Localization of a radiation source using data from a space-based information and measuring system: Bayesian approach to data processing

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We have developed a new approach to data processing for a space-based information and measuring system observing the Earth's surface and locating sources of pulsed optical radiation on the Earth. In this approach, the direct (non-iteration) algorithm for calculating the initial estimate of the source's coordinates is connected with the procedure of updating the estimate by the method of Bayes recursion. The direct method for the initial estimate of the coordinates excludes the problem of possible divergence of calculated results. Application of Bayes recursion enables one to obtain final high accurate estimates and simplifies methodological analysis of measurements by constructing the covariation matrix of estimation errors (uncertainties) together with estimation of the coordinates.

1. In modern optics, together with other important applied problems, there is a problem of studying the Earth and its ambient atmosphere from the space in the optical range of the radiation spectrum. To carry out such investigations, one applies systems of global observations. The systems include groups of satellites with devices intended for sensing the Earth's surface and a ramified network of ground-based and airborne information tools.[1] This is a very complicated and expensive equipment on the basis of which space-borne information and measuring systems (SB IMS) are being developed. The SB IMS development and application in research purposes for solving concrete problems pay for themselves in the cases when one can guarantee the reliability of the results obtained. The latter, finally, is determined by the accuracy and reliability of measurements on sensing the Earth from space. Here we deal with indirect measurements whose error in some cases is determined rather by the method and algorithms of data processing than by the performance characteristics of the system's equipment.

In this paper, we consider a particular but practically important problem of data processing in localizing a ground-based (or near-ground-based) source of pulsed optical radiation with a space-based information and measuring system. There exist several approaches to solving this problem which yield satisfactory results (see, for instance, Ref. 2 and its bibliography), but the demand of further increase of the accuracy and reliability of algorithms and their metrological support is still urgent.

We propose a new scheme for processing data of remote sensing to localize sources of optical radiation with a space-based information and measuring system. The scheme has some advantages as compared with other approaches. Its salient feature is combination of the Kalman filter[3] in the variant corresponding to the static problem of recursion Bayesian estimation with the algorithm of source coordinates' estimation by the direct (non-iterative) method of intersecting spherical surfaces (MISS).[4] Direct methods are reliable, efficient in calculation, and yield results sufficiently close to the "true values,B i.e., to the linear estimates of the source's coordinates, that are the best regarding the criterion of minimum variance. The Kalman filter opens the possibility for further refinement of the results obtained by MISS in the process of acquiring new data about the object from different space-borne instruments. Simultaneously, the error covariation matrix (according to ISO terminology, "uncertaintyB[5] of estimation) is being calculated and renewed. The covariation matrix describes the accuracy characteristics of the method and allows one to form an opinion about its competitive abilities and expedience of application in one or another applied task. The matrix is necessary in certification of the method, models, and algorithms of data processing. The problem of developing and certifying the methods, models, and algorithms is central both in remote sensing and in the SB IMS metrology.[6]

2. The variant of Kalman's filter that is used in the proposed scheme of data processing is in fact a Bayesian procedure of random values estimation. The Bayesian approach to statistical estimation of the probability distribution is, as it is well known, "polytheistic.B It includes both subjective (the choice of the initial distribution) and objective (further refinement of the chosen distribution on the basis of new data obtained from an experiment) factors. In this approach, the word "BayesianB means the fact that the *a priori* data are refined by the formula of recursion which formally looks like the Bayesian theorem from elementary probability theory.[7] Let $\mathbf{X}$ be the vector of parameters that are to be estimated in the given problem, v_1, v_2, ... be the results of measurements whose probability distribution depends on $\mathbf{X}$. By $p(\mathbf{X})$ we denote the *a priory* probability of the vector of parameters $\mathbf{X}$ (the parameters are considered as random

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values) and let $p(\mathbf{X}|v_1, \ldots, v_{m-1})$ be the *a posteriori* probability corresponding to the measurement results $v_1, \ldots, v_m$. Then the recursion formula

$$p(\mathbf{X}|v_1, \ldots, v_m) \approx p(\mathbf{X}|v_1, \ldots, v_{m-1})\ell(v_m|\mathbf{X}),\quad m = 2, 3, \ldots, n \tag{1}$$

is valid with the initial condition

$$p(\mathbf{X}|v_1) = p(\mathbf{X})\ell(v_1|\mathbf{X}).$$

Here $\ell(v_i|\mathbf{X}) = p(v_i|\mathbf{X})/p(v_i)$ is the likelihood function $\mathbf{X}$ for a given v_i $(i = 1, \ldots, m)$, i.e., conditional probability that the experiment yields the value v_i. The values $\mathbf{X}$ for which the likelihood function reaches its maximum are likely. These are the values that are chosen as estimates of the true values of $\mathbf{X}$ parameters.

