TO THE THEORY OF RADIO-SIGNAL SCATTERING IN THE ATMOSPHERE

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The characteristics of a radio signal singly scattered in the atmosphere and in the ionosphere are considered theoretically for monostatic radar sensing. A method of establishing relationships between the received signal spectrum and the spectral fluctuations of the characteristics of a medium is proposed. The radar selectivity is analyzed.

INTRODUCTION

Radio-wave backscattering on small-scale fluctuations of a medium provides a basis for radar sensing, which harnesses the radar equation that establishes the relationship between the correlation function of a received signal and the parameters of the medium. The Born approximation and the additional assumption that the spatial correlation length of inhomogeneities of the medium is small[1,2] provide the basis for the radar equation. This assumption is fairly realistic. It agrees well with the experimental data on scattering by the dielectric constant fluctuations $\delta\varepsilon(\mathbf{r}, t)$ caused by the atmospheric turbulence or the thermal motion of the ionospheric plasma.[1,2]

From the theoretical viewpoint it seems to be important to establish the relationship between the signal and the characteristics of the medium without additional assumptions, that is, for arbitrary values of the spatial correlation length. In a number of experiments (see, for example, Refs. 3 and 4) backscattered signals had abnormally high power levels that had not yet been explained theoretically. It is possible that the effects of spatial coherence of the fluctuations of the medium should be taken into account.

In the present paper, the results are given of theoretical consideration of backscattered signals and their relationships with the characteristics of the medium without additional limitations on the spatial correlation length. We examine here the monostatic radar sensing.

RELATIONSHIP OF THE SCATTERED SIGNAL WITH THE FLUCTUATIONS OF THE MEDIUM

The derivation of all further equations is based on the Born single scattering approximation.[1,2] It is valid for sufficiently weak fluctuations of the parameters of the medium and weak scattered field compared with the initial wave field. For monostatic sensing geometry of the experiment, the envelope of a received signal can be written to within the constant factors in the form[2,5]

$$u(t) = \int h(t-t')\, a\left(t'-\frac{2r}{c}\right) e^{i2k_0 r}\, \frac{\delta\varepsilon\,(\mathbf{r}, t'-r/c)}{r^2}\, g(\mathbf{e}_r)\, \mathrm{d}\mathbf{r}\, \mathrm{d}t'. \quad (1)$$

Here, $\mathbf{e}_r = \mathbf{r}/r$ is the unit vector, $g(\mathbf{e}_r) = f_s(\mathbf{e}_r)\, f_i(\mathbf{e}_r)\, \mathbf{P}_s[\mathbf{e}_r, [\mathbf{e}_r, \mathbf{P}_i]]$, f_i and f_s are the directional patterns of an antenna, $\mathbf{P}_s$ and $\mathbf{P}_i$ are the unit polarization vectors of the antenna in the transmission and reception regimes, respectively, and k_0 is the wave number of a sensing signal. The complex envelope of the sensing signal $a(t)$ and the transfer function of a receiver $h(t)$ have narrow-band spectra that meet the requirements $\Delta\Omega_h, \Delta\Omega_a \ll k_0 c$. The above equation was derived for the far zone of the antenna $r\lambda_0 \gg D^2$, where D is the diameter of the antenna aperture.

Equation (1) provides a basis for the derivation of the radar equations for the backscattering method that connect the characteristics of the medium with the parameters of transmitting and receiving systems. It is not commonly used even in the stage of a preliminary analysis where the quadratic (energetic) characteristics of the signal and field are examined immediately.[1,2] However, it is of interest to examine the general properties of the problem with the help of the initial equation. For this reason, below we analyze Eq. (1) in more detail.

Within the framework of the Born approximation the main spatial physical mechanism of the signal formation is scattering on spatial harmonics $\delta\varepsilon$ that satisfy the specular reflection principle and the Bragg condition.[1,5] Therefore, it is preferable to establish the explicit relationship of the scattered signal with the Fourier spectrum of fluctuations. The complexity of transition to the spectral representation is caused by the fact that the radar is not within the Fraunhofer zone of the scattering volume that is specified by the

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directional pattern of the antenna and the duration of a and h. It increases with the distance r; therefore, the standard procedure of approximate replacing the spherical wavefront by a plane wavefront throughout the scattering volume is inapplicable here. In the derivation of the relationship of the correlation function of the signal with the Fourier spectrum of fluctuations it is usually assumed[1,2,5] that the spatial correlation length is so small that this replacement is valid within its limits. Let us demonstrate that the sought-after relationship can be established without any additional assumption.

