MICROWAVE TOMOGRAPHY OF INHOMOGENEOUS MEDIA

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New methods for solving a problem of tomography of inhomogeneous media using coherent or incoherent monochromatic or broadband microwave radiation are considered. Efficiency of these methods is confirmed by the results of imitation and field experiments.

1. INTRODUCTION

Intensive development of tomography is conditioned primarily by its wide application to medicine.[1–3] Now tomographic methods become basic means of high-precision diagnostic of various pathological changes of a human organism. These methods are based on the possibility of sensing of objects from various directions by radiation of different wavelengths. By the physical nature of radiation, X-ray, emission, nuclear magnetic resonance (NMR), and ultrasonic tomography can be classified. Variations of the parameters of transmitted radiation yield information about the internal structure of an object, for example, about the distribution of the material density.

Each type of tomography differs by its advantages and disadvantages. Among disadvantages are high expense of equipment, incomplete safety of operation, and complexity of interpretation of the observed data. Great difficulties arise in tomography of inhomogeneities whose densities differ only slightly. For example, in the X-ray tomography, various contrasting substances should be injected. This is acceptable not for all kinds of tissue. It often appears, especially at initial stages of pathology evolution, that diagnostics can be made by its water content. Among the methods in use, ultrasonic diagnostics is most sensitive to these variations but within narrow limits. Tissue with high moisture content is transparent for ultrasound. In this connection, it becomes necessary to develop alternative tomography methods.

It is well known that with the change of water content of tissue its dielectric constant is varied. Often the substance density still has no time to change, but dielectric constants of healthy tissue and pathological one have already contrasted noticeably. To identify arising inhomogeneities, it seems promising to use the microwave radiation, which is sufficiently sensitive to the dielectric constant variations. In addition, it can be successfully used for active impact: direct therapy of a number of illnesses in medicine or destruction of internal inhomogeneities in introscopy of materials and products. Encouraging circumstance is a developed element base of radio electronics (generators, transmitters, panoramic measuring devices, microprocessors, and so on). However, now there are no microwave tomographs in operation. There are only a few works that discuss possible approaches.[4–6]

In our opinion, the main reason that is a barrier to the development of the microwave tomography is interpretation of measurable distortions of the parameters of radiation interacted with inhomogeneous media. Investigations show that during the propagation of microwave radiation, for example, through the biological tissue the effects of multiple interactions of waves (scattering, absorption, reflection, diffraction, depolarization, and so on) are essential. When the characteristic sizes of inhomogeneities are comparable with the wavelengths of employed radiation, the theory of multiple interaction is highly complicated and cannot be used to obtain a simple solution to an inverse problem of reconstruction of the structure of inhomogeneities.

A number of new methods that approach a solution of the problem of the microwave tomography are discussed in the present paper.

2. APPLICATION OF COHERENT RADIATION

One of the main problems that makes the development of the microwave tomography difficult is the multiple interactions of waves with inhomogeneities. It can be overcome after its adequate description or by elimination of the effect of these inhomogeneities. Moreover, application of coherent radiation is more promising.

There are many approaches to the description of multiple interactions, among which, in our opinion, of greatest interest is the method based on the tentative subdivision of an investigated volume into the finite number of discrete elements with homogeneous dielectric constants.[6] The tomography problem reduces to the solution of a system of nonlinear ill-posed algebraic equations that call for application of special regularization methods. Great progress has not yet been made here.

On the way of elimination and reduction of multiple interactions, the results are better. Thus, in Ref. 7 the method is suggested based on the use of the pulsed radiation. It comprises the selection of so-called "first-transmitted photons," that is, a part of radiation

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that comprises the leading front of the scattered radiation and from the physical consideration should be connected with single scattering. The experiments with the laser radiation confirmed the efficiency of this method for the increase of contrasts. The attempt of Toida et al.[8] is well known to decrease the role of multiple interactions on the basis of simultaneous use of several neighboring sensing frequencies. Under specific conditions, interference patterns of overlapped multiple interactions at these frequencies canceled each other and the role of the single interactions is increased. In our opinion, the method of double focusing[9] is most promising for the separation of the single interactions and solution of the microwave tomography problem.

