Active image restoration with regard to relative movement of a target

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Received July 26, 1999

The method is synthesized for active image restoration under conditions of amplitude-phase distortions in the spatial spectrum of a signal. The methods allow for spatiotemporal modulation of the signal due to target rotation and use no reference source in the target image plane. The proposed solution is also suitable for achieving super-resolution over the angular coordinate, including the case of constructing a 3D image.

The method of active restoration of target images which assumes compensation for multiplicative distortions in the spatial spectrum of a signal has been considered in Refs. 1–4. Such distortions can be caused by both imperfections of elements of the antenna–feeder circuit and turbulence of the medium the signal propagates through.

In this paper this method is further developed. Its new modification takes into account a complex nature of the target motion.

Actually, any movement of a target can be presented as a set of tangential, radial, and rotational movements. In this case it appears that the rotational movement "itself" provides the basis for active restoration.

The target image $\dot{\mathbf{E}}(\mathbf{r})$ is, as earlier, considered as the vertical complex spatiotemporal distribution of a signal in the image plane of the target upon its exposure to a plane wave oriented along the sight line. Such an approach is convenient, since it allows us to ignore the volume character of a target. Thus, keeping in mind that the information on the radial structure is in the argument of the complex function $\dot{\mathbf{E}}(\mathbf{r})$, we can reduce the problem to the Kirchhoff integral over the surface located in the image plane of the target.

For convenience, we use the Fraunhofer approximation, implying that with the corresponding substitutions we can use the Fresnel approximation.

So, let the target turn by a small angle $\Delta\theta$ with respect to the normal to the sight line within a short time interval Δt, not exceeding the time of the signal coherence. Then, as in Refs. 1–3, $\mathbf{r}$ is the vector in the image plane of the target, $\boldsymbol{\rho}$ is the vector in the aperture plane. Let also

$$\phi_t(\mathbf{r}) = \dot{\mathbf{E}}(\mathbf{r}) \ . \tag{1}$$

At rotation by the angle $\Delta\theta$, the phase at every point $\mathbf{r}$ changes according to the equation of a straight line

$$\Delta\phi_t(\mathbf{r}) \equiv 2\mathbf{k} \tan(\Delta\theta)\, \mathbf{r}, \tag{2}$$

where for simplicity (but without loss of generality) it is assumed that the center of rotation lies on the sight line; $k = 2\pi/\lambda$ is the wave number; λ is the wavelength.

Then at the time Δt the signal

$$\dot{E}(r, \Delta t) = E(r) \exp\{j[\phi_t(r) + \Delta\phi_t(r)]\} = E(r) \exp\{j[\phi_t(r) + 2k \tan(\Delta\theta)\, r]\} \tag{3}$$

is formed in the image plane of the target. At small $\Delta\theta$ and Δt, Eq. (3) can be written as

$$\dot{E}(r, \Delta t) = E(r) \exp\{j[\phi_t(r) + 2k\, \Delta\theta\, r]\} \ . \tag{3'}$$

Taking into account that

$$\Delta\theta = \omega_{rot}\, \Delta t \ , \tag{4}$$

where ω_{rot} is the speed of target rotation, we obtain from Eq. (3′) that

$$\dot{E}(r, \Delta t) = E(r) \exp\{j[\phi_t(r) + 2k\, \omega_{rot}\, \Delta t\, r]\} \ . \tag{5}$$

In the Fraunhofer approximation, omitting the factors which are inessential for further consideration, we obtain the following amplitude–phase distribution in the plane of the receiving aperture at the time t:

$$\dot{\varepsilon}(\rho, t) = \exp[j\varphi_a(\rho)]\, A(\rho) \int dr \exp(-j2\pi r\rho/\lambda R)\, \dot{E}(r, t) \ , \tag{6}$$

or, taking into account the definition of the spatial distribution of the valid signal

$$\dot{\varepsilon} = \int dr \exp(-j2\pi r\rho/\lambda R)\, \dot{E}(r), \tag{7}$$

we have

$$\dot{\varepsilon}(\rho, t) = A(\rho) \exp[j\varphi_a(\rho)]\, \dot{\varepsilon}(\rho) \ . \tag{6'}$$

Hereinafter $A(\rho)$ denotes amplitude distortions and $\varphi_a(\rho)$ the multiplicative phase distortions, which are constant during the time of consideration.

