INFLUENCE OF THE CYLINDER SHAPED ICE PARTICLES ORIENTATION ON THE EXTINCTION MATRIX

B.V. Kaul' and D.N. Romashov

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk
Received June 10, 1997

In this paper we analyze the propagation of polarized and unpolarized light through an optically anisotropic medium. In this study we have investigated the polarization state and energy losses of the beam passed through such a medium as a function of the incident beam polarization, the direction of the beam incidence with respect to the axes describing the medium anisotropy as well as of the optical depth of the medium along the beam. To achieve this task we have carried out model calculations of the extinction matrix elements for anisotropic media of two types . The first type assumes the medium to be a layer of cylinder shaped crystals whose long axes are randomly oriented in a horizontal plane, while in the medium of the second type the long axes of cylinders are additionally grouped about a preferred direction in the horizontal plane. The model calculations made have shown that at optical depths that are realistic for actual cirrus clouds the changes in the polarization state of the beam passed through such a cloud may surely be neglected. At the same time the calculations have revealed a strong dependence of the off-diagonal elements of the extinction matrix on the incidence angle of the beam for both types of anisotropic media and on the azimuth orientation of the plane of incidence for the medium of the second type.

1. INTRODUCTION

In natural crystal clouds particles very often take certain orientation. Among the mechanisms of such an orientation there is the action of aerodynamic forces when particles fall down under the gravity force, of course, the action of other forces is also possible. The crystal cloud particles are anisometric, that means that they have different dimensions along different axes and, correspondingly they have different values of polarizability along some axes. The orientation of particles according to a certain order naturally gives rise to anisotropy of the optical properties of the ensemble of such particles as a whole. The anisotropy of optical properties of the ensemble manifests itself in certain peculiarities of light scattering by such an ensemble. These peculiarities are often observed for sun light propagated through such media in the form of sun pillars, halo, mock sun, and other optical phenomena. Explanations of these phenomena have been given in many publications a review of which may be found in Ref. 1. In recent years there is observed an increased interest in studies of light scattering by individual crystal particles and ensembles of such particles as well.[2–4]

The primary goal of this paper is to try to understand the transformations of the initial, not scattered, radiation that may occur during its propagation through the anisotropic media. The matter is that the energy losses of a beam propagated through an anisotropic medium should depend on the direction of the beam incidence on the medium, with respect to the axes characteristic of the medium anisotropy, and, in the general case on the polarization state of the incident beam. Moreover, the polarization state of the beam propagated through the medium may depend on the optical depth of the propagation path within the medium. Below, after a brief overview of the theoretical results concerning this problem, we shall estimate their importance for light scattering in crystal clouds by calculating the extinction matrices for the ensembles of ice cylinders.

2. EXTINCTION MATRIX AND THE GENERALIZED BOUGER LAW

The components, E_1 and E_2, of the electric field of an electromagnetic wave propagated through a layer dz of a scattering medium are related to the corresponding quantities E_1^0 and E_2^0 of the wave incident onto the medium layer by the following expression[5]

0235-6880/97/12 931-06 $02.00

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \left(\mathbf{I} - 2\pi\, N\, k^{-2} \mathrm{d}z \begin{pmatrix} \tilde{A}_{11}(0) & \tilde{A}_{12}(0) \\ \tilde{A}_{21}(0) & \tilde{A}_{22}(0) \end{pmatrix} \right) \begin{pmatrix} E_1^0 \\ E_2^0 \end{pmatrix}, \qquad (1)$$

where $\mathbf{I}$ is the unit matrix of 2 by 2 dimension; N is the number of particles in a unit volume; k is the wave number of radiation that propagates along the z direction (the direction $\mathbf{e}_3$, such that, and $\mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3$, $\mathbf{k} = k\mathbf{e}_3$); $\tilde{A}_{ij}(0)$) are the elements of matrix of the forward scattering amplitudes averaged over the particles ensemble. According to Ref. 5 this equation well describes such optical phenomena as birefringence, rotation of the polarization plane, linear and circular dichroism.

It may be shown that if we describe the electromagnetic field in terms of the Stokes vector the relationship similar to the Eq. (1) takes the following form

$$\mathbf{S}(0) = (\mathbf{I} - \boldsymbol{\varepsilon}(z)\, \mathrm{d}z)\mathbf{S}_0\,, \qquad (2)$$

where $\mathbf{S}(0)$, $\mathbf{S}_0$ are the Stokes vectors of the radiation passed through the layer dz and of the incident radiation, respectively; $\mathbf{I}$ is the unit matrix of 4 by 4 dimension; $\boldsymbol{\varepsilon}(z)$ is the extinction matrix of the medium.

