Retrieval of the slope distribution of a rough water surface from the image of a grid test-object

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A laboratory method of rough water surface diagnostics based on the use of a grid test-object is considered. The optical image of the object consists of a set of distorted lines with randomly arranged nodes. In the case of topological correspondence between the image and object, each node in the grid image corresponds to a node on the object, which permits one to easily identify the node. It is shown that based on the nodal coordinates in the distorted and undistorted image, one can determine the magnitude and direction of the surface slope at the point corresponding to the node in the distorted image. The theoretical prerequisites of the method are considered and results of numerical calculations are presented.

In laboratory studies of the wave state of a water surface, optical methods for obtaining diagnostics of the wave state of the water surface find wide application. These include, in particular, methods based on an analysis of images of various test-objects located above or below the water and observed through the rough surface in refracted or reflected illumination.

As was shown in Ref. 1, for the appropriate choice of the test-object it is possible to determine in a very simple way from its image both the "instantaneous" and statistical characteristics of the wave state. The simplest object allowing one to record realizations of the slopes of a rough surface is an "optical wedge" – a test-object with a linear brightness distribution. The brightness distribution in the image of such an object contains a component proportional to the spatial distribution of the slopes of the water surface in the direction of maximum variation of brightness of the object.

The principle of the use of a "wedge" type object lies at the basis of the method for determining the energy spectrum of the wave state from the image of the sea surface under conditions of illumination by scattered skylight.[2,3] This same principle lies at the basis of a number of laboratory techniques for examining wind-driven waves (see, e.g., Refs. 4 and 5).

It is obvious that information about the wave state of a water surface is contained not only in the energy parameters (brightness) of the image of the test-object but also in its geometrical characteristics. Probably the simplest example here is a point test-object, whose image is in general a set of randomly scattered points. Here the difference in the coordinates of an arbitrary "glint" in the image and a "reference" point, which can be obtained by observing through the air–water interface when at rest, has a linear dependence on the gradient vector of the water surface at the point where the glint is located. Thus, an analysis of the image of these glints makes it possible to extract information about a rough surface at a discrete set of points of the surface.

Information about the wave state contained in the image of a point test-object is usually insufficient to reconstruct the slope field on a required segment of the water surface. A possible solution here is to increase the number of point objects. However, in this case it becomes necessary to make the objects differ from one another in some way, which is not a simple task; besides, this would entail the necessity of building receivers capable of responding to these differences, which is probably a still more complicated problem. A method free of these drawbacks is the method based on an analysis of images of a grid test-object. Such an object consists of a set of narrow stripes, for definiteness, of dark color on a bright background, spaced at equal intervals in two mutually perpendicular directions (Fig. 1).

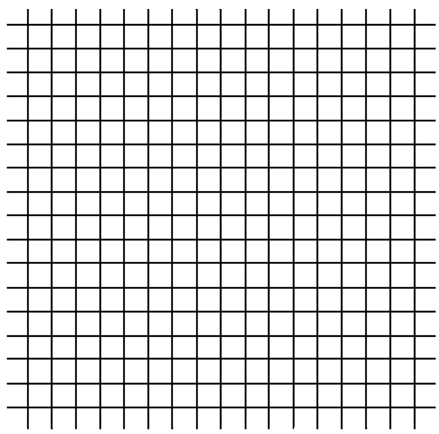

Fig. 1. Image of a grid test-object when observing through a flat air–water interface.

The image of the object consists of a set of curved lines with randomly arranged nodes. Depending on the degree of distortion, the image can be either topologically equivalent to the object (Fig. 2) or not (Fig. 3). The character of the image depends on the

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magnitude of the sea state of the rough surface and the depth of the object.

