TO THE THEORY OF SOUNDING WITH CHIRPS

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A method for sounding of an underwater acoustic channel with broadband chirps with preliminary signal processing by the frequency compression method is analyzed on the basis of the theory of linear systems. The dependence of a recorded spectrum on group delays in the channel and current frequency of sounding signal is obtained. It is shown that the result of processing of an individual sampling of received signal under several commonly used assumptions is mathematically equivalent to sounding with narrow-band complex signal whose delays determine the maxima of recorded signal modulus.

Signals with linear frequency modulation (chirps) are widely used in acoustic and radar sounding of natural media including underwater sound channel,[1] the upper atmosphere, and the Earth's ionosphere.[2] Their abundance is conditioned by the fact that chirps have large bases (bandwidth-duration products), which allows the diagnostics capabilities of sounding systems to be increased. There are different methods of chirp processing either in the temporal or in the frequency domain.

A successive description of widespread method for long chirp processing typically named "frequency compression method" is given in this paper.

The necessity of special consideration of this method stems from the fact that usually an analysis of long chirps (with large frequency deviation) is based on consideration of the chirps with relatively narrow bandwidths. In so doing, the radiated chirp with large frequency deviation is considered as a set of individual narrow-band chirps whose propagation is analyzed by the methods used for the narrow-band chirps.

The essence of the frequency compression method is that in a receiver the chirp is multiplied by a reference frequency-modulated signal having the same rate of frequency variation with a certain temporal delay. After multiplication of the reference signal by the received chirp a low-pass filter separates the signal at difference frequency, which contains the information about the propagation channel. Because the frequency difference between the received chirp and the reference signal is proportional to the propagation time, a spectral analysis of the filtration result after strobing with a temporal window permits one to obtain the "delay spectrumB of the received chirp on the reference signal frequency corresponding to the strobing interval center.

Therewith, portions of the chirp emitted at different times and coming simultaneously with different frequencies could enter the receiver. These portions are spaced through the spectrum, but the recorded spectrum is in fact a superposition of different portions of the emitted chirp; hence, it is profitable to analyze in detail the recorded spectrum considering the propagation channel and a real data processing scheme.

Let us consider in detail the frequency compression method. Let the transmitted chirp takes the form

$$u(t) = a(t)\cos\varphi(t) = a(t)\cos(\omega_0 t + \beta t^2/2), \qquad (1)$$

where $a(t)$ is the envelope with duration of several seconds or minutes, ω_0 is the initial cyclic carrier frequency and β is the rate of the cyclic frequency change. In this case, the frequency deviation is $\Delta\omega = \beta T$, where T is the chirp duration. As a rule, the employed chirps have large bases, the product of the chirp duration on the total bandwidth $\beta T > \omega_0$, and the total chirp base is about 10^5–10^6.

In the receiver the signal $u_1(t)$ transmitted through the channel with impulse transfer characteristic $h(x)$

$$u_1(t) = \int\limits_0^\infty h(x)\, u(t - x)\, \mathrm{d}x \qquad (2)$$

is multiplied by the reference signal $u(t - t_0)$ (delayed by t_0 from the transmitted signal). The resulting signal after transmission through the low-pass filter (with the impulse transfer characteristic $h_f(t)$) produces the signal

$$u_2(t) = \int\limits_0^\infty h(x)\, r(t, x)\, \mathrm{d}x \qquad (3)$$

at the filter output, where

$$r(t, x) = \int_0^\infty h_f(z)\, u(t - t_0 - z)\, u(t - x - z)\, \mathrm{d}z. \quad (4)$$

The function $r(t, x)$ specifies the result of low-pass filtration of the product of two chirps containing difference and summed frequencies. The low-pass filter bandwidth F_t is usually chosen much less than ω_0, so the summed frequencies are practically completely cut off.