If the Stokes parameters are defined as

$$\begin{aligned} I &= E_1E_1^* + E_2E_2^*; \quad Q = E_1E_1^* - E_2E_2^*; \\ U &= E_1E_2^* + E_2E_1^*; \quad V = -i\,(E_1E_2^* - E_2E_1^*), \end{aligned} \qquad (3)$$

then the extinction matrix takes the following form[6]

$$\boldsymbol{\varepsilon} = \frac{2\pi\, N}{k^2} \begin{pmatrix} \mathrm{Im}(\tilde{A}_{11} + \tilde{A}_{22}) & \mathrm{Im}(\tilde{A}_{11} - \tilde{A}_{22}) & \mathrm{Im}(\tilde{A}_{12} + \tilde{A}_{21}) & \mathrm{Re}(\tilde{A}_{12} - \tilde{A}_{21}) \\ \mathrm{Im}(\tilde{A}_{11} - \tilde{A}_{22}) & \mathrm{Im}(\tilde{A}_{11} + \tilde{A}_{22}) & \mathrm{Im}(\tilde{A}_{12} - \tilde{A}_{21}) & \mathrm{Re}(\tilde{A}_{12} + \tilde{A}_{21}) \\ \mathrm{Im}(\tilde{A}_{12} + \tilde{A}_{21}) & \mathrm{Im}(\tilde{A}_{21} - \tilde{A}_{12}) & \mathrm{Im}(\tilde{A}_{11} + \tilde{A}_{22}) & \mathrm{Re}(\tilde{A}_{22} - \tilde{A}_{11}) \\ \mathrm{Re}(\tilde{A}_{12} - \tilde{A}_{21}) & -\mathrm{Re}(\tilde{A}_{12} + \tilde{A}_{21}) & \mathrm{Re}(\tilde{A}_{11} - \tilde{A}_{22}) & \mathrm{Im}(\tilde{A}_{11} + \tilde{A}_{22}) \end{pmatrix}. \qquad (4)$$

Here the values $\tilde{A}_{ij}$ are, as in the above formulas, the amplitudes of the forward light scattering averaged over the particle ensemble. These complex values $\tilde{A}_{ij}$ are the functions of the particulate matter refractive index, particle shape, and particle orientation. In many practical cases, when no anisotropy takes place in the refractive index and particles are randomly oriented, the operation of averaging over the particle ensemble reduces the extinction matrix to the product of a scalar value (the extinction coefficient) by the unit matrix. However, if there is a macroscopic anisotropy of a medium, for instance orientation of anisometric particles along some preferred direction, the elements of the extinction matrix should necessarily depend on the direction along which the incident radiation propagates through the medium. The transformations, that radiation experiences in an elementary volume, are described by Eq. (2), while the propagation of radiation through the medium is described by the radiation transfer equation[6,7]

$$(\mathbf{k}\nabla)S_i(\mathbf{r},\, \boldsymbol{\Omega}) = -\sum_{j=1}^{4} \alpha_{ij}\, S_j(\mathbf{r},\, \boldsymbol{\Omega}) + $$
$$+ \sum_{j=1}^{4} \int_{4\pi} \mathrm{d}\Omega'\, D_{ij}(\boldsymbol{\Omega},\, \boldsymbol{\Omega}')\, S_j(\mathbf{r},\, \boldsymbol{\Omega}')\,, \qquad (5)$$

where $S_i(\mathbf{r}, \boldsymbol{\Omega})$ is the Stokes parameter of radiation at the point $\mathbf{r}$; the radiation with the wave number k propagates along the direction Ω; $\mathbf{k}\nabla$ is the operator that has the meaning of the derivative along the direction $\mathbf{k}$; α_{ij} are the elements of the matrix $\boldsymbol{\alpha} = k\boldsymbol{\varepsilon}$; $D_{ij}(\boldsymbol{\Omega}, \boldsymbol{\Omega}')$ are the elements of the scattering phase matrix that may easily be expressed in terms of the scattering amplitudes $A_{ij}(\boldsymbol{\Omega},\boldsymbol{\Omega}')$ for the scattering process from the direction $\boldsymbol{\Omega}'$ to $\boldsymbol{\Omega}$. If the task to be achieved is the estimation of only coherent component of the radiation propagated through a medium within a small solid angle d$\boldsymbol{\Omega}$, for instance, to identify the transformations of a laser radiation at its propagation in an anisotropic medium, then one surely may neglect the second term in equation (5). Note, that in that case only incoherent fraction of multiply scattered radiation is omitted. As to the forward scattered light it is coherent and it is just the interference of the direct and forward scattered wave that determines the transformations of the incident radiation summed up in the transmitted wave. Let the radiation be propagating along the z direction. By assuming that $\mathbf{k}\nabla = k\dfrac{\mathrm{d}}{\mathrm{d}z}$, $\mathbf{S}(z = 0) = \mathbf{S}_0$ and omitting the second term in Eq. (5) we may write the latter in the following form