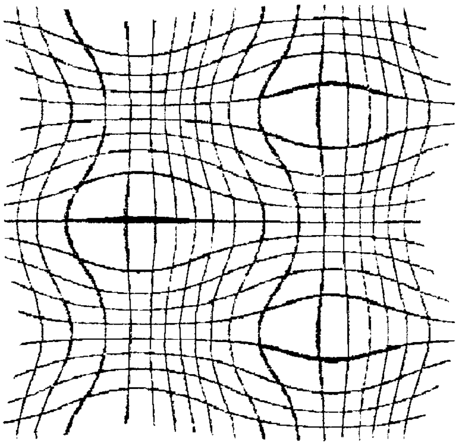


Fig. 2. Image of a grid when observing through a rough surface for $\lambda = 40$ cm.

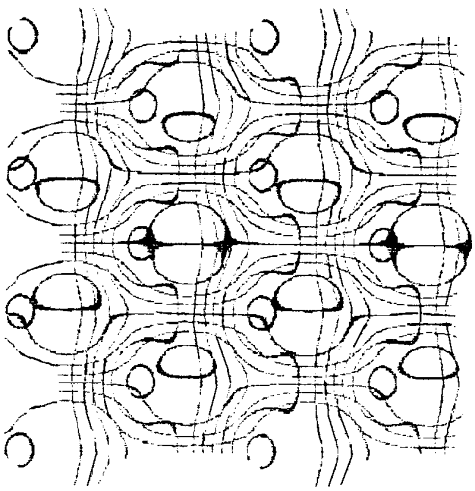


Fig. 3. Image of a grid when observing through a rough surface for $\lambda = 25$ cm.

If the inequality $ap \le 1$ ($a = h/4$, where h is the depth of the object and p is the maximum value of the second derivative of the surface), then the image has the form shown in Fig. 2. In this case each node in the image of the grid corresponds to one node on the object, which allows it to be "identified." The procedure of reconstructing information about the slopes of a rough surface, as in the problem of the test-object, is the following: it is necessary to determine the coordinates of the corresponding nodes in the distorted and undistorted images and on this basis calculate the magnitude and direction of the slope of the surface at the point corresponding to the node in the distorted image.

Note that in contrast to the method of wave-state diagnostics in Ref. 4, here we have the possibility to reconstruct complete information about the slope field of the rough surface at discrete, randomly located points. Reconstructing the slopes at an arbitrary point on the surface is an interpolation problem.

Let us turn now to the quantitative side of the problem. Figure 4 depicts the observation scheme. The observation system (OS) is located at a height H above the water surface and in its most general form consists of a camera-obscura with a mosaic of photosensitive elements in the image plane located at the distance f from the plane of the entrance pupil. A grid test-object is located at a depth h under the water having cells of, for simplicity, rectangular shape. We will derive relations linking the coordinates $\mathbf{R}$ of an arbitrary node of the grid and coordinates $\boldsymbol{\rho}$ of corresponding node in the image plane. It follows from Fig. 4 that

$$\mathbf{R} = h\mathbf{z}_0 + \mathbf{r} - \frac{h}{\gamma_0}\,\boldsymbol{\Omega}_0^0, \qquad (1)$$

$$\boldsymbol{\rho} = -f\mathbf{z}_0 + \frac{f}{\gamma_i}\,\boldsymbol{\Omega}_i^0, \qquad (2)$$

$$\mathbf{r} = H\mathbf{z}_0 - \frac{H}{\gamma_i}\,\boldsymbol{\Omega}_i^0, \qquad (3)$$

where $\mathbf{r}$ is the position vector of a point on the water surface, $\gamma_{i,0} = (\mathbf{z}_0{\cdot}\boldsymbol{\Omega}_{i,0}^0)$, where $\boldsymbol{\Omega}_{i,0}^0$ are unit vectors in the directions of the rays.

