DETERMINATION OF THE OUTER SCALE OF THE ATMOSPHERIC TURBULENCE FROM THE DATA OF ACOUSTIC MEASUREMENTS

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Based on our numerical investigations, a method of remote determination of the outer scale L_0 of atmospheric turbulence from the ratio of acoustic radiation powers scattered at angles $\theta_1 \le 30°$ and $\theta_2 > \theta_1$ from the range $30° \le \theta_2 \le 70°$ has been proposed. The method is capable of measuring L_0 in the range from several tens of centimeters to 20 m with satisfactory accuracy. Efficiency of this method is illustrated by a numerical experiment. Its accuracy is estimated, and limits of applicability are specified.

The outer scale of atmospheric turbulence is a low-frequency boundary of the inertia interval of the structure function of the refractive index of an acoustic wave. This parameter is used to predict the turbulent extinction of acoustic wave propagating on forward and ranging paths, which makes the problem of its experimental determination especially urgent.

A method of the outer turbulence scale determination from the ratio of acoustic signal powers scattered in the forward hemisphere has been proposed in the paper. Its efficiency is illustrated by a numerical example. The accuracy of the method is also evaluated, and applicability limits are specified.

Analytic expression for the phase function of scattering of acoustic radiation by the atmospheric turbulence was derived in Ref. 1. Our numerical calculations have revealed some specific features of the behavior of the phase function of sound scattering in the forward hemisphere. They are illustrated by Fig. 1 and are as follows: At frequencies $F \lesssim 10$ kHz the phase function of sound scattering by the temperature and wind velocity fluctuation is practically independent of the intensity of atmospheric turbulence (the difference between the scattering phase functions for strong and weak turbulence is less than 2%). With a further increase of frequency, the difference also increases, and at $F = 20$ kHz it reaches about 20%. The ratio of the phase function of scattering at $\theta_1 \le 30°$ to that at $\theta_2 > \theta_1$, where $30 \le \theta_2 \le 70°$, depends only on the outer scale of turbulence L_0

$$g_{1,2} = \frac{g(\mu_1)}{g(\mu_2)} = \frac{\mu_1^2\,[2L_0^2(1-\mu_2)+\lambda^2]^{-11/6}(1+\mu_1)}{\mu_2^2\,[2L_0^2(1-\mu_2)+\lambda^2]^{-11/6}(1+\mu_2)}, \quad (1)$$

where $g(\mu_l)$ is the scattering phase function; $\mu_l = \cos\theta_l$; $l = 1, 2$; and, λ is the acoustic wavelength.

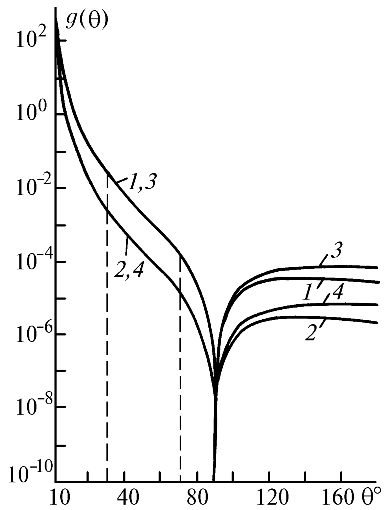

FIG. 1. Dependence of the phase function of acoustic radiation scattering on the outer scale and intensity of the atmospheric turbulence (F = 1 kHz) *for strong turbulence (1, 2) with* C_T^2 = 0.9 K^т·m$^{-т/3}$ *and the rate of dissipation of the turbulent energy* ε=0.1m^{т}s^{-3} *and weak turbulence with* $C_T^т$ = 0.0т K^т·m$^{-т/3}$ *and* ε = 0.003 m^{т}s^{-3}; L_0=5 *(1, 3) and* т0 m *(2, 4).*

Expression (1) was derived with the use of analytic formula for the scattering phase function obtained in Ref. 1. In the derivation, the contribution from the temperature pulsations, being insignificant for the examined range of variation of scattering angles, was ignored. It follows from Eq. (1) that the outer scale of atmospheric turbulence can be determined from the measured ratio of the phase functions of scattering at the above-indicated angles θ_1 and $\theta_т$. The calculated dependence of the ratio of the scattering phase functions on the outer scale of the turbulence is shown in Fig. т on a logarithmic scale at a frequency of 1 kHz.