Then at the time $(t + \Delta t)$ we have in the plane of the receiving aperture

0235-6880/99/12 1084-03 $02.00

$$\dot{\varepsilon}(\rho,\ t+\Delta t) = A(\rho)\ \exp\ [j\varphi_a(\rho)] \times$$
$$\times \int \mathrm{d}r\ \exp\ (-\ j2\pi r\rho/\lambda R)\ \dot{E}(r,\ t+\Delta t) =$$
$$= A(\rho)\ \exp\ [j\varphi_a(\rho)] \int \mathrm{d}r\ \exp\ (-\ j2\pi r\rho/\lambda R)\ E(r) \times$$
$$\times \exp\ \{j[\phi_t(r) + 2k\ \omega_{\mathrm{rot}}\ \Delta t\ r]\},$$

and, taking into account Eq. (1) and the shift theorem,

$$\dot{\varepsilon}(\rho,\ t+\Delta t) = A(\rho)\ \exp\ [j\varphi_a(\rho)] \times$$
$$\times \int \mathrm{d}r\ \exp\ [-\ j2\pi r\rho/(\lambda R)\ (\rho - 2R\ \omega_{\mathrm{rot}}\ \Delta t)]\ \dot{E}(r) =$$
$$= A(\rho)\ \exp\ [j\varphi_a(\rho)]\ \dot{\varepsilon}(\rho - 2R\ \omega_{\mathrm{rot}}\ \Delta t)\ . \qquad (8)$$

Introducing the designations

$$\varphi(\rho) = \arg \dot{\varepsilon}(\rho) \qquad (9)$$

and

$$\psi(\rho,\ t) = \arg \dot{\varepsilon}(\rho,\ t)\ , \qquad (10)$$

from Eqs. (6′) and (8) we derive the following two sets of equations for the phase and amplitude:

$$\begin{cases} \psi(\rho,\ t) = \varphi_a(\rho) + \varphi(\rho), \\ \psi(\rho,\ t+\Delta t) = \varphi_a(\rho) + \varphi(\rho - 2R\ \omega_{\mathrm{rot}}\ \Delta t), \end{cases} \qquad (11)$$

$$\begin{cases} \varepsilon(\rho,\ t) = A(\rho)\ \varepsilon(\rho)\ , \\ \varepsilon(\rho,\ t+\Delta t) = A(\rho)\ \varepsilon(\rho - 2R\ \omega_{\mathrm{rot}}\ \Delta t)\ , \end{cases} \qquad (12)$$

where ε is the absolute value of $\dot{\varepsilon}$.

Let us first solve Eq. (11) for $\varphi(\rho)$. Upon subtraction of the first equation of the set (11) from the second one, we have

$$\psi(\rho,\ t+\Delta t) - \psi(\rho,\ t) = \varphi(\rho - 2R\ \omega_{\mathrm{rot}}\ \Delta t) - \varphi(\rho)\ . \ (13)$$

Having divided both sides of Eq. (13) by Δt, we obtain

$$\frac{\psi(\rho,\ t+\Delta t) - \psi(\rho,\ t)}{\Delta t} = \frac{\varphi(\rho - 2R\ \omega_{\mathrm{rot}}\ \Delta t) - \varphi(\rho)}{\Delta t}\ . \quad (14)$$

Let us designate

$$\Delta\rho = -\ 2R\ \omega_{\mathrm{rot}}\ \Delta t\ . \qquad (15)$$

With regard to this designation, Eq. (14) takes the form

$$-\frac{\psi(\rho,\ t+\Delta t) - \psi(\rho,\ t)}{\Delta t} =$$
$$= -\ 2R\ \omega_{\mathrm{rot}}\ \frac{\varphi(\rho - 2R\ \omega_{\mathrm{rot}}\ \Delta t) - \varphi(\rho)}{\Delta\rho}\ . \qquad (16)$$

We can see that as Δt tends to zero, $\Delta\rho$ also tends to zero according to Eq. (15). Thus, from Eq. (16) we derive

$$\lim_{\Delta t \to 0} \frac{\psi(\rho,\ t+\Delta t) - \psi(\rho,\ t)}{\Delta t} =$$
$$= -\ 2R\ \omega_{\mathrm{rot}} \lim_{\Delta t \to 0} \frac{\varphi(\rho + \Delta\rho) - \varphi(\rho)}{\Delta\rho}\ . \qquad (17)$$