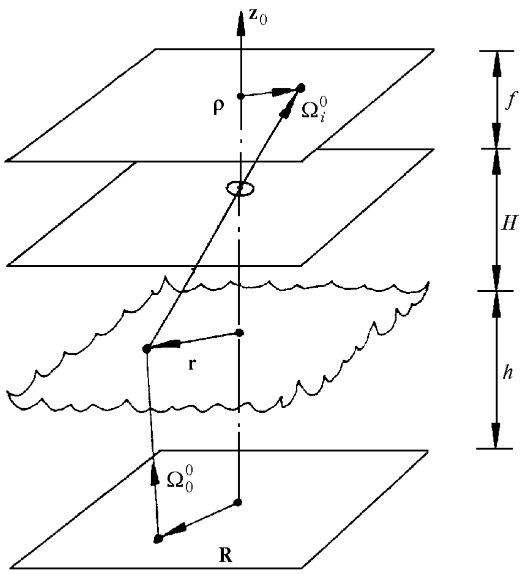


Fig. 4. Observation scheme.

A relation linking the vectors of the incident and refracted rays can be obtained from the refraction law; it has the following form[6]:

$$\boldsymbol{\Omega}_0 = \frac{1}{m}\,[(\boldsymbol{\Omega}_i - A\ \mathbf{q}(\mathbf{r})], \qquad (4)$$

where $A = \sqrt{m^2 - 1 + \gamma_i^2} - \gamma_i$, m is the refractive index of water, $\boldsymbol{\Omega}_{i,0}$ are the projections of the vectors $\boldsymbol{\Omega}_{i,0}^0$ onto the plane z = const.

Relations (1)–(4) were derived under the condition of smallness of the slopes $q^2 \ll 1$, and also neglecting elevations ζ of the surface, which is valid for H, $h \gg \zeta$.

Let us express the parameters entering into relations (1)–(4) in terms of the coordinates of a node in the image:

$$\mathbf{\Omega}_i = \frac{\boldsymbol{\rho}}{\sqrt{\rho^2 + f^2}}\,; \quad \gamma_i = \frac{f}{\sqrt{\rho^2 + f^2}}\,;$$

$$\mathbf{r} = -\boldsymbol{\rho}\,\frac{H}{f}; \quad \gamma_0 = \frac{1}{m}\sqrt{m^2 - \frac{\rho^2}{\rho^2 + f^2}}\,. \qquad (5)$$

From relations (1)–(5) we obtain by way of some uncomplicated transformations an equation linking the coordinates of a node (grid point) on the object with the coordinates of a node on the image:

$$\mathbf{R} = -\boldsymbol{\rho}\,\frac{H}{f}\left(1 + \frac{h}{mH\alpha_\rho}\right) + h\,\frac{\alpha_\rho - 1/m}{\alpha_\rho}\,\mathbf{q}\left(-\boldsymbol{\rho}\,\frac{H}{f}\right), \quad (6)$$

where

$$\alpha_\rho = \sqrt{1 + \frac{\rho^2}{f^2}\left(1 - \frac{1}{m^2}\right)}\,.$$

For an undisturbed surface Eq. (6) yields a relation between the coordinates of the node $\mathbf{R}$ on the object and the coordinates $\boldsymbol{\rho}_0$ in its undistorted image:

$$\mathbf{R} = -\boldsymbol{\rho}_0\,\frac{H}{f}\left(1 + \frac{h}{mH\alpha_0}\right), \qquad (7)$$

where

$$\alpha_0 = \sqrt{1 + \frac{\rho_0^2}{f^2}\left(1 - \frac{1}{m^2}\right)}\,.$$

Thus, we have obtained two equations which determine the coordinates of the nodes in a distorted image and the undistorted image of a grid test-object. Equating the right-hand sides of Eqs. (6) and (7), we obtain one equation which determines the value of the gradient vector of the rough water surface:

$$\mathbf{q}\left(-\boldsymbol{\rho}\,\frac{H}{f}\right) = \frac{\alpha_\rho H}{hf(\alpha_\rho - 1/m)} \times$$
$$\times\left[\boldsymbol{\rho}\left(1 + \frac{h}{mH\alpha_\rho}\right) - \boldsymbol{\rho}_0\left(1 + \frac{h}{mH\alpha_0}\right)\right]. \qquad (8)$$