The method of double focusing is based on simultaneous direct focusing of the radiation field onto the given point of the volume and secondary focusing of the radiation scattered by inhomogeneities onto the same point of the volume.[9] The intensity of received signal represents the convolution of the instrumental function of the system and the volume distribution of the dielectric constant inhomogeneities. Scans of the focusing point of the investigated region reduce the problem of its tomography to the inversion of convolution. In Ref. 9 the method of double focusing was formulated for closed transceiving aperture. In the present paper, this method is generalized for finite apertures. The experimental measurements are presented with application of the double focusing method for convergent beams.

In accordance with Ref. 9, to focus physically the radiation in the vicinity of the given point $\mathbf{r}_0$ located in a sounding volume, the distribution of currents in the form

$$I_0(\mathbf{r}_s) \equiv \frac{1}{i(2\pi)^2}\frac{\mathrm{d}}{\mathrm{d}n}\, G^-(\mathbf{r}_s - \mathbf{r}_0)$$

should be obtained over a radiating aperture.

In this case, the wave field in other arbitrary point $\mathbf{r}$ of the volume is determined as

$$E_0(\mathbf{r}) = \iint\limits_{S_1} I_0(\mathbf{r}_s)\, G^+(\mathbf{r} - \mathbf{r}_s)\,\mathrm{d}s \equiv \delta_1(\mathbf{r} - \mathbf{r}_0).$$

Here, $G^{\pm}(\mathbf{r}) = \exp(\pm\, ikr)/r$ are the Green's functions of the divergent and convergent waves and $k = \sqrt{\varepsilon} \times \times 2\pi/\lambda_0$ is the wave number for the medium whose dielectric constant $\varepsilon = \varepsilon' + i\varepsilon''$ in general is complex. In case of closed aperture, the field is mostly localized in the vicinity of the point $\mathbf{r}_0$. For the aperture of finite size, the field distribution is less concentrated about the focusing point and is elongated along the radiation propagation direction (Fig. 1). Radiation localization pattern has the comb-shaped form with the minimum size of the order of the wavelength in the transverse direction in the vicinity of the focusing point.

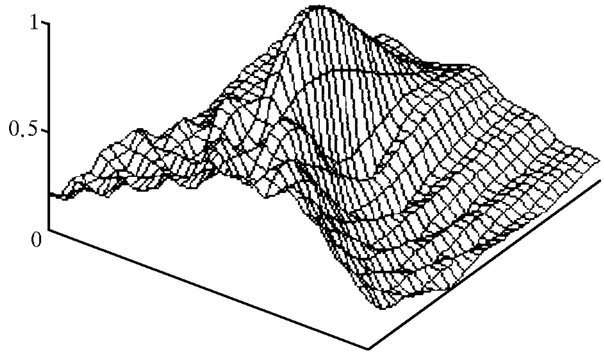

FIG. 1. Field localization pattern for the focused radiation and aperture of the finite size.

The illumination field is scattered by inhomogeneities of the medium in all possible directions. Scattered field received by the aperture S_2 may be focused onto the point $\mathbf{r}_0$ with the help of the weight function $I_0(\mathbf{r}_s)$. For the orthogonal beams of the direct and secondary focusing, the radiation will be localized in the narrowest region. In this case, the signal is

$$I(\mathbf{r}_0) = \iiint \sigma(\mathbf{r})\; Q(\mathbf{r} - \mathbf{r}_0)\;\mathrm{d}v, \qquad (1)$$

where the instrumental function of the system is $Q(\mathbf{r} - \mathbf{r}_0) = \delta_1(\mathbf{r} - \mathbf{r}_0)\delta_2(\mathbf{r} - \mathbf{r}_0)$. In the approximation of weak inhomogeneities, the dielectric constant distribution is written as $\varepsilon(\mathbf{r}) = \varepsilon_0 + \sigma(\mathbf{r})k^{-2}$ and its reconstruction reduces to the inversion of convolution (1). It is important to emphasize that orthogonal intersection of the partial localization regions (see Fig. 1) provides fairly strong localization when the multiple interaction effects are strongly attenuated.

