On the role of turbulent diffusion in vertical distribution of aerosol particles

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Based on calculation of the kinetic energy of an aerosol particle conditioned by turbulent diffusion in the atmosphere, the diffusion effects on the aerosol particle vertical distribution have been investigated qualitatively. The Gibbs method shows that turbulent diffusion, whose intensity decreases with the altitude, favors the equilibrium existence of aerosol layers. The stabilization altitude of the layers depends on aerosol particle sizes and the rate of the diffusion coefficient decrease. It is found that the increase in the turbulent diffusion intensity leads to the expansion of the range of particle sizes, which can be in equilibrium at stratospheric heights. This can be one of the mechanisms of the negative feedback in stabilization of the stratospheric temperature. The obtained conclusions are qualitatively proved by results of the stratospheric aerosol lidar sounding in Tomsk and Yakutsk.

It is a well-known fact, that the Gibbs distribution is a powerful instrument for investigation of the high-altitude profiles of atmospheric parameters, starting from the barometric formula, describing the vertical distribution of pressure and density, to construction of the vertical profile of the ozone distribution.[1] The Gibbs distribution application to studies of the stratospheric aerosol vertical distribution requires the knowledge of the aerosol particle Hamiltonian. It is clear that the accounting for kinetic energy of the aerosol particle translational motion in the Hamiltonian is of no use, because in order to obtain the kinetic energy comparable with the potential one, the particle at stratospheric heights ($h \geq 10$ km) should have velocities of $\sim \sqrt{2gh}$ (g is the gravity acceleration), i.e., comparable with the sound velocity. In this paper, we have made an attempt of estimating the aerosol particle kinetic energy conditioned by the turbulent diffusion in the atmosphere, as well as investigating qualitatively the diffusion effects on the vertical distribution of aerosol particles.

To calculate the kinetic energy of a particle, we use the well-known approach.[2] The particle (in further estimations we assume its spherical shape with the radius a), being under the action of a random force $\mathbf{F}$ and the Stokes resistance force $6\pi\eta a\mathbf{V}$ ($\mathbf{V}$ is the particle velocity; η is the molecular* air viscosity) meets the motion equation, the projection of which on the arbitrarily chosen X-axis has the form

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = F_x - 6\pi\eta a\frac{\mathrm{d}x}{\mathrm{d}t}. \quad (1)$$

* In the equation of the aerosol particle motion, just the molecular viscosity should be used instead of the turbulent one, since the particle size is certainly much less than the inner turbulence scale, i.e., being inside the least vortex, the particle does not interact with other vortices.

Multiplying both parts of Eq. (1) by x and averaging over the ensemble of particles, we obtain

$$\frac{m}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\langle x^2\rangle - m\langle(\mathrm{d}x/\mathrm{d}t)^2\rangle = -3\pi\eta a\frac{\mathrm{d}}{\mathrm{d}t}\langle x^2\rangle, \quad (2)$$

where the angular brackets mean the value averaged over the ensemble. Since it is known that $\langle x^2\rangle = Dt$ and $(\mathrm{d}x/\mathrm{d}t)^2 = V^2/3$, the expression for the particle kinetic energy follows from Eq. (2):

$$W = (9/2)\pi\eta aD. \quad (3)$$

Figure 1 presents the isopleths of equalized heights for kinetic and potential energy values of submicron particles in the plane “particle radius – turbulent diffusion coefficient.”

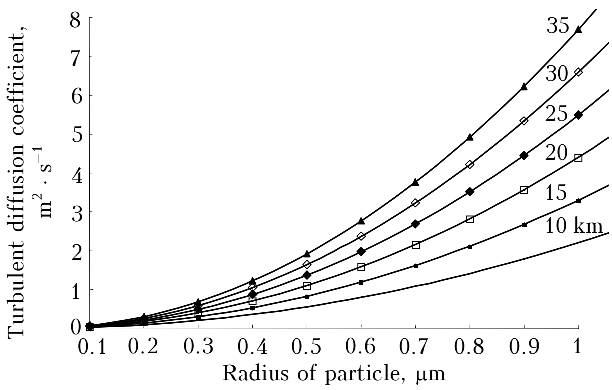

Fig. 1. Dependence of the equalization height of aerosol particle kinetic and potential energies on the particle size and intensity of the turbulent diffusion.

As is seen, kinetic energy of the particle turbulent diffusion significantly contributes to its Hamiltonian. The dynamic range of the last parameter was chosen in conformity with the data from Ref. 3, the air viscosity was taken equal to 132 μP that corresponds to the winter stratosphere, and ρ was equal to 1 g · cm^{-3}.

The Gibbs height distribution for particles of a radius can be written as

$$f_a(h) = \frac{1}{Z}\exp\left[-\frac{mgh + (9/2)\pi\eta a D(h)}{kT}\right], \quad (4)$$

where $Z = \int_0^\infty dh \exp\left[-\frac{mgh + (9/2)\pi\eta a D(h)}{kT}\right]$ is the statistical sum.