We rewrite Eq. (8) in the coordinate system associated with the air–water interface:

$$\mathbf{q}(\mathbf{r}) = \frac{\beta_r}{h(\beta_r - 1/m)} \times$$
$$\times\left[\mathbf{r}_0\left(1 + \frac{h}{mH\beta_0}\right) - \mathbf{r}\left(1 + \frac{h}{mH\beta_r}\right)\right], \qquad (9)$$

where

$$\beta_r = \sqrt{1 + \frac{r^2}{H^2}\left(1 - \frac{1}{m^2}\right)}\,,\ \beta_0 = \sqrt{1 + \frac{r_0^2}{H^2}\left(1 - \frac{1}{m^2}\right)}\,;$$

$$\mathbf{r} = -\boldsymbol{\rho}\,\frac{H}{f}\,,\ \mathbf{r}_0 = -\boldsymbol{\rho}_0\,\frac{H}{f}\,.$$

Expression (9) takes an especially simple form in the case when the observation angle of the object is not large (r, $r_0 \ll H$) and the observation system is located sufficiently far from the water surface ($H \gg h/m$):

$$\mathbf{q}(\mathbf{r}) = \frac{1}{a}\,(\mathbf{r}_0 - \mathbf{r}), \qquad (10)$$

where $a = h\,(m - 1)/m$.

To check the adequacy of the proposed method of reconstructing the slope field of a rough water surface, we performed some numerical calculations. We modeled the situation of an air–water interface prescribed by the sum of three harmonic waves having the same amplitude σ and wavelength λ and propagating in directions differing by 120°. The modeling was carried out in several steps.

In the first step the problem was solved of formation of the distorted image of the test-object, consisting of a panel with dimensions 0.8×0.8 m with the figure of a grid imprinted on it consisting of 17 stripes along two orthogonal directions (grid step 5 cm, stripe width 2 mm). The depth of the object was 2 m. The image of the test-object when observing through the calm water surface is shown in Fig. 1. The initial (undistorted) image of the object, created on a display with the usual scale transformation from real coordinates to screen coordinates, was scanned line-by-line with a scanning step of 1 pixel. At each point on the screen the screen coordinates $\mathbf{r}^e$ were transformed into the real coordinates $\mathbf{r}$. In accordance with the formula $\mathbf{r}_0 = \mathbf{r} + a\mathbf{q}(\mathbf{r})$ we determined real coordinates of points in the object plane, which then were transformed into the screen coordinates. Next the screen brightness at the point $\mathbf{r}_0$ was determined (this brightness is nonzero if the point belongs to the figure of the grid and is zero if it lies outside it). The distorted image obtained in this way consisted of 640×480 points defining the resolution of the VGA display. Figure 2 shows an image obtained when observing through a surface with $\lambda = 40$ cm and $\sigma = 4$ mm. (The surface elevation function is shown in Fig. 5 in the form of a contour plot.)

In the second step the problem arose of determining the gradient vector of the slopes of the water surface at the nodes of the distorted image. The image of the object was displayed on the screen and by means of a cursor that could be repositioned by the operator the coordinates of all the nodes in the distorted image of the grid $\mathbf{r}_i$ were determined and recorded in the form of a square matrix M. In addition, the matrix M_0 was created, containing information about the coordinates of the nodes $\mathbf{r}_i^0$ of the undistorted image (a regular grid). Next, according to the formula

$$\mathbf{q}_i(\mathbf{r}) = (\mathbf{r}_i^0 - \mathbf{r}_i)/a$$

using the data contained in M and M_0 the values of the gradients of the slopes were determined successively at all nodes of the distorted grid. The results obtained at this step were recorded in the form of the square matrices Q_x and Q_y containing the values of the slope components q_x and q_y at the nodes of the distorted grid. Note that the process of "deciphering" the nodes of the distorted grid by hand is quite laborious; however, the proposed method of analysis of wave processes can be automated.