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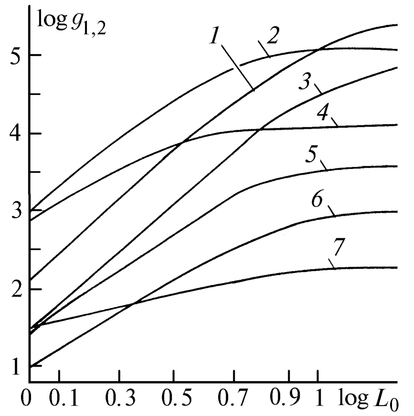

FIG. 2. Dependence of the ratio of the scattering phase functions on the outer scale of the atmospheric turbulence at scattering angles $\theta_1 = \text{т}$ *(curves 1 and 3),* 5 *(curves 2, 5, and 6), and* 10° *(curves 4 and 7) and* $\theta_{\text{т}} = 30$ *(curve 5),* 40 *(curves 1, 5, and 7),* 50 *(curve 3), and* 70° *(curves 2 and 4).*

In calculations the smaller angle of scattering was equal to т° considering that the angular width of the main lobe of the directional pattern of existing sodars varies between 1–т.5° (see Refs. т–4). It is seen from the figure that the ratio of the phase functions of scattering at the angles $\theta_1 = \text{т}°$ and $\theta_{\text{т}} = 40°$ ensures the maximum sensitivity of the method.

Results of calculations have shown that at $\theta_1 = 5°$, L_0 can be successfully determined in the region from several tens of centimeters to 10 m by the method suggested here. At $\theta_1 = \text{т}°$, the maximum measurable value of L_0 is equal to т0 m. Then the curves saturate, and with a further increase of L_0 the ratio of the scattering phase functions remains practically unchanged.

From Eq. (1) it follows that

$$L_0 = \frac{\lambda}{\sqrt{\text{т}}} \times$$

$$\times \sqrt{\frac{[g_{1,\text{т}}\mu_{\text{т}}^{\text{т}}(1+\mu_{\text{т}})]^{6/11} - [\mu_1^{\text{т}}(1+\mu_1)]^{6/11}}{(1-\mu_{\text{т}})[\mu_1^{\text{т}}(1+\mu_1)]^{6/11} - (1-\mu_1)[g_{1,\text{т}}\mu_{\text{т}}^{\text{т}}(1+\mu_{\text{т}})]^{6/11}}} \quad (\text{т})$$

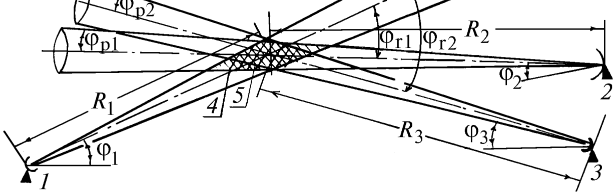

FIG. 3. Geometry of the experiment. Here 1 is the sound source, 2 and 3 are the receivers, and 4 and 5 are the scattering volumes.

To substantiate the practical applicability of the method, let us perform numerical experiment. The geometry of the experiment is shown in Fig. 3. To select a scattered signal, the scattering angle must be greater than the angular halfwidth of the directional patterns of receiving antennas φ_{p1} and $\varphi_{p\text{т}}$. This yields the receiver's field-of-view angles

$$\begin{aligned} &\varphi_u + \varphi_{p1} \le \varphi_{r1} \le 30° - \varphi_{p1}, \\ &30° + \varphi_{p\text{т}} \le \varphi_{r\text{т}} \le 70° - \varphi_{p\text{т}}. \end{aligned} \quad (3)$$

With the use of the equation of acoustic bistatic sounding, the powers of acoustic signals received at points *2* and *3* can be written as

$$P_1(\varphi_{r1}, z) = \frac{P_0 A_{\text{т}}}{R_{\text{т}}^{\text{т}}} \frac{V_1}{A_1} g(\varphi_{r1})\, \beta_{ex}(z) \times$$

$$\times \exp\left\{ - \int_{R_1} \beta_{ex}(r)\, dr - \int_{R_{\text{т}}} \beta_{ex}(r)\, dr \right\}, \quad (4)$$

$$P_{\text{т}}(\varphi_{r\text{т}}, z) = \frac{P_0 A_3}{R_3^{\text{т}}} \frac{V_{\text{т}}}{A_{\text{т}}} g(\varphi_{r\text{т}})\, \beta_{ex}(z) \times$$

$$\times \exp\left\{ - \int_{R_1} \beta_{ex}(r)\, dr - \int_{R_3} \beta_{ex}(r)\, dr \right\}, \quad (5)$$

where P_0 is the transmitted acoustic power, A_1 is the effective area of the transmitting aperture, $A_{\text{т}}$ and A_3 are the effective areas of the receiving apertures, V_1 and $V_{\text{т}}$ are the effective volumes of scattering, and β_{ex} is the acoustic radiation extinction coefficient. Placing the receivers at the same distance $R_{\text{т}} = R_3$ from the scattering volume, for the powers of acoustic signals received at points *2* and *3* we obtain

$$\frac{P_1(\varphi_{r1}, z)}{P_{\text{т}}(\varphi_{r\text{т}}, z)} = \frac{g(\varphi_{r1}, z)\, V_1}{g(\varphi_{r\text{т}}, z)\, V_{\text{т}}}. \quad (6)$$