The Kalman filter differs from the classical Bayesian procedure in the following: instead of the recursion refinement of the probability distributions, it solves a more "mundaneB problem of recursion estimation of values taken by the random vector $\mathbf{X}$ for these probability distributions, without explicit reference to the distributions. The key point is the fact that the components of the vector $\mathbf{X}$ and its estimate $\hat{\mathbf{X}}$ lie in the probability space of random values with the finite second moment, i.e., in the probability space $L_2(\Omega, \mu)$ (Ω is the space of elementary events ω; μ is its probability measure). But this is a Hilbert space, and one can apply the projection theorem, i.e., estimation genesis by the criterion of minimum variance.[8,9] According to this theorem, in any closed subspace M of the space $L_2^n(\Omega, \mu)$, there exists a unique vector $\hat{\mathbf{X}}$ such that

$$\|\mathbf{X} - \hat{\mathbf{X}}\| \le \|\mathbf{X} - M\| \text{ for all } M \in \mathsf{M}.$$

This vector is a projection of $\mathbf{X}$ onto M, $(\mathbf{X}-\hat{\mathbf{X}})\perp\mathsf{M}$. In our consideration, M is the subspace of data, $\hat{\mathbf{X}}$ is the optimum estimate or, more exactly, estimator of $\mathbf{X}$ based on the measurement data $\mathbf{V}(\omega)$. As follows from the projection theorem, the estimator $\hat{\mathbf{X}}$ exists and is determined uniquely by the relation

$$\hat{\mathbf{X}} = g \circ \mathbf{V} \equiv g(\mathbf{V}(\omega)) \text{ and } \|\mathbf{X} - \hat{\mathbf{X}}\| \le \|h \circ \mathbf{V} - \mathbf{X}\|, \tag{2}$$

where g is the estimating function and h is an arbitrary Baire (Borel-measurable) function, both defined on Ω. The real output of the estimation tool is the estimating function g and covariation matrix of the estimation errors

$$P \triangleq E[(\mathbf{X} - \hat{\mathbf{X}})\ (\mathbf{X} - \hat{\mathbf{X}})^{\mathrm{T}}]. \tag{3}$$

Here e is the mathematical expectation operator and T is the matrix transposition sign (in this case, this is a column matrix); Δ is the composition sign.

3. The estimation procedure under consideration is based on the following theorem (it is called the Bayes renewal theorem).[9] Let there be an estimator for minimum variance $\hat{\mathbf{X}}_1$ of a vector $\mathbf{X}$ by a vector $\mathbf{V}_1$ and the covariation matrix P_1 corresponding to it. Further, let a new information $\mathbf{V}_2$ about the vector $\mathbf{X}$ be obtained by linear measurement against the background of an additive noise W: $\mathbf{V}_2 = H\mathbf{X} + W$. The noise has no internal structure, i.e., its bias equals zero and it correlates neither with the measured value $\mathbf{X}$, nor with any past information about it:

$$E[W] = 0,\ E[\mathbf{X}W^{\mathrm{T}}] = 0,\ E[\mathbf{V}_1 W^{\mathrm{T}}] = 0.$$

Then the new estimator of the minimum variance of the vector $\mathbf{X}$ based on the data vector $\mathbf{V} = [\mathbf{V}_1^{\mathrm{T}} \,\vdots\, \mathbf{V}_2^{\mathrm{T}}]^{\mathrm{T}}$ is uniquely defined by the relation

$$\hat{\mathbf{X}}_2 = \hat{\mathbf{X}}_1 + P_1 H^{\mathrm{T}}(HP_1H^{\mathrm{T}} + \Sigma)^{+}(\mathbf{V}_2 - H\hat{\mathbf{X}}_1), \tag{4}$$

and the covariation matrix P_2 by the relation

$$P_2 = P_1 - P_1H^{\mathrm{T}}(HP_1H^{\mathrm{T}} + \Sigma)^{+}\, HP_1. \tag{5}$$