After the low-pass filtration, instead of Eq. (4) we obtain the following (simplified) formula:

$$r(t, x) = \frac{1}{2}\int_0^\infty h_f(z)\, a(t-z; t_0, x)\, \cos\varphi(t - z, x)\, \mathrm{d}z, \quad (5)$$

where the designations $a(t-z;\, t_0, x) = a(t-z-x) \times a(t - z - t_0)$ are used and the cosine argument $\varphi(t, x)$ is given by the relation

$$\varphi(t, x) = \psi(x) + t\gamma(x) =$$

$$= [\omega_0\,(x - t_0) + \beta(t_0^2 - x^2)] + t\beta(x - t_0). \quad (6)$$

For $x - t_0 \ll T$ the narrow-band signal (as a function of $(t - z)$) is in the integrand of Eq. (5) with the carrier frequency equal to $\beta(x - t_0)$ and the envelope equal to the product of the chirp envelopes. In fact, this is the controlling condition for the selection of t_0. Because the range of variation of x (delays) depends on the signal propagation channel and is known *a priori*, t_0 is chosen so that the quantity $\beta(x - t_0)$ – the difference frequency – falls into the filter bandwidth, which is much less then the initial frequency and the total signal bandwidth. The latter means that the pulse duration $a(t - z;\, t_0, x)$ equal to $T - (x - t_0)$ differs from T only insignificantly, that is, at the filter input the difference frequency signal is truly narrow-band. In some cases in which very short delays are analyzed, t_0 can be chosen negative; in so doing not only delayed but also advanced frequencies enter the receiver. This situation, although exotic in the acoustic range, can be considered separately, but is beyond the scope of this paper.

After temporal strobing of the signal $u_2(t)$ by the window $w_k(t) = w(t - t_k)$, where t_k is the temporal window center (of duration Δ) and transition to the spectrum we derive the relation

$$S_k(\Omega) = \frac{1}{2\pi}\int_0^\infty h(x)\, \mathrm{d}x \int_0^\infty \mathrm{e}^{-i\Omega t}\, w_k(t)\, r(t, x)\, \mathrm{d}t =$$

$$= \int_0^\infty h(x)\, S_k(\Omega, x)\, \mathrm{d}x, \quad (7)$$

which describes the result of application of the frequency compression method. The position of the strobe $w(t)$ on the temporal axis specifies samples of duration Δ centered at $t_k = t_0 + (k - 1/2)\Delta$, $k = 1, \ldots, n$. The strobe duration is chosen from the condition of the channel stationarity and its dispersive properties. Although in water the material dispersion is practically absent, a waveguide leads to the appearance of the waveguide dispersion. This in its turn leads to the appearance of the coherence band, that is, the frequency band for which the dispersive distortions can be ignored. Therefore, the strobing window is chosen so that in the window the frequency advance does not exceed the coherence bandwidth. We use this condition as well. Because the total signal duration may be several tens of minutes, that time exceeds the stationarity period of the medium; therefore, the duration of strobe window is chosen so that the changes of the medium for these time intervals can be neglected. The spectral function $S_k(\Omega)$ of the signal fragment cut out by the strobe window characterizes the properties of the propagation channel at the time t_k.

If we proceed in Eq. (5) to the spectral function of the low-pass filter and to the spectrum of the function $a(t;\, t, x)$ (for the variable t), after a number of standard transformations we obtain the following relation:

$$r(t, x) = \frac{\pi}{2}\int_{-\infty}^\infty \mathrm{e}^{-i\omega t}\, A(\omega, x)\, [H_f(\omega + \gamma)\, \mathrm{e}^{i\psi + i\gamma t} +$$

$$+ H_f(\omega - \gamma)\, \mathrm{e}^{-i\psi - i\gamma t}]\, \mathrm{d}\omega. \quad (8)$$

Note that here, in accordance with Eq. (6), φ and γ are the functions of the variable x. The function $A(\omega, x)$ is the Fourier transform of $a(t;\, t_0, x)$ over the variable t.

Substituting Eq. (8) in the relation for the function $S_k(\Omega, x)$ determined by Eq. (7), we obtain

$$S_k(\Omega, x) = \frac{1}{4}\int_{-\infty}^\infty \mathrm{e}^{it_k(\omega - \Omega)}\, A(\omega, x)\, [F_+(\Omega - \gamma, \omega) +$$

$$+ F_-(\Omega + \gamma, \omega)]\, \mathrm{d}\omega. \quad (9)$$

Here, for convenience, the designation

$$F_\pm(\Omega \mp \gamma, \omega) = W(\Omega - \omega \mp \gamma)\, H_f(\omega \pm \gamma)\, \mathrm{e}^{\pm i\phi \pm it_k\gamma}$$

is used, where $W(\omega)$ is the Fourier transform of the function $w(t)$ – the temporal window.

