THEORETICAL MODEL OF SOUND GENERATION DUE TO PHASE CHANGES IN A LIQUID AEROSOL PARTICLE

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Results of theoretical model calculations of sound generation by water drops in the process of their evaporation and explosive fragmentation have been presented in this paper. Explosive fragmentation of drops has been shown to decrease the efficiency of conversion of the light energy absorbed by drops into the acoustic wave energy. This fact is primarily explained by the decrease of vapor influx into air due to the decreased rate of drop evaporation after its fragmentation. The available experimental data on acoustic signals produced by exploding water particles have been interpreted.

Phase changes in the liquid aerosols are caused by surface evaporation, explosive boiling up, supercritical changes, and secondary condensation. These lead to perturbation of the air density around particles and generation of acoustic waves. In this paper, sound generation by water drops in the process of evaporation and explosion is modeled theoretically.

A general formulation of the problem of sound generation in a channel of high-power laser radiation propagating in an aerosol medium is based on the equations of thermohydrodynamics for a biphase medium considering the effect of laser radiation. In actual practice the aerosol merely perturbs the gas dynamic parameters of the air. In this case, the problem reduces to the study of the equation for an air medium, in which the aerosol contributes only to the function of a thermal source, whereas vapors of the particle substance, formed in the process of aerosol interaction with radiation, are considered solely in the equation of state.

The corresponding linearized equations of thermohydrodynamics have the form[1]

$$\frac{\partial \rho'}{\partial t} + \rho_0 \operatorname{div}(\mathbf{v}') = 0, \qquad \rho_0 \frac{\partial \mathbf{v}'}{\partial t} + \nabla p' = 0, \qquad (1)$$

$$\rho_0 \frac{\partial u'}{\partial t} = \lambda_T \Delta T' - p_0 \operatorname{div}(\mathbf{v}') + Q(\mathbf{r}, t),$$

$$p' = R_a \rho_0 T' + R_v \rho'_v T_0 + R_a \rho'_a T_0.$$

The initial conditions are $\rho'_v(0) = v'(0) = u'(0) = T'(0) = 0$.

Here, perturbations of the parameters of the air medium are indicated by the prime, their unperturbed values are denoted by the subscript "0," ρ_a and ρ_v are the air and vapor densities, respectively, p is the pressure, v is the mass velocity, Q is the thermal source function, R_a and R_v are the gas constants for the air and vapor, respectively, T is the temperature of the medium, λ_T is the coefficient of thermal conductivity of air, and u is the intrinsic energy.

By standard manipulations, the initial system of equations of thermodynamics reduces to two self-consistent equations

$$\frac{\partial T}{\partial t} = \frac{\lambda_T}{C_p \rho_a} \Delta T + \frac{1}{C_p \rho_a} \frac{\partial p}{\partial t} + \frac{c_s^2 R_v}{C_p \rho_a R_a \gamma_1} \frac{\partial \rho_v}{\partial t} + \frac{Q}{C_p \rho_a};$$

$$c_s^2 \Delta \left(p + \lambda_T \frac{\partial T}{\partial t} \right) - \frac{\partial^2 p}{\partial t^2} = - \frac{\partial}{\partial t} \left[(\gamma_1 - 1) Q(\mathbf{r}, t) - \frac{c_s^2}{\gamma_1} \frac{\partial \rho_v}{\partial t} \right], \qquad (2)$$

where c_s is the sound velocity.

By virtue of the above-mentioned remark that perturbations of the gas dynamic parameters of air caused by aerosol are small, we can neglect the terms describing coupling, $\partial p/\partial t$ and $\partial T/\partial t$, respectively, in Eqs. (2). As a result, the problem reduces to a solution of two independent equations: the equation of thermal conductivity in gaseous medium and the inhomogeneous wave equation for pressure. The latter equation has the form:

$$c_s^2 \Delta p - \frac{\partial^2 p}{\partial t^2} = \frac{\partial}{\partial t} \left[(\gamma_1 - 1) Q(\mathbf{r}, t) - \frac{c_s^2}{\gamma_1} \frac{\partial \rho_v}{\partial t} \right] \equiv - \frac{\partial}{\partial t} F(\mathbf{r}, t). \qquad (3)$$

The general solution of Eq. (3) in the wave zone is written as

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$$p(\mathbf{r},\,t) = \frac{1}{4\pi\, r\, c_s^2} \int\limits_{V_r} \frac{\partial F(\mathbf{r}',\, t - r/c_s)}{\partial\, t}\, \mathrm{d}V',$$

where integration is performed over the volume V_R of sound generation.

The term $\partial Q/\partial t$ appearing in the source function $F(\mathbf{r},\,t)$ considers the contribution of thermal mechanisms to sound generation by the particle. Because temporal scale of thermal conductivity in a gas is much larger than the characteristic time of interaction,[1] we hereafter will ignore the "thermal" term in the function $F(\mathbf{r},\,t)$. Hence it follows that

$$p(r,\,t) = \frac{1}{4\pi\, r}\frac{1}{\gamma_1}\frac{\partial^2 M_v}{\partial\, t^2}\;.$$

Here M_v is the total mass of vapor entering the medium during the interaction.