Here $\Sigma = E[WW^{\mathrm{T}}]$, and $(^{+})$ is the pseudo-inverse operator; it is defined for a non-degenerate matrix A by the relation[10] $A^{+} = (A^{\mathrm{T}}A)^{-1}A^{\mathrm{T}}$. It is not required to know the vector $\mathbf{V}_1$ in calculating of the estimate $\hat{\mathbf{X}}_2$ by the vector $\mathbf{V}_2$.

If all the matrices P_1, P_2, Σ, and $\mathrm{HP_1H^T}+\Sigma$ are regular, the equations (4) and (5) are simplified. In this case, using the lemma of matrix inversion[9,11]

$$(A_{11} - A_{12}A_{22}^{-1}A_{21})^{-1} = \\ = A_{11}^{-1} + A_{11}^{-1}A_{12}(A_{22} - A_{21}A_{11}^{-1}A_{12})^{-1}A_{21}A_{11}^{-1}$$

and the well known rule $(AB)^{-1} = B^{-1}A^{-1}$, one can transform Eqs. (4) and (5) to

$$P_2 = [P_1^{-1} + H^{\mathrm{T}}\,\Sigma^{-1}H]^{-1};$$
$$\hat{\mathbf{X}}_2 = P_2H^{\mathrm{T}}\,\Sigma^{-1}\,\mathbf{V}_2 + P_2P_1^{-1}\,\hat{\mathbf{X}}_1\,. \tag{6}$$

The presented theorem implicitly describes the scheme of recursion estimation. If $\hat{\mathbf{X}}_1$ and P_1 are known and, in addition, information $\mathbf{V}_2$ is obtained, one can calculate $\hat{\mathbf{X}}_2$ and P_2. Then, if $\mathbf{V}_3$ is obtained, one can calculate $\hat{\mathbf{X}}_2$ and P_3, and so on. Here, capital letters denote matrices and random vectors. Concrete realizations of random vectors are denoted by lower case letters $\mathbf{x}$, $\mathbf{v}$, etc. In practice, one calculates not the vector $\hat{\mathbf{X}}$ but one of its realizations $\hat{\mathbf{x}}$ (estimation) defined through the realization $\mathbf{v} = [v_1 \ldots v_m]^{\mathrm{T}}$ of the vector $\mathbf{V}$ by use of the estimating function g:

$$\hat{\mathbf{x}} = g(v_1 \ldots v_m).$$

Estimation is performed by the same equations (3)–(5) but $\mathbf{X}$ and $\mathbf{V}$ are substituted for by $\mathbf{x}$ and $\mathbf{v}$.

The fact that the covariation matrix P_2 can be determined before obtaining the data $\mathbf{v}_2$ and calculation $\mathbf{x}_2$ seems to be curious. This can be explained by the fact that P_2 is not a characteristic of the estimate $\hat{\mathbf{x}}_2$ but a characteristic of the estimation process. This is an element of the algorithm's certification.

The incoming information can be of different types and characterized by different matrices H_i. Matrices Σ also can be different. Generalizing the equations of the recursion estimation with the allowance for this fact, let us write them in the form

$$\hat{\mathbf{x}}_i = \hat{\mathbf{x}}_{i-1} + P_{i-1}H_i^{\mathrm{T}}(H_iP_{i-1}H_i^{\mathrm{T}} + \Sigma_i)^+ (\mathbf{v}_i - H_i\hat{\mathbf{X}}_{i-1}),$$
$$i = 2, \dots, m; \quad (7)$$

$$P_i = P_{i-1} - P_{i-1}H_i^{\mathrm{T}}(H_iP_{i-1}H_i^{\mathrm{T}} + \Sigma_i)^+ H_iP_{i-1},$$
$$i = 2, \dots, m. \quad (8)$$

The problem is formulated here as applied to the static problem of the recursion estimation: data of different measurements $\mathbf{V}_i$ are used to improve estimates of the same vector $\mathbf{X}$ which does not depend on time. In our consideration, this corresponds to the case of a stationary source. The situation with a moving source allows a similar consideration but the equations (7), (8) must be changed for more general equations of dynamic Kalman filtering.

