*c*). Time interval between the significant points along the abscissa is 1/60 s.

Figure 3 presents the structure function for the cases presented in Fig. 2. It is seen that the scaling region, where the tendency toward stable growth of the structure function is seen, makes about 32 significant points that exceeds the size of the fluctuation correlation zone by approximately 3 times.

The measurements have shown that the Hurst parameter is characterized by the high stability at a definite number of passages along the cylinder and at fixed values of the thermal and gasdynamics parameters of the turbulent medium. Its deviations from the mean, over the realizations, values did not exceed 0.04. The Hurst parameter has a tendency to grow with the increase in the number of passages and in the beam width that is connected with the averaging of the effects of small-scale inhomogeneities.

Some particular values of the parameters obtained in the experiment for the beams with diameter of 5 and 15 mm both for horizontal and vertical directions are presented in the Table.

The characteristics of these oscillations are different, since the turbulence is not isotropic in the cylinder. Anisotropy is conditioned by the fact that ordered motion upwards for the air heated by the electric elements is not completely eliminated by its mixing with cold air injected into the cylinder along the horizontal direction.

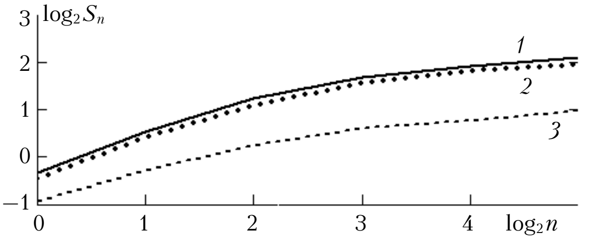

Fig. 3. The structure function of fluctuations of the laser beam gravity center for different numbers of beam passages along the cylinder: 8 (*1*), 6 (*2*), and 4 passages (*3*).

If the intensity of turbulent processes in the cylinder was amplified by increasing the temperature difference between hot and cold air, the values of H dropped. However, due to the growth of noise level in the recorded signals, the test accuracy of H considerably decreased. Effective analysis of the fractality is only possible, under these conditions, by use of faster recorders.

In parallel to the experimental study, we have carried out numerical modeling of the jitter effects for beam propagated through the cylinder using the random screen method. Two one-dimensional phase screens were considered, located near the reflecting surfaces of flat mirrors of the multi-pass cylinder cell moving along the direction perpendicular to the cylinder axis. Their correlation functions were set by the expression[9]:

$$B(r) = \left\langle n_1^2 \right\rangle \exp\left(-|r/l|^q\right),$$

where n_1 is the quantity characterizing intensity of the refractive index fluctuations in the cylinder; r is the spacing between the screens, l is the inhomogeneity size.

Table

Number of passages	Root-mean-square displacements, rel. units	Hurst parameter	Root-mean-square displacements, rel. units	Hurst parameter
	Beam width, mm			
	5		15	
	Displacements along horizontal direction			
8	5.11	0.37	4.45	0.48
6	3.56	0.35	3.52	0.48
4	1.59	0.27	1.90	0.38
	Displacements along vertical direction			
8	9.38	0.35	7.06	0.39
6	4.89	0.29	5.61	0.38
4	1.94	0.18	2.24	0.26

The numerical modeling has shown that the best fit to the experimental results was achieved, if the exponent q was taken to be equal to 1.75. The power of the phase fluctuation spectrum was determined by the correlation function according to Wiener–Khinchin theorem and the individual realizations of phase screen structure were generated. Then we have considered the laser beam diffraction on each of the screens. Figure 4 presents fluctuations of the beam center of gravity simulated in such a way, assuming the experimental conditions.

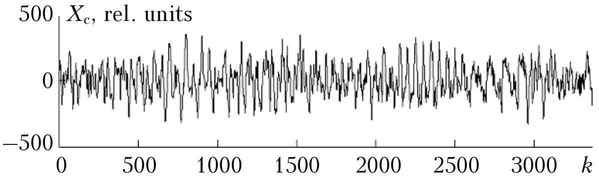

Fig. 4. Simulated fluctuations of the beam center of gravity.

Analysis of these fluctuations has shown that their structure, as well as the structure of the beam oscillations observed in the experiment, has fractal properties. The Hurst parameter for beam fluctuations was estimated to be $H = 0.56$, that somewhat exceeds the value obtained in the experiment. Most likely, this discrepancy results from that the model of the refractive index fluctuations used is quite approximate.

Thus, the experimental data, confirmed by the numerical modeling, indicate that fractal characteristics in laser radiation fluctuations earlier revealed in field experiments on the near-ground paths are not the result of random concurrence of different physical factors and reflect a deep connection between the fractal structure of inhomogeneities and fractality of the radiation fluctuations.

Acknowledgments

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