The heights of monodispersed aerosol concentration extrema are roots of equation

$$\frac{d}{dh}D(h) = -\frac{8a^2\rho g}{27\eta}. \quad (5)^*$$

When deriving Eq. (5), we have neglected the altitude dependence of T and η against the background of much more sharp altitude dependence of the potential energy and the turbulent diffusion coefficient. As follows from Eq. (5), the extrema of aerosol concentration should be disposed in the regions of the turbulent diffusion coefficient decreasing with the height. According to the present-day representations, the turbulence intensity decreases with height, therefore function $f_a(h)$ should have a maximum. Let us estimate the position of the maximum based on the widespread model[4] supposing the exponential character of decreasing of the turbulent effect intensity with height. Assuming

$D(h) = D(h_0)\exp[-(h - h_0)/H]$, where $H = 10$ km,

for $h > h_0 \simeq 10$ km, we find the position of the maximum

$$h_m = H \ln\frac{27D_1\eta}{8a\rho g H} \quad (6a)$$

and the condition of its existence in the region $h > h_0$ (stratosphere):

$$\frac{D(h_0)}{a^2} > \frac{8\rho g H}{27\eta}, \quad (6b)^{**}$$

where $D_1 = D(h_0)\exp(h_0/H)$.

As it follows from Fig. 2, the increase in the turbulent diffusion coefficient leads to expansion (to large particles) of the dynamic range of sizes of the aerosol having the concentration maximum in the stratosphere, and to decrease of the slope in the dependence of the maximum height on the particle size.

Note that maximum of the height distribution of monodispersed aerosol is very sharp (Fig. 3).

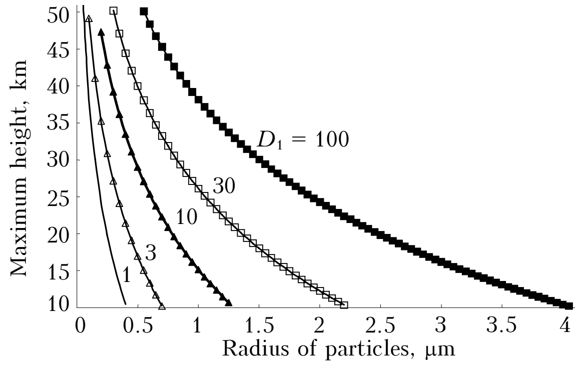

Fig. 2. Dependence of the maximum height on the particle size at different values of the turbulent diffusion coefficient, m^2/s.

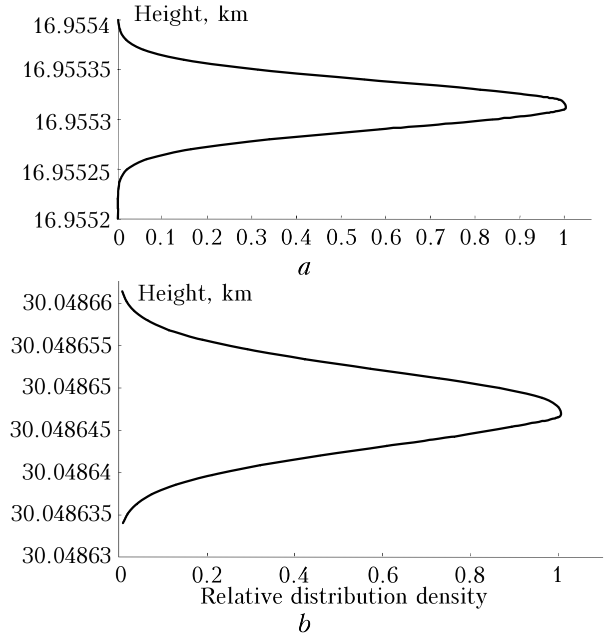

Fig. 3. Height distribution of the monodispersed aerosol: the coefficient of diffusion of 3 m^2/s at a radius of 0.5 μm (*a*); the coefficient of diffusion of 100 m^2/s at a radius of 1.5 μm (*b*).

The distribution half-width makes decimeters or even centimeters that allows approximating the altitude distribution of the monodispersed fraction by the Dirac delta with the carrier concentrated at the maximum height. It is important that the maximum sharpness is stipulated not by strong spatial variability of the field of turbulent diffusion coefficient in the vertical direction (its characteristic scale is four or five orders of magnitude greater than the half-width of the distribution density maximum). It is connected with the fact that changes of both potential and kinetic energies of an aerosol particle at small height variations are very high as compared to kT. Actually, when denoting the exponent in Eq. (4) through $-\varphi(h)$, expanding it into Maclaurin series in the vicinity of the root of Eq. (5) h_0, and allowing for the obvious relation $\varphi(h_0) = 0$, we come to the following expression for distribution $f_a(h)$:

* For the particles with radius of 1 μm and density of ~1 g · cm^{-3} the right part of Eq. (5) is equal to ~20 m · s^{-1}.