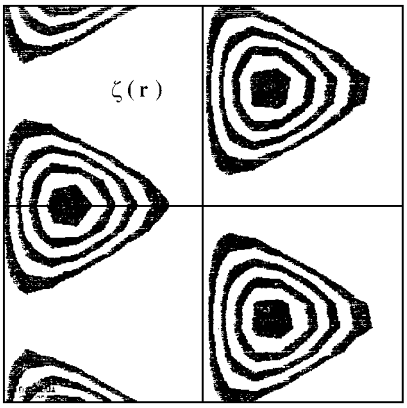
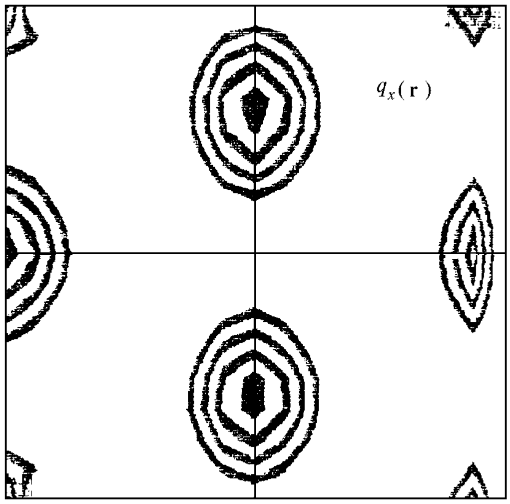


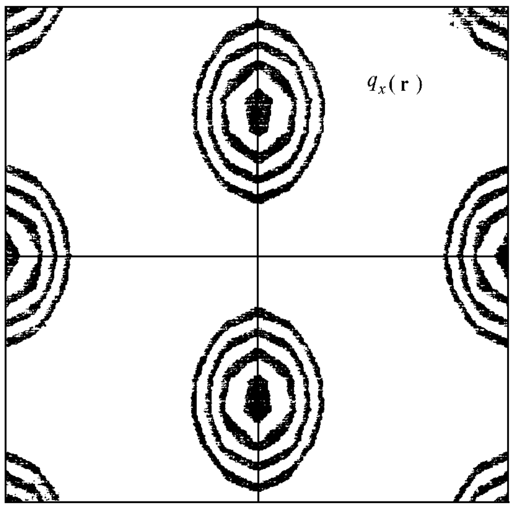
Fig. 5. Contours of the surface elevation function (λ = 40 cm).

In the third step the problem was solved of interpolating the values of the matrices Q_x and Q_y to the nodes of a regular grid. To determine the values of the function q_x (or q_y) at some point, we used information on the values of this function at the four nearest nodes of the distorted grid. In this case, the point turned out to lie inside an irregular quadrilateral. The question of which quadrilateral a point belonged to was key here and was solved by a special calculational scheme. As the interpolation formula we used the incomplete quadratic form $z = axy + bx + cy + d$. The results obtained at this step were recorded in the form of the matrices $Q_{x,y}^0$, whose elements, in contrast to the matrices $Q_{x,y}$, were determined at the nodes of a regular grid.

In the fourth step the problem was solved of constructing two-dimensional distributions of the components q_x and q_y of the gradient vector of the rough surface; in particular in the form of lines of equal level. To solve this problem, we used data contained in the matrices $Q_{x,y}^0$. Note that in constructing the graphs we made use of traditional and well-known interpolation schemes. Figure 6*a* plots the spatial distribution of the x component of the slopes, and Fig. 7*a* shows the spatial distribution of the y component (so as not to clutter the figures, only the positive half-waves are shown). Figures 6*b* and 7*b* plot theoretical distributions of the functions $q_x(\mathbf{r})$ and $q_y(\mathbf{r})$. Comparing the distributions obtained by processing the modeled images with the theoretical distributions, one can convince oneself of their good agreement. "Edge" errors are a result of the fact that some nodes of the regular grid fall outside the distorted image. The maximum error of reconstruction of the slopes, as analysis shows, in the example under consideration does not exceed 12%. This error can be decreased if one uses a test-object with less distance between the nodes.



a



b

Fig. 6. Slope field $q_x(\mathbf{r})$ of a water surface (λ = 40 cm): (*a*) reconstructed, (*b*) theoretical.