To check the efficiency of the method, we developed an experimental model with orthogonally oriented transmitting and receiving antennas at $\lambda_0 = 3$ cm. The radiation was focused with two lenses made of plaster. In Fig. 2, the result of tomography is shown of point inhomogeneities in the form of two identical polystyrene balls with diameters of 1 cm spaced at a distance of 7 cm. The inhomogeneities are fairly well localized with small distorting effects on each other.

The above-described method can be used for multifrequency sounding signal. Wide spectra of these signals should provide the deep penetration of radiation in inhomogeneous media, thereby improving radiation localization in the given regions. The multifrequency (broadband) approach is based on the preliminary reconstruction of the transfer function $H(\omega)$ of the bulk inhomogeneities, which connects the spectra of emitted signal $U_0(\omega)$ and signal $U(\omega) = H(\omega)U_0(\omega)$ scattered by inhomogeneities. After double focusing, the transfer function $H(\omega)$ reduces to the signal $I(\mathbf{r}_0)$, which represents convolution (1). The solution of equation of convolutions gives the reconstructed distribution of the dielectric constant at each frequency over the investigated volume. Application of multifrequency pulsed signals will be useful in case of the directed electromagnetic impact on the inhomogeneities.

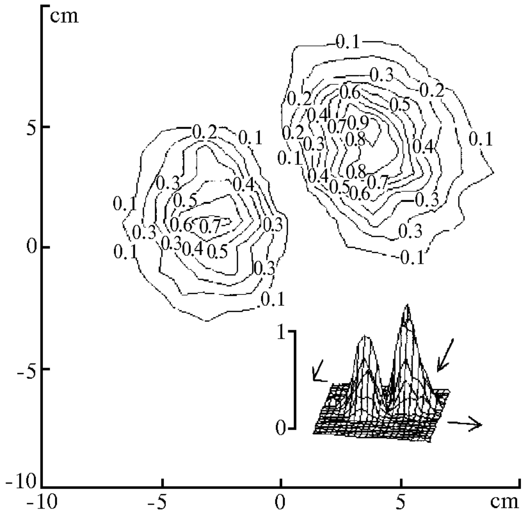

FIG. 2. Tomogram of two inhomogeneities at $\lambda_0 = 3$ cm.

3. APPLICATION OF THE INCOHERENT RADIATION

The interest to application of incoherent radiation to the tomography has been increased recently. Many sources of radiation have lower degree of coherence. In addition, due to the interaction with highly inhomogeneous media the radiation coherence breaks significantly. As a result, local regions of inhomogeneities act as sources of primary and secondary incoherent radiation. The distribution of their intensities characterizes the distribution of inhomogeneities. The tomography problems[10] are the problems of this class.

Let us consider a problem of passive tomography of an object with incoherent distribution of its emission intensity in a weakly absorbing medium. This distribution should be reconstructed from the scans with the antenna having a narrow directional pattern at different sighting angles[11] ψ. The approximation of weak absorption means that radiation is attenuated primarily due to its spherical divergence.

The equation for the power radiated by the volume with incoherent currents in cylindrical system of coordinates can be written as

$$P = \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{2\pi} \frac{I(\rho,\varphi,z)\,\rho\,d\rho\,d\varphi\,dz}{|\mathbf{R}_0 - \rho|^2},$$

where I is the density of radiation intensity distribution over the volume, P is the received power, R_0 is the distance from the center of the volume to the point of radiation reception, and $\rho_0 = R_0\sin\psi$ is the impact parameter. In general, the power of the received radiation P depends on the observation angle. Taking the expansion of the observed parameter in spherical harmonics of the observation angle and considering that the radiation intensity is constant along the z axis, we proceed from the integral over φ to the integral over ψ. As a result, for the spherical harmonic of the power received by the sharply directed antenna at the sighting angle ψ we obtain the formula

$$P_n(\rho_0) = 2\int_{\rho_0}^{\infty} \frac{I_n(\rho)\exp\{in(\pi/2-\psi)\}}{\sqrt{1-(\rho_0/\rho)^2}}\,T_n(\rho_0/\rho)\,d\rho. \quad (2)$$

Here, $I_n(\rho)$ is the spherical harmonic of the density of the radiation intensity emitted by inhomogeneities and $T_n(x)$ is the first order Chebyshev polynomial of the first kind. After inversion of equation (2), the desired two-dimensional distribution of the radiation intensity can be obtained by summation of the Fourier transform with the coefficients $I_n(\rho)$.