** For the vertical variability scale of the turbulent diffusion coefficient of ~10 km and density of ~1 g · cm^{-3}, the right part of Ineq. (6b) is equal to ~3 THz.

$$f_a(h) = \sqrt{\frac{\lambda}{\pi}}\frac{1}{l_0}\exp\left[-\lambda\frac{(h-h_0)^2}{l_0^2}\right], \qquad (7)$$

where $l_0 = \sqrt{D(h_0)/\ddot{D}(h_0)}$ is the spatial scale of the turbulent diffusion coefficient variability; $\lambda = 3D(h_0)/(8uH) \gg 1$ is the dimensionless parameter, whose large values allow the transition of the gaussoid (7) into the Dirac delta. Possibly, the observed pronounced high-altitude localization of the Junge layer is connected just with this transition.

It is of interest, how the stated reasons agree with results of lidar sounding of the stratospheric aerosol, which in practice is the only source of information on the aerosol component vertical distribution in the stratosphere. Unfortunately, the overwhelming majority of data represent the results of the single-frequency sounding free of polarization selection in the receiving channel. Therefore, these data cannot be used for restoration of the stratosphere aerosol microstructure parameters, that makes impossible to test the above-mentioned reasons quantitatively. (Note that reconstruction of the vertical distribution at least of the particle number density and the mean square radius would allow estimating the vertical profiles of the turbulent diffusion coefficient in the stratosphere, but this information is not available nowadays.) At the same time, the layer structure of profiles of the stratospheric aerosol optical activity clearly observed in Fig. 4, is in good agreement with the conclusion on strong connection of the location of the aerosol layer height, sizes of its particles, and the rate of the turbulent diffusion coefficient decrease with the height.

Turbulent diffusion is a specific selector of different height fractions of the aerosol stratospheric component. The results of lidar sounding in Yakutsk, which is known as a region with zones of anomalously low turbulence in winter period, confirm this conclusion. Two profiles in Fig. 5 are typical representatives of such class of profiles, which are regularly observed in this region during the period from October to April.

Certainly, the presented results cannot be considered as the direct experimental verification of the above-stated calculations and estimations. Such verification requires a specially organized complex experiment and, as it was mentioned above, opens wide prospects for obtaining regular data on turbulent diffusion in the stratosphere. However, the qualitative agreement of the observed lidar profiles with the above-described mechanism of the turbulent diffusion influence on vertical aerosol distribution seems quite reasonable.

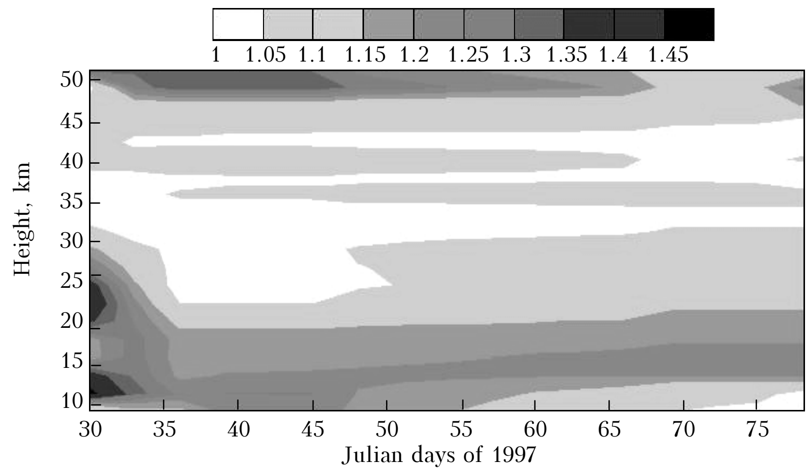

Fig. 4. Altitude–time section of ratio of the full backscattering to the molecular one according to the lidar sounding data in Tomsk.

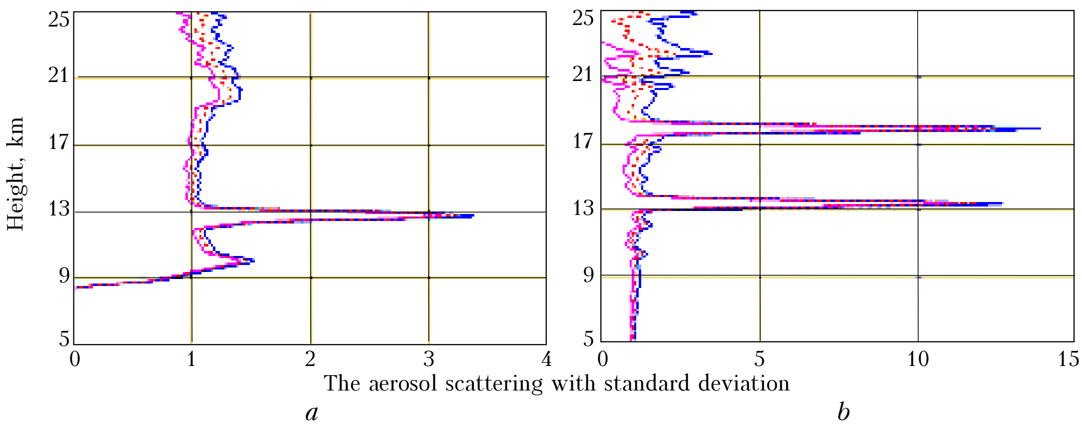

Fig. 5. Results of lidar sounding in Yakutsk: February 16, 2006 (*a*), October 5, 2005 (*b*).

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