As a result, taking into account the definition of a derivative, we have

$$\frac{\partial\psi(\rho,\ t)}{\partial t} = -\ 2R\ \omega_{\mathrm{rot}}\ \frac{\partial\varphi(\rho)}{\partial\rho}\ . \qquad (18)$$

Upon integration of Eq. (18), we find

$$\varphi(\rho) = \frac{-\ 1}{2R\ \omega_{\mathrm{rot}}} \int \mathrm{d}\rho\ \frac{\partial\psi(\rho,\ t)}{\partial t}\ . \qquad (19)$$

Equations (18) and (19) give the sought solution for the phase of the spatial spectrum of signals, which is free from multiplicative phase distortions $\varphi_a(\rho)$.

The set of equations (12) in its turn can be solved for the absolute value of the spatial spectrum of the valid signal $\varepsilon(\rho)$. Toward this end, it is first convenient to take the logarithm of the equations:

$$\begin{cases} \ln\ \varepsilon(\rho,\ t) = \ln\ A(\rho) + \ln\ \varepsilon(\rho)\ , \\ \ln\ \varepsilon(\rho,\ t+\Delta t) = \ln\ A(\rho) + \ln\ \varepsilon(\rho - 2R\ \omega_{\mathrm{rot}}\ \Delta t)\ . \end{cases} \qquad (20)$$

Subtracting again the first equation of the set (20) from the second one, we find

$$\ln\ \varepsilon(\rho,\ t+\Delta t) - \ln\ \varepsilon(\rho,\ t) =$$
$$= \ln\ \varepsilon(\rho - 2R\ \omega_{\mathrm{rot}}\ \Delta t) - \ln\ \varepsilon(\rho)\ . \qquad (21)$$

Repeating the transformations analogous to Eqs. (14)–(17), we obtain from Eq. (20) that

$$\frac{\partial\ \ln\ \varepsilon(\rho,\ t)}{\partial t} = -\ 2R\ \omega_{\mathrm{rot}}\ \frac{\partial\ \ln\ \varepsilon(\rho)}{\partial\rho}\ , \qquad (22)$$

wherefrom

$$\ln\ \varepsilon(\rho) = \frac{-\ 1}{2R\ \omega_{\mathrm{rot}}} \int \mathrm{d}\rho\ \frac{\partial\ \ln\ \varepsilon(\rho,\ t)}{\partial t} \qquad (23)$$

or, finally,

$$\varepsilon(\rho) = \exp\left[\frac{-\ 1}{2R\ \omega_{\mathrm{rot}}} \int \mathrm{d}\rho\ \frac{\partial\ \ln\ \varepsilon(\rho,\ t)}{\partial t}\right]. \qquad (24)$$

Equations (22)–(24) give the sought solution for the absolute value of the spatial spectrum of signals, which is free from multiplicative amplitude distortions $A(\rho)$.

Thus, we have the absolute value [Eq. (24)] and phase [Eq. (18)] of the spatial spectrum of the signal (actually, the amplitude–phase distribution of the field at the antenna aperture), which are free from multiplicative distortions. So, then we can calculate (estimate) the image to be restored $\dot{E}(r)$ itself. To do this, we should invert, with the corresponding substitutions, the integral transformation of the form (7). This transformation, expressing the Fraunhofer diffraction, is the spatial Fourier transform accurate to the scale factors.

As to the practical implementation of the proposed method, it is clear that time separation of the signal can be obtained by introducing delay lines, and operations of integration and differentiation can be replaced by summation and subtraction, respectively.

The proposed solution can also apply to the problems of 3D vision, similar to that considered in Ref. 5, where the high resolution over the angular coordinate is achieved by inverse synthesis of the aperture, and high resolution over the range is achieved by using complex signals, the product of whose spectral widths by the duration exceeds unity. Let us only note that in contrast to Ref. 5, the high resolution over the range here is by no means a guarantee of high resolution over the angular coordinate; the latter is achieved without invoking the concept of a point-like reference source.

So, in this paper the method for active image restoration under conditions of amplitude–phase distortions in the spatial spectrum of signals is synthesized. This method takes into account the spatiotemporal modulation of the signal due to target rotation and does not use a reference source in the target image plane.

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