The main expression to analyze is Eq. (7). Because the impulse response $h(x)$ has in fact the limited carrier, the function $a(t;\, t_0, x)$ differs from zero on sufficiently long interval of variation of t for all x over which the integral is taken in Eq. (7). In this instance, the function $A(\omega, x)$ as the Fourier transform of the function $a(t;\, t_0, x)$ has a very narrow spectrum compared with the bandwidth of the low-pass filter and the spectral width of the strobing window. Taking an advantage of this property and taking the spectral functions of the temporal window w and of the low-pass filter H_f outside the integral in Eq. (8) (assuming $\omega = 0$), we obtain less cumbersome formula

$$S_k(\Omega, x) = \frac{1}{4}\, a(t_k, t_0, x)\, \mathrm{e}^{-it_k\Omega}\, [F_+(\Omega - \gamma) +$$

$$+ F_-(\Omega + \gamma)]. \tag{10}$$

Here, the functions $F_\pm(\Omega)$ are defined by the same relations when $\omega = 0$. Note that in the derivation of Eq. (10) we cannot neglect the dependence on ω of the phase factors of the integrand in Eq. (8), because due to the multiplier t_k adjacent to ω and changing within the limits of the total duration of the chirp the phase values can be compared to π.

The windows used in practice usually have the narrow bandwidth of the order of several units or fractions of hertz. Moreover, $F(\Omega \pm \gamma)$ are localized within the region of zero argument; therefore, in Eq. (10) for the function $S_k(\Omega, x)$ the first term is concentrated in the region of positive Ω whereas the second term – in the region of negative Ω; therewith, the condition $S_k(-\Omega, x) = S_k^*(\Omega, x)$ is fulfilled because $S_k(\Omega, x)$ is the Fourier transform of the real function. Let us restrict ourselves to the consideration of $S_k(\Omega, x)$ for positive Ω, retaining only the first term in Eq. (10).

Let us proceed from the impulse response of the channel to the impulse transfer function in Eq. (7) by taking the Fourier transform for x and changing the integration variable x by

$$y = \Omega/\beta - (x - t_0)$$

in the interior integral. Taking $x - t_0 > 0$, the equation for $S_k(\Omega)$ may be written as

$$S_k(\Omega) =$$

$$= \frac{1}{4}\int\limits_{-\infty}^{\infty} H(\omega) \int\limits_{-\infty}^{y_0} a(t_k, y)\, F(\beta y)\, \mathrm{e}^{(i\Omega t_k + i\omega(\Omega/\beta - y + t_0))}\, \mathrm{d}y\, \mathrm{d}\omega, \tag{11}$$

where $y_0 = \Omega/\beta + t_0$. The carrier $F(\beta y)$ (the range of variation of the argument where the function differs significantly from zero) coincides with the spectrum carrier $W(\beta y)$ that, as already inducted above, is concentrated near the zero argument. Therewith, the minimum frequency Ω measured by an analyzer exceeds the window spectrum bandwidth. Therefore, the limits of integration over y in Eq. (11) enclose the function carrier $F(\beta y)$, that is, the integral over y is the direct Fourier transform, because the upper limit of integration can be expanded to ∞. After the transformation, we obtain

$$S_k(\Omega) = \frac{\pi}{2}\mathrm{e}^{i\varphi} \int\limits_{-\infty}^{\infty} H(\omega)\, B(\beta\,(t_k - y_0) + \omega + \omega_0)\, \mathrm{e}^{-i\omega y_0}\, \mathrm{d}\omega, \tag{12}$$

where $B(x)$ is the result of the Fourier transform of the integrand in Eq. (11).

In this designations, after processing of the kth sample the form of $S_k(\Omega)$ is analogous to the relation for the communication channel response to the transfer of the signal with the spectrum B with the only difference that not only the exponent, but also the parameter B depends on Ω. Therewith, the spectrum B differs significantly from zero in the narrow band of frequency ω.

It is well known that the registration of several communication channel responses to the quasimonochromatic signal is possible for acoustic sounding. This means that $H(\omega)$ can be represented as a sum of the impulse transfer functions $H_l(\omega)$ corresponding to different propagation modes.