The problem of the solution of integral equation (2) appeared during processing of the projections recorded at various observation angles in the X-ray or emission active tomography.[1–3] The possibility of solution of this equation was demonstrated with the help of various integral transforms. This requires much computation time to provide the stability in the sense of the measurement noise. We developed an alternative method of solution of equation (2), based on proceeding to the integral convolution equation, making the substitution of the variables $\rho_0 = a\exp(\tau_0)$ and $\rho = a\exp(\tau)$ and taking the Fourier transform.[11]

An advantage of this method is the application of the Fourier transform, which is widely used in various problems and for which the efficient methods of numerical realization have been developed. Analytic representation of the Fourier transform of the kernel of integral equation also decreases the level of reconstruction errors. Moreover, this method realizes variable distribution step more comfortable for exact integration. For small ρ_0, the integral is calculated with the smaller step size. As the impact parameter increases, the step size also increases. Finally, regularization algorithms for the integral convolution equations are well developed. The results of imitation modeling confirmed the efficiency of the method for various forms of inhomogeneities.

Various processes can be the sources of incoherent radiation in the surrounding space. For example, in Refs. 12 and 13 it was predicted theoretically that in the atmosphere dissociation of water vapor upon exposure to radioactive radiation may be accompanied by the increase of concentration of atomic hydrogen manifested through the occurrence of the characteristic line of the microwave atmospheric emission.

To the right of Fig. 3, the joint measurements of the two-dimensional spatiotemporal spectrum of the intensity distribution of incoherent radiation of atomic hydrogen at a wavelength of 21 cm are shown carried out during the experiments performed by the Tomsk State University, the Siberian Physical-Technical Institute at the Tomsk State University, and the Institute of Atmospheric Optics of the SB RAS that confirm this prediction. The two maxima with different intensities can be clearly seen in the direction toward the Siberian Chemical Integrated Plant (SCIP).

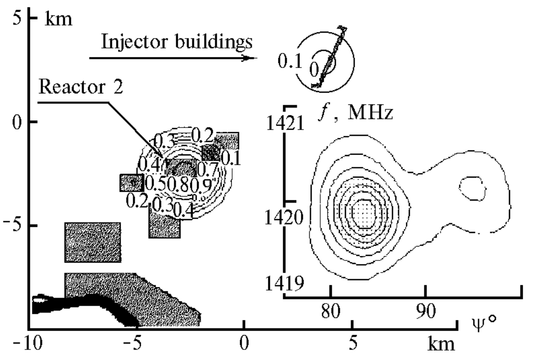

FIG. 3. Two-dimensional energy distribution and tomogram of distribution of sources of the microwave radiation at a wavelength of 21 cm.

To the left of Fig. 3, the spatial structure of the microwave radiation source is shown reconstructed by us from the experimental data. The assumption was used that each local source is axisymmetric. The scheme of location of buildings was borrowed from Ref. 14, where the data of the Landsat N–5026 NPIC were used. This approach can be used for thermography of forest fires and solution of a wide class of ecological problems.

In case of tomography of strongly absorbing media, the above-described methods are inapplicable: it is not true that a wave propagates along straight-line trajectories; in fact, the notion of trajectory (ray) itself should be revised. Further development of the tomography methods of strongly absorbing media requires not only the increase of the radiation power, but also revision of the mathematical apparatus used for reconstruction of the structure of the media. From all variety of possible approaches the preference, evidently, should be given to the methods that permit one to obtain the solution in a closed form. In Ref. 15 for solving the formulated problem we suggested the methods based on the introduction of the notion of amplitude trajectory by which we mean such virtual curve along which the wave amplitude

$$c = \exp\,(-L),\; L = \int n\mathrm{d}l$$

is least attenuated during its propagation. Here, n is the coefficient of the linear absorption by the medium. Then in accordance with the Ferma principle the amplitude trajectory of wave propagation can be found from the equation