4. Now let us turn directly to the process of localization of a source with an information and measuring space-based system and apply the above-mentioned formalism of data processing. In source localization, we use pairwise differences of time moments t_i of a signal's arrival at different space platforms (SP) included into the SB IMS:

$$d_{ik} = c(t_i - t_j),\ i, j = 1, \dots, N, \quad (9)$$

where c is the speed of light in free space. Non-iteration (direct) methods for estimation of the source's coordinates $\mathbf{x} = [x\ y\ z]^{\mathrm{T}}$ by these data are reduced to "solvingB the system of equations which is inconsistent in the general case. Below, it is represented in a matrix form:

$$A\mathbf{x} \approx R\mathbf{d} - \Delta . \quad (10)$$

Here A is the system matrix consisting of the coordinates x_i, y_i, z_i, of the SP detecting a signal; Δ and $\mathbf{d}$ are data vectors; R is the distance from the source to one of the SPs chosen as a reference point. Let it be the 1st SP. Then, in the geocentric coordinate system, for a round SP orbit of constant radius $R_i = \mathrm{const}$, we have

$$A = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ \dots & \dots & \dots \\ x_N - x_1 & y_N - y_1 & z_N - z_1 \end{bmatrix},$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ \dots \\ d_{N-1} \end{bmatrix},\ \Delta = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \dots \\ \Delta_{N-1} \end{bmatrix},$$

where $d_i \equiv d_{i+1}$ and $\Delta_i = d_i^2/2$. Sometimes it is convenient to perform calculations in a coordinate system connected with the reference satellite. The new system is obtained by a translation of the initial one when the origin is superposed with the reference SP. Here

$$A := \begin{bmatrix} x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \\ \dots & \dots & \dots \\ x_N & y_N & z_N \end{bmatrix}; \quad \Delta_i := \frac{1}{2}\,[R_{i+1}^2 - (d_i)^2] .$$

The sign of the approximate equality stands in Eq. (10) because its right-hand part contains a noise.

The estimate of the vector of the source's coordinates $\mathbf{x} = [x\,y\,z]^{\mathrm{T}}$ can be obtained from these equations for $N \ge 4$. Calculating initial estimates $\hat{\mathbf{x}}_1$ and P_1, let us restrict ourselves to the minimum number $N = 4$. If R is known, the solution of equation (10) in this case takes the form

$$\mathbf{x} \approx A^{-1}(\Delta - R\mathbf{d}). \quad (11)$$

But we do not know R. However, taking into account that the equality $\mathbf{x}^{\mathrm{T}}\mathbf{x} \approx R^2$ is valid in the coordinate system connected with the reference SP, we deduce a quadratic equation in R from Eq. (11). Its solution is

$$\hat{R} = (-\,b/a) \pm \sqrt{(b/a)^2 + (c/a)} . \quad (12)$$

Here

$$a = 1 - \mathbf{d}^{\mathrm{T}}(AA^{\mathrm{T}})^{-1}\,\mathbf{d}; \quad b = \mathbf{d}^{\mathrm{T}}(AA^{\mathrm{T}})^{-1}\Delta;$$
$$c = \Delta^{\mathrm{T}}(AA^{\mathrm{T}})^{-1}\Delta .$$

Only one of these two solutions is physically permeable ($R > 0$), namely, the solution with the (+) sign before the square root. Substituting Eq. (12) into Eq. (11) and returning to the initial geocentric system of coordinates ($\mathbf{x} := \mathbf{x} + \mathbf{x}_1$), we obtain the initial estimate

$$\hat{\mathbf{x}}_1 = A^{-1}\,(\Delta - \hat{R}\mathbf{d}) \quad (13)$$

for the following improvement by use of the Bayesian procedure.

The covariation matrix $P \equiv \mathrm{cov}\{\tilde{x}\}$ of errors of $\mathbf{x}$ estimation by the presented algorithm was defined in Ref. 2. For the case when the errors in determining different d_{ij} do not correlate and are characterized by the same variance σ^2, the approximate expression for the matrix has the form

$$P_1 = \sigma^2\,R_1^2\,(A^{\mathrm{T}}A)^{-1}, \quad R_1^2 = \hat{\mathbf{x}}_1^{\mathrm{T}}\,\hat{\mathbf{x}}_1. \quad (14)$$

It is the expression that we take as the initial covariation matrix.