Let us now proceed to the calculation of the integral in Eq. (12), taking advantage of the transformations that are typically used for the impulse quasimonochromatic signals. First, we change the variable ω by $-\omega$ and take advantage of the property of the impulse transfer function of the communication channel

$$H(-\omega) = H^*(\omega) = \Sigma H_l^*(\omega).$$

Second, we reduce the conjugate transfer function to the spectral band $B(\beta(t_k - y_0) + \omega + \omega_0)$.

For this purpose, we expand the function $H(\omega)$ around the frequency

$$\omega_k = \omega_0 + \beta(t_k - y_0), \tag{13}$$

which is the center of the carrier $B(\beta(t_k - y_0) + \omega + \omega_0)$. The phase of transfer function is fast varying quantity, while the amplitude $H_l^*(\omega)$ depends only weakly on ω. Therefore, for the considered frequency band we restrict ourselves to the phase expansion retaining only the linear term. The function $|H_l^*(\omega)|$ is considered constant. As a result, we obtain

$$S_k(\Omega) = \frac{\pi}{2}\, \mathrm{e}^{-i(\psi + \omega_k y_0)} \sum_l H_l^*(\omega_k) \times$$

$$\times \int\limits_{-\infty}^{\infty} B(\omega_k - \omega)\, \mathrm{e}^{-i(\omega_k - \omega)(y_0 - \tau_l)}\, \mathrm{d}\omega, \tag{14}$$

where τ_l is the group delay defined as the derivative of the transfer function phase $H_l(\omega)$ at the frequency ω_k. The integral in Eq. (14) is the inverse Fourier transform, that is,

$$S_k(\Omega) = \frac{\pi}{2} e^{i(\psi + \omega_k y_0)} \sum_l H_l^*(\omega_k)\, b(\Omega/\beta + t_0 - t_l). \qquad (15)$$

Here, $b(x) = a(t_k, x)F(\beta x)$.

It is evident from Eq. (15) that at the analyzer output the signals are recorded that correspond to the propagation modes for sounding by a quasimonochromatic pulse with the complex envelope $b(\Omega/\beta)$. Thus neglecting dispersive distortion, the signal waveform is determined only by the form of the spectrum analyzer window $w(t)$ and the transmission coefficient of the low-pass filter. The lth signal center position on the axis Ω is determined from the condition $y_0 = \tau_l$ and corresponds to the frequency

$$\Omega_l = \beta\ (\tau_l - t_0). \qquad (16)$$

Equation (16) establishes the relation between the recorded signal delay of the lth mode and the analyzer variable Ω and permits us to match the range of variations of the signal group delays to the working range of the analyzer by the corresponding choice of t_0. In addition, our analysis shows that the recorded spectrum maxima are related with the channel group delays only when the condition $2\pi F_w < \beta(\tau_l - t_0) < 2\pi F_f$ is satisfied rather than for arbitrary delays, where F_w is the strobe window bandwidth and F_f is the low-pass filter bandwidth.

The subscript k in Eq. (13) defines the current frequency ω_k for which the lth mode chirp characteristics are calculated. The difference from the pulsed sounding consists in the fact that during the individual kth sample analysis time, ω_k changes by the working spectrum bandwidth $\Delta\Omega$. The parameter $\Delta\Omega$ usually can be neglected in comparison with ω_k.. Therefore, for every l the signal characteristics are for the frequency $\omega_k = \omega_0 + \beta t_k$. If required, for each mode with the delay τ_l we can determine the carrying frequency from Eqs. (16) and (13). Changing k from 1 to N corresponds to the chirp frequency variation over the entire sounding band.

Thus, $S_k(\Omega)$ describes the dependence of the signal level on the delay τ_l and carrying frequency ω_k. The registration of the modulus $S_k(\Omega)$ is the result of application of the considered sounding method. The dependence $S_k(\Omega)$ (on τ_l) at the moment t_k is the signal waveform at the current frequency ω_k.

As a result, we can draw the following conclusions:

1) The result of processing of an individual sample of received signal is formally similar to channel sounding with a narrow-band pulse signal, the characteristics of which are determined by the temporal window used to select the samples. The group delays of the effective pulse signal b determine the recorded spectrum maxima.

2) The proposed method for processing permits one to model the waveform and recording spectrum phase structure in case of sounding of a real medium with a broadband chirp by the frequency compression method.

3) If necessary, the considered method permits one to take into account the channel dispersive distortions, in particular, arising from the dispersion of absorption, which results in waveform distortion as well.[3]

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