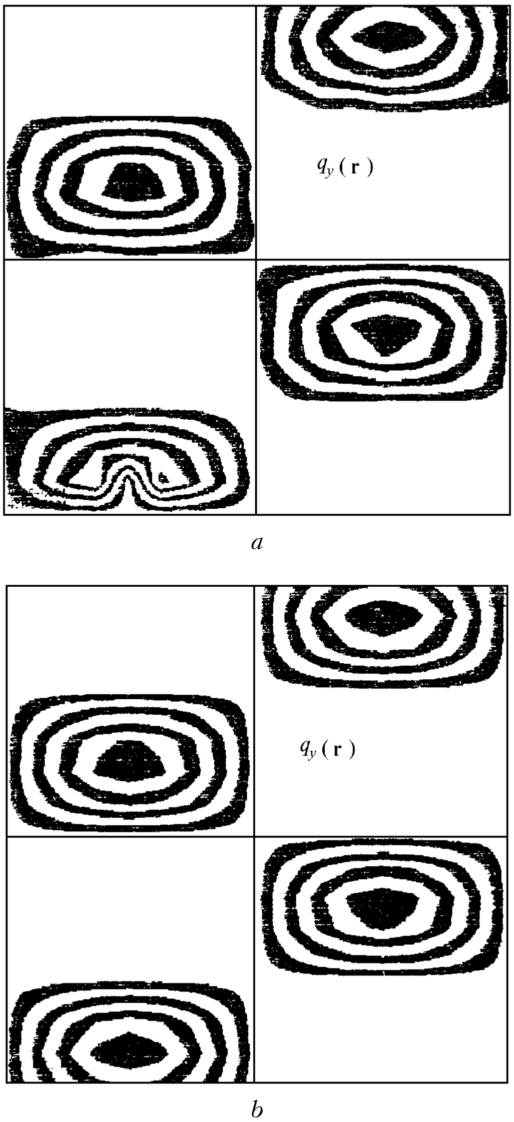


Fig. 7. Slope field $q_y(\mathbf{r})$ of a water surface (λ = 40 cm): (*a*) reconstructed, (*b*) theoretical.

In conclusion we stress again that the proposed method of reconstructing the spatial distribution of slopes of a rough water surface works well only under the condition of sufficiently "smooth" and "continuous" distortions of the image of the test-object. Only in this case is the topological character of the image equivalent to that of the object, and the main requirement on the possibility of solving the inverse problem is that an exact one-to-one correspondence of the nodes in the image and on the object be rigorously obeyed. This circumstance limits the region of applicability of the method to laboratory conditions. Note, however, that in the case when one point on the object corresponds to several points in the image (see Fig. 3), the inverse problem can still be solved, but this would require the quite tedious work of "identifying" the nodes in the image. This work can be done by a human. It could be automated, but this would also be a difficult task.

The optical method proposed here for recording wave phenomena on a water surface was used as an experimental check of a theoretical model of the transformation of surface waves on a two-dimensional inhomogeneous flow. The experiments were carried out in a large hydrophysical basin of the Institute of Applied Physics, RAS, with length 20 m, width 4 m, and depth 2 m. As the test-object we used a panel with dimensions 1.5×1.5 m with a figure imprinted on it consisting of a system of stripes with a spacing of 5 cm. The depth of the object was 1.64 m. The optical receiver was based on a CCD television camera located at a height of 0.67 m. Deciphering of the coordinates of the nodes in the image and calculation of the values of the slope field of the water surface were performed on a personal computer using formulas (8) and (9). The experiments, whose results are reported in Ref. 7, demonstrate the practical operability of the proposed method of reconstructing the slope field of a rough water surface.

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