The variations of density of the medium are due to the evaporating drop, therefore,

$$\frac{\partial^2 M_v}{\partial\, t^2} = \frac{\partial}{\partial\, t}\int\limits_{S_0} [\,j(t)]\,\mathrm{d}S', \qquad t < t_d\,, \tag{4}$$

where is the time of drop fragmentation, $j(t)$ is the vapor mass flux from the surface of the evaporating drop, and integrating is performed over its surface S_0. After explosive fragmentation of the drop into an ensemble of fragments the total vapor flux in the medium is determined by flux summation over all fragments of the drop

$$\frac{\partial^2 M_v}{\partial\, t^2} = \sum\left[\frac{\partial}{\partial\, t}\int\limits_{S_i} [\,j(t)]\,\mathrm{d}S'\right], \qquad t > t_d\,. \tag{5}$$

It follows from expressions (4) and (5) that explosive fragmentation of the drop may cause variations of the acoustic signal amplitude primarily due to change of the rate of particle evaporation after its fragmentation. This is primarily explained by the fact that, as is well known, the rate of drop evaporation (when the radiation intensity remains unchanged) decreases with the drop size due to increase of thermal losses into the air. In addition, after explosive fragmentation of the drop into fragments, the evaporation regime changes from gas kintetic to diffusion one.[2] This is due to change of the condition of evaporation, because after the drop fragmentation, separating fragments are in the dense vapor–air medium, which essentially reduces the rate of their evaporation.

In Ref. 3 we have already pointed out that the drop evaporation law can be written in the form $\frac{\partial a}{\partial t} = \beta I(t)$ before and after explosive fragmentation, which corresponds formally to the regular quasistationary regime of evaporation.[3] Here β has a sense of the differential efficiency of evaporation. In this case, the ratio of the coefficients β_1/β_2 is equal to ~2.2 when a small homogeneously absorbing water drop explodes, where β_1 is the value of β before explosion (evaporation of the initial drop), and β_2 is that after fragmentation of the drop (Fig. 1).

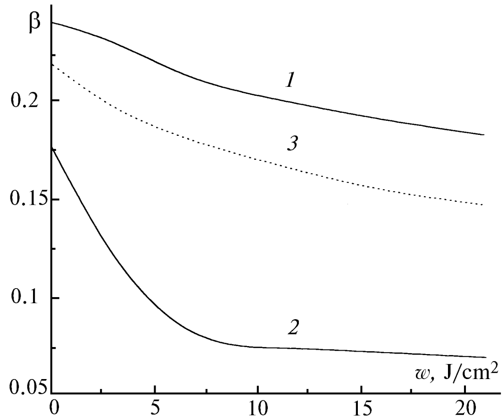

FIG. 1. Dependence of differential efficiency of evaporation β *on the laser energy density in the process of evaporation of a water drop* (a_0 = 3 μm) *before explosion (1) and after it (2). Curve 3 is for regular evaporation of drops.*

On the assumption that the drop evaporation is weak before its explosion, the amplitude of a sound signal can be written as

$$p_1 \approx \frac{c_s^2}{\gamma_1}\, a_0^2\, \beta_1\, I_0\, \frac{\mathrm{d}\, g(t)}{\mathrm{d}\, t}\,,\;\; t < t_d\,,$$

where I_0 is the peak laser pulse intensity, and $g(t)$ is the function describing the pulse shape. Hence, after the drop fragmentation we obtain

$$p_2 \approx \frac{c_s^2}{\gamma_1}\, a_0^2\, \beta_2\, I_0\, \frac{\mathrm{d}\, g(t)}{\mathrm{d}\, t}\,,\;\; t > t_d\,.$$

In doing so, we assume $a_d = a(t_d)\, n_d^{1/3}$, where a_d is the average size of the drop fragments, and n_d is their number. Hence it follows that $p_1'/p_2' = \beta_1/\beta_2$, i.e., fragmentation of a large drop into smaller fragments leads to the decrease of the total vapor influx into the medium and consequently to the decrease of the acoustic signal.

This process is vividly illustrated by Fig. 2, in which dependence of the efficiency of conversion of radiation energy into that of acoustic wave $\eta = W_{ac}/W_{ab}$ is shown for different initial size of a drop in energetic coordinates. The efficiency of conversion is defined as the ratio of the acoustic wave energy $W_{ac} = \int\limits_{V_i}\left(\frac{p\,V'}{\gamma_1 - 1} + \frac{\rho\,\mathbf{v}^2}{2}\right)\mathrm{d}V'$ to the total energy

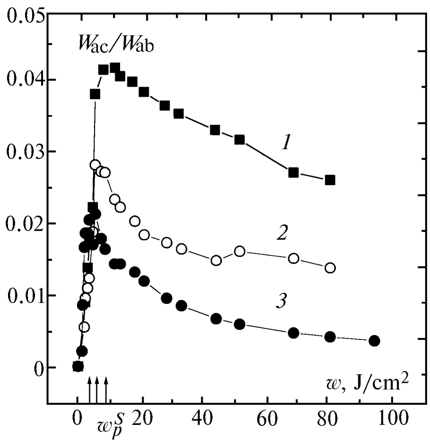

FIG. 2. Dependence of efficiency of conversion of light energy absorbed by a drop into the acoustic energy on the laser pulse energy density at a_0 = 92 *(1)*, 48 *(2)*, *and* 15 μm *(3). The curves are the result of processing of experimental data.*[4]

absorbed by the drop $W_{ab} = \pi\, a_0^2\, K_a(a_0)\, w_p$. Here V_i is the volume of sound generation, $K_a(a_0)$ is the absorption efficiency, w_p is the laser pulse energy density. It is seen from Fig. 2 that the curves have their maxima at the fixed value of the parameter w_p^S determined by the threshold of complete fragmentation of the initial drop. In this case, the larger is the drop, the higher is the threshold w_p^S.

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