5. Let us illustrate our approach to data processing in estimating coordinates of a light source by the example of the model experiment described in Ref. 12.

Table 1. Geocentric coordinates (x_i, y_i, z_i) of SP detecting a signal, distances of SP from the source (r_i), differences of the distances between the source and different SP pairs (d_i), and errors of their determination ε_i

i	x_i, m	y_i, m	z_i, m	r_i, m	d_i, m	ε_i, m
1	8420304	9408516	22121259	19173926		32
2	18764166	7611333	15451391	19641065		281
3	22434093	332881	12057050	20804192		277
4	8823081	−7520353	22679712	20872552		−140
5	−10525984	12972770	19227134	22156824	*	−250
6	17713457	18284404	−828144	22665355		−150
7	−12949786	8880350	20055279	22685992		−192
8	−8746072	−10085138	21692580	23636650		142
9	−9567691	22643850	6669867	23936548		−23
10	* $d_i = r_{i+1} - r_1 + \varepsilon_i$, $i = 1, \ldots, 6$,					−354
11	$d_i = r_i - r_2 + \varepsilon_i$, $i = 7, \ldots, 9$,					9
12	$d_i = r_{i-6} + r_2 - \varepsilon_i$, $i = 10, \ldots, 12$					−200

The signal from a source is detected with nine SPs ($N = 9$) whose coordinates in the geocentric coordinate system at the moment of signal detecting are presented in Table 1. The same table presents differences of the distances between the source and different pairs of SPs d_{ij} which are used in the calculations ($i = 2, \ldots, N$, $j = 1$ or 2 depending on the choice of the reference SP) and errors of their determination ε_i. The source's coordinates are $x = 2879593$ m, $y = 2249784$ m, $z = 5218818$ m. The errors are considered to be random values. The presented ε_i are samples from a (discrete) random Gaussian process with zero mean value and the variance $\sigma^2 = 210$ m generated by a computer (in the MathCAD PLUS 7.0 PRO program pnorm(m, μ, σ)).

The initial estimate of the source was performed by the direct method using Eqs. (12)–(14) with the values

$$A = H1 := \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{bmatrix};$$

$$d = d1 := \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}, \quad \Lambda = \Lambda 1 := \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \end{bmatrix}.$$

Let us present the obtained results which give an insight about the accuracy characteristics of the direct method initializing the Bayesian recursion process:

$$\hat{\mathbf{x}}_1 = \begin{bmatrix} 2880910 \\ 2250222 \\ 5220836 \end{bmatrix};$$

$$P_1 = \begin{bmatrix} 13323448 & 759467 & 18646607 \\ 759467 & 95278 & 1029493 \\ 18646607 & 1029493 & 26228992 \end{bmatrix}.$$

Further improvement of the initial estimates obtained by the direct method was performed by the Bayes formula (6) in two steps. At the first improvement step, we used the values

$$H2 = \begin{bmatrix} x_5 - x_1 & y_5 - y_1 & z_5 - z_1 \\ x_6 - x_1 & y_6 - y_1 & z_6 - z_1 \\ x_7 - x_1 & y_7 - y_1 & z_7 - z_1 \end{bmatrix};$$

$$d2 = [d_4 \; d_5 \; d_6]^{\mathrm{T}}, \; \Lambda 2 = [\Lambda_4 \; \Lambda_5 \; \Lambda_6]^{\mathrm{T}};$$

at the second step

$$H3 = \begin{bmatrix} x_7 - x_2 & y_7 - y_2 & z_7 - z_2 \\ x_8 - x_2 & y_8 - y_2 & z_8 - z_2 \\ x_9 - x_2 & y_9 - y_2 & z_9 - z_2 \end{bmatrix};$$

$$d3 = [d_7 \; d_8 \; d_9]^{\mathrm{T}}, \; \Lambda 3 = [\Lambda_7 \; \Lambda_8 \; \Lambda_9]^{\mathrm{T}};$$

at the third step

$$H4 = \begin{bmatrix} x_4 - x_2 & y_4 - y_2 & z_4 - z_2 \\ x_5 - x_2 & y_5 - y_2 & z_5 - z_2 \\ x_6 - x_2 & y_6 - y_2 & z_6 - z_2 \end{bmatrix};$$

$$d4 = [d_4 \; d_5 \; d_6]^{\mathrm{T}}, \; \Lambda 2 = [\Lambda_4 \; \Lambda_5 \; \Lambda_6]^{\mathrm{T}}.$$