$$n(r)\; r \sin \alpha(r) = n(r_0)\; r_0 \sin \alpha(r_0) \equiv p,$$

which is an analog of Snell's law in geometric optics. Here, $\alpha(r)$ is the angle of the trajectory slope at the distance r from the symmetry center, r_0 is the initial distance to the symmetry center, and p is the impact parameter of the trajectory. In connection with this, for the attenuation on the optimal trajectory we have

$$L(p) = 2 \int\limits_{r_{\min}}^{r_0} \frac{(n(r))^2\; r\mathrm{d}r}{\sqrt{[n(r)\; r]^2 - p^2}}\,, \tag{3}$$

where $r_{\min}$ is the radius of the point of trajectory turn determined from the relation $r_{\min} n(r_{\min}) = p$. For the known angular distance between the end points of the optimal trajectory, the impact parameter can be found as $p = \mathrm{d}L/\mathrm{d}\psi$. As a result of inversion of the Abel equation for the profile of the linear absorption coefficient $n(r)$ in case of axisymmetric layered medium, we obtain the solution of the form

$$\ln\left(\frac{r}{r_0}\right) = \frac{1}{\pi} \int\limits_{nr}^{n(r_0)\,r_0} \mathrm{d}\psi\; \ln\left\{\frac{p + \sqrt{p^2 - [nr]^2}}{nr}\right\},$$

where the quantity $n(r_0)$ at the distance r_0 from the center of symmetry is taken to be known. With its help for the given value of the parameter nr, the corresponding value of r is determined, and this finally is equivalent to the reconstruction of the desired dependence $n = n(r)$.

The results of imitation modeling on the profile reconstruction confirmed high efficiency and stability of the proposed solution in the sense of the measurement noise. This method can be used for the construction of tomographic systems of sounding of biological tissue, for remote diagnostic of the disperse composition of atmospheric pollutants including radioactive contamination in case of sensing in absorption lines, and others. The sounding radiation may be optical, radio, or acoustic. Coherence of radiation is not obligatory.

4. CONCLUSION

The above-considered methods have not yet been adequately developed to obtain the complete solution to the problem of microwave tomography of inhomogeneous media, but mutually supplementing each other, they permit, in our opinion, to approach significantly to its solution.

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REFERENCES

1. A.N. Tikhonov, V.Ya. Arsenin, and A.A. Timonov, *Mathematical Problems of Computer Tomography* (Nauka, Moscow, 1987), 160 pp.
2. G.A. Fedorov and S.A. Tereshchenko, *Computer Emission Tomography* (Energoizdat, Moscow, 1990), 184 pp.
3. J.E. Greenleaf and R.C. Bahn, IEEE Trans. Biomed. Eng. **BME–18** (1981).
4. V.R. Anpilogov, Zarub. Radioelektron., No. 1, 45–50 (1996).
5. K. Nikita and N. Usunoglu, *Trans. Black Sea Region Symposium on Applied Electromagnetism*, Athens (1996), BISY 13.
6. E.N. Voronin, Radioelektronika **36**, No. 8, 3–11 (1993).
7. S. Andersson-Engels, R. Berg, S. Svanberg, and O. Jarlman, Optics Letters **5**, No. 21, 1179–1181 (1990).
8. M. Toida, T. Ichimura, and H. Inaba, CLEO, 548–550 (1990).
9. V.P. Yakubov and M.L. Masharuev, Izv. Vyssh. Uchebn. Zaved., Ser. Fizika, No. 4, 87–92 (1997).
10. A.G. Sel'skii and A.M. Fisher, Radiotekhnika, No. 9, 85–89 (1995).
11. V.P. Yakubov and D.V. Losev, Atmos. Oceanic Opt. **10**, No. 2, 110–113 (1997).
12. E.T. Protasevich, Atmos. Oceanic Opt. **7**, No. 5, 367–368 (1994).
13. S.T. Penin and L.K. Chistyakova, Atmos. Oceanic Opt. **10**, No. 1, 45–49 (1997).
14. L.P. Rikhvanov, *Global and Regional Biological Problems* (Publishing House of the Tomsk Politechn. Univ., Tomsk, 1997), 384 pp.
15. V.P. Yakubov and D.V. Losev, Atmos. Oceanic Opt. **9**, No. 10, 867–870 (1996).