Equation (1) specifies the relationship between the scattered signal and the fluctuations of the medium in spatiotemporal representation. In accordance with Eq. (1), $u(t)$ is the result of averaging of $\delta\varepsilon$ over the unit sphere with the weighting function g and of subsequent scanning with a narrow-band filter in the variables r and t'. In so doing, the exponent shifts the filtration band toward higher-frequency spatial harmonics so that only a narrow spectral interval centered at $k \cong 2k_0$ makes significant contribution to the signal. The function $\delta\tilde{\varepsilon}(\mathbf{r}, t) = \delta\varepsilon(\mathbf{r}, t)/r$ entering Eq. (1) can be represented in the form of the Fourier integral over the wave vectors $\mathbf{k}$ and hence Eq. (1) can be written as

$$u(t) = \int h(t - t')\, a(t' - 2r/c)\, e^{i2k_0 r}\, dt'\, r dr \times$$

$$\times \int I(\mathbf{k})\, \delta\tilde{\varepsilon}(\mathbf{k}, t' - r/c)\, d\mathbf{k}, \quad (2)$$

where

$$I(\mathbf{k}) = \int \exp(ir\mathbf{k}\, \mathbf{e}_r)\, g(\mathbf{e}_r)\, d\mathbf{e}_r. \quad (3)$$

Moreover, only the wave vectors with $k \cong 2k_0$ should be considered in integral (3).

The integral (3) can be calculated by the method of stationary phase. Indeed, the function $g(\mathbf{e}_r)$ varies slowly compared with the exponent, because the characteristic angular width of the directional pattern is $\Delta\theta \cong \lambda_0/D$. This method is valid for $(\Delta\theta)^2 kr \gg 1$ (see Ref. 6), which practically coincides with the far field approximation $r\lambda_0 \gg D^2$, because $k \cong 2k_0$. Thus, using the method of stationary phase we remain within the framework of approximations commonly used in the derivation of Eq. (1). In the first approximation of the method of stationary phase, we obtain the following formula for Eq. (3):

$$I \approx 2\pi i/(kr)\, [g(-\mathbf{e}_k)\, e^{-ikr} - g(\mathbf{e}_k)\, e^{ikr}]. \quad (4)$$

Here, the first term dominates, because the second term has positive phase and practically makes no contribution to Eq. (2) when the integral is taken over r.

After substitution of Eq. (4) into Eq. (2) with spectral representation of all functions, we derive the formula for the received signal spectrum

$$u(\omega) = 8\pi^3 i h(\omega) \int (2k_0 + (\omega + \nu)/c)\, a(\nu)\, d\nu \times$$

$$\times \int g(-\mathbf{e}_k)\, \delta\tilde{\varepsilon}\, ((2k_0 + (\omega + \nu)/c)\, \mathbf{e}_k, \omega - \nu)\, d\mathbf{e}_k. \quad (5)$$

This formula specifies the relationship between the scattered signal and the fluctuation characteristics of the medium in spectral representation. It specifies the selectivity of sensing in the explicit form, namely:

1. The range of frequencies of the fluctuations of the medium to be analyzed is completely determined by spectral widths of the sensing signal and receiving filter and is equal to their sum, namely $\Delta\Omega = \Delta\Omega_h + \Delta\Omega_a$.

2. The condition of specular reflection which states that only the spatial harmonics of the medium whose wave vectors lie within the angular sector bounded by the angles of specular reflection of the antenna directional pattern contribute to the scattered signal.

3. The Bragg condition which states that the wave numbers of the fluctuations $\delta\varepsilon$ that make the contribution to the received signal lie in the interval $2k_0 \pm \Delta k$, where $\Delta k = \Delta\Omega/c$. The selection by the wave vectors is illustrated by Fig. 1.

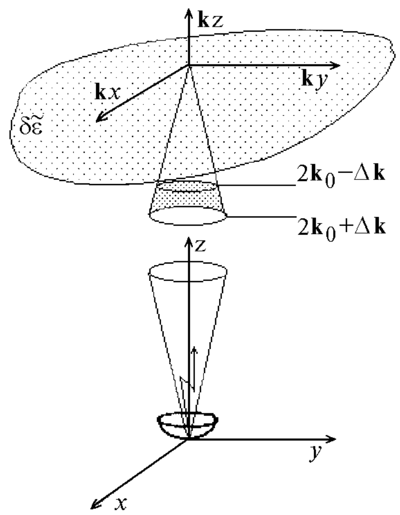

FIG. 1. Selectivity of the scattering process.

CONCLUSION

The characteristics of singly scattered signal for monostatic sensing have been considered in this paper. The method suggested here allows the relationship between the linear characteristics of the radiation field – the received signal spectrum – and the spectral characteristics of the fluctuations of the medium to be obtained without any additional limitations on the fluctuation characteristics. The equation derived by this method is one of the forms of the Born approximation and yields additional information about the properties of single scattering.

It specifies in the explicit form the selectivity of sensing of the characteristics of the medium: only the fluctuations of the medium whose wave vectors **k** lie within the angular sector bounded by the angles of specular reflection of the antenna directional pattern and whose absolute values are in the interval $2k_0 \pm \Delta k$ contribute to the received signal; the frequency range of fluctuations is determined by the spectral widths of the transmitted signal and receiving filter.

The obtained formula can be used for the derivation of the radar equation of monostatic sensing of the atmosphere and the ionosphere in cases of deterministic and statistic formulations of the problem of diagnostics of the medium without any limitations on the spatial correlation length of fluctuations of the medium.

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