The relative error δL_0 in determining L_0 was estimated with the use of the well-known formulas for estimation of the measurement error and was calculated from the relation

$$\delta L_0 = \frac{6}{11} g_{1,\text{т}}^{6/11} \frac{(a_1 a_3 - a_{\text{т}} a_4)\,(a_3 - a_4\, g_{1,\text{т}}^{6/11})}{(a_3 - a_4\, g_{1,\text{т}}^{6/11})^{\text{т}}\,(a_1\, g_{1,\text{т}}^{6/11} - a_{\text{т}})} \times$$

$$\times \sqrt{\delta^{\text{т}} P_1 + \delta^{\text{т}} P_{\text{т}} + \delta^{\text{т}} V_1 + \delta^{\text{т}} V_{\text{т}}}\,100\%, \quad (7)$$

where

$$\begin{aligned} &a_1 = [\mu_1^{\text{т}}(1 + \mu_1)]^{6/11}; \quad && a_{\text{т}} = [\mu_{\text{т}}^{\text{т}}(1 + \mu_{\text{т}})]^{6/11}; \\ &a_3 = (1 - \mu_{\text{т}})\, a_1; && a_4 = (1 - \mu_1)\, a_{\text{т}}; \end{aligned} \quad (8)$$

δP_1 and $\delta P_{\text{т}}$ are the relative errors in measuring the powers of acoustic signals received at points *2* and *3*; and, δV_1 and $\delta V_{\text{т}}$ are the relative errors of determining the effective scattering volumes. Formula (7) was derived with the use of Eqs. (т) and (6). The calculated error in determining L_0 is shown in Fig. 4 for 5 and 10% errors in measuring the powers of acoustic radiation and calculating the scattering volumes. It is seen from the figure that the error decreases as θ_1 decreases with a simultaneous increase of $\theta_\text{т}$. Thus, at $\theta_1=5^0$ and $L_0=1$ m it is less than 10% for the entire range of variation of the examined values of $\theta_\text{т}$. The error increases reaching 30% for $L_0=8$ m and 4т% for $L_0=10$ m. At $\theta_1 = \text{т}°$, $\delta L_0 \sim 5\%$ for $L_0 = 1$ m, and $\delta L_0 = \text{т}8\%$ for $L_0 = \text{т}0$ m.

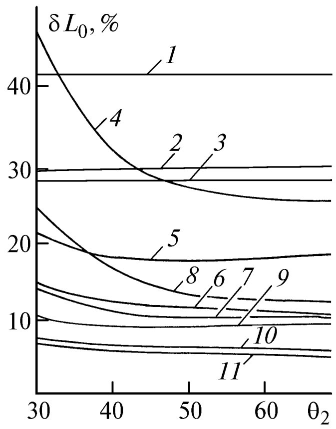

FIG. 4. Estimates of the errors in determining the outer scale of the atmospheric turbulence from the data of acoustic measurements with 5 *(curves 9–11) and* 10% *(curves 1–8) errors in measuring the acoustic radiation power and calculating the scattering volumes at* $\theta_1=\text{т}$ *(curves 3, 5, 7, 9, and 11) and* 5° *(curves 1, 2, 4, 6, 8, and 10) for* $L_0=1$ *(curves 6, 7, 10, and 11)*, 5 *(curves 4 and 8)*, 8 *(curve 2)*, 10 *(curves 1, 5, and 9), and* т0 m *(curve 3).*

Thus, numerical analysis has shown that the method suggested here can be used in practice for reliable determination of the outer scale of the atmospheric turbulence up to т0 m with satisfactory accuracy.

Analytic expressions for the scattering volumes were derived in Ref. 5 for rectangular directional patterns and in Ref. 6 for conical directional patterns of the source and receivers of acoustic radiation. For identical half widths of rectangular directional patterns of the source and receivers $\varphi = \varphi_u = \varphi_{p1} = \varphi_{p\text{т}}$, we derive

$$g_{1,\text{т}} = \frac{P_1(\varphi_{r1}, z)}{P_\text{т}(\varphi_{r\text{т}}, z)} \left[\frac{\tan\varphi_\text{т}(\tan\varphi_1 + \tan\varphi_3)\cos(\varphi_{r1}/\text{т})}{\tan\varphi_3(\tan\varphi_1 + \tan\varphi_\text{т})\cos(\varphi_{r\text{т}}/\text{т})}\right]^3 \times$$

$$\times \frac{\sin(\varphi_{r1}/\text{т})}{\sin(\varphi_{r\text{т}}/\text{т})}. \quad (9)$$

Formula (9) relates the ratio of the scattering phase functions to the ratio of acoustic signals received at points *2* and *3* for the angles of elevation φ_1, $\varphi_\text{т}$, and φ_3 of the transmitter and receivers.

In conclusion it should be noted that the outer scale of the atmospheric turbulence was obtained for von Karman's model of the spectrum of pulsations of the refractive index. Relationships among the outer scales for different models are given in Ref. 7.

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