The values $\hat{R} := R2$ and $\hat{R} := R3$ at each step were taken first from the preceding calculation cycle and then re-calculated (in the satellite framework) by the formulas

$$R2 := \sqrt{\hat{\mathbf{x}}_2^{\mathrm{T}} \, \hat{\mathbf{x}}_2} \quad \text{and} \quad R3 := \sqrt{\hat{\mathbf{x}}_3^{\mathrm{T}} \, \hat{\mathbf{x}}_3}.$$

Here, as one should expect, the values appeared to be close to r_1 and r_2 (m):

$$R2 = 19173919 \approx r_1 \text{ and } R3 = 19641098 \approx r_2.$$

The final result of calculations is represented in the form

$$\hat{\mathbf{x}}_4 = \begin{bmatrix} 2879531 \\ 2249922 \\ 5218796 \end{bmatrix}; \quad P4 = \begin{bmatrix} 4972 & 2429 & 3487 \\ 2429 & 18725 & 10640 \\ 3487 & 10640 & 25206 \end{bmatrix}.$$

The estimation values $\hat{x}$, $\hat{y}$, $\hat{z}$, $\hat{r}_1$(= $R2$), $\hat{r}_2$(= $R3$) after two improvement cycles by the Bayesian procedure differ from the true values (given in the formulation of the model problem) x, y, z, $r_1 = \sqrt{x_1^2 + y_1^2 + z_1^2}$, $r_2 = \sqrt{x_2^2 + y_2^2 + z_2^2}$ by not very large values $\delta x = -62$, $\delta y = 137$, $\delta z = -23$, $\delta r_1 = -7$, $\delta r_2 = 33$ m. In percent, the values are very small, e.g., $\delta x/x \approx 6 \cdot 10^{-6}$ %, but localization problems, as a rule, need just absolute, not relative values.

The diagonal elements of the covariation matrix P yield estimation variances $(\delta x)^2$, $(\delta y)^2$, $(\delta z)^2$ for conditions of the given experiment. In accordance with the results obtained for P_4, we have

$$\sigma_x = 71, \quad \sigma_y = 139, \quad \sigma_z = 159 \text{ m}.$$

These are estimation characteristics. They certificate the estimation method, and not an individual estimate obtained by this method. The values $|\sigma x|$, $|\sigma y|$, $|\sigma z|$ not obligatorily coincide with σ_x, σ_y, σ_z but must not differ from them too much.

6. The paper presents a new approach to processing of data of a space-based information and measuring system for localization of ground-based optical radiation sources from the space. It has some advantages as compared with other known approaches. The first advantage is in the absence of the problem on calculation results' convergence. This is guaranteed by application of a direct (non-iteration) method for the initial estimation of a source's coordinates in initialization of the further process of the results' refinement. The second advantage is in high accuracy of the final results on estimating source coordinates what is caused by execution of several cycles of Bayesian recursion. For a typical situation considered in the paper (detection of a signal from an optical pulsed radiation source by nine SPs at round orbits of 26000 km radius), the method is characterized by the following standard deviations of coordinates' estimate: $\sigma_x = 71$, $\sigma_y = 139$, $\sigma_z = 159$ m. The third advantage is in the fact that one can simultaneously calculate the estimate for a source's coordinates and the covariation matrix of uncertainties in estimating x, y, and z., i.e., metrological characteristics of the method for a given configuration of the source and "constellationB of SPs detecting the signal.

The considered method of data processing for remote sensing of the Earth's surface in localization of optical radiation sources with a space-based information and measuring system requires a larger body of calculations as compared with the direct method for estimating source's coordinates described in Ref. 2. However, it is not only more accurate and reliable but, at the same time, more convenient for a metrologically proved estimates of the results obtained. In particular, it can be applied to metrological investigations and periodic tests of other methods, more rapid but less accurate.

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