$$\frac{\mathrm{d}\mathbf{S}(z)}{\mathrm{d}z} = -\,\boldsymbol{\varepsilon}(z)\,\mathbf{S}(z)\,. \qquad (6)$$

This equation is a vector-matrix form of a system of ordinary differential equations of the first order for functions $I(z)$, $Q(z)$, $U(z)$, and $V(z)$. A solution to this system may be sought as an iteration series[8]

$$\mathbf{S}(z) = \left\{ \mathbf{I} - \int_{z_0}^{z} \boldsymbol{\varepsilon}(z_1)\mathrm{d}z_1 + \int_{z_0}^{z} \boldsymbol{\varepsilon}(z_1)\mathrm{d}z_1 \int_{z_0}^{z_1} \boldsymbol{\varepsilon}(z_2)\mathrm{d}z_2 - \right.$$
$$\left. - \int_{z_0}^{z} \boldsymbol{\varepsilon}(z_1)\mathrm{d}z_1 \int_{z_0}^{z_1} \boldsymbol{\varepsilon}(z_2)\mathrm{d}z_2 \int_{z_0}^{z_2} \boldsymbol{\varepsilon}(z_3)\mathrm{d}z_3 \right\} \mathbf{S}_0\,. \qquad (7)$$

One can easily see that series (7) coincides, at $\boldsymbol{\varepsilon}$ independent of z, with the definition of exponential function whose argument is a matrix

$$\exp(-\mathbf{A}) = \sum_{n=0}^{\infty} \frac{(-\mathbf{A})^n}{n!} , \tag{8}$$

where $\mathbf{A}$ is, in this particular case, the matrix $\mathbf{A} = (z - z_0)\boldsymbol{\varepsilon}$.

Taking this comment into account a solution to the above equation may be written in the following form:

$$\mathbf{S}(z) = \exp \{-\boldsymbol{\varepsilon}(z - z_0)\}\mathbf{S}_0 . \tag{9}$$

Formula (9), in its form, coincides with the Bouger law and thus it may be considered as a generalization of the latter to the case of light propagation through a homogeneous anisotropic medium. In the case of an isotropic medium the extinction matrix reduces to the product of a unit matrix by a scalar value, and the equation (9) takes the form of usual Bouger law written for each of the Stokes parameters.

In an optically thin anisotropic medium, that is when the condition

$$\varepsilon_{ii}(z - z_0) \ll 1 , \tag{10}$$

holds one may take only the first term in the expansion (8) and write the equation for Stokes vector as follows:

$$\mathbf{S}(z) = \left[\mathbf{I} - (z - z_0)\,\boldsymbol{\varepsilon}\right] \mathbf{S_0} . \tag{11}$$

Equation (11) coincides, in its form, with the equation (2) that describes transformations of radiation in an elementary volume. In the general case of a homogeneous medium it is advisable to use the expansion of the exponent (9) over its argument powers, while in the case of an inhomogeneous medium one should calculate the iteration integral (7).

3. NUMERICAL ESTIMATIONS FOR DIFFERENT MODEL MEDIA

In order to estimate the effect of cloud particle orientation on the extinction of incident radiation, as well as on the polarization state of radiation propagated through the cloud we have calculated extinction matrices for the ensembles of round ice cylinders. We have performed the calculations using lognormal type of particle size-distribution function at three values of modal radii of the distribution ($r_{\mathrm{m}} = 1$; 10; and 125 μm). In all the three cases the distribution parameter σ was taken to be 0.15, the cylinder length was $l = 4a$, where a is the cylinder radius. The incident radiation was taken to have the wavelength $\lambda = 0.53$ μm and the complex refractive index of ice was taken to be $m = 1.3 - i2.5{\cdot}10^{-9}$. Birefringence of the ice was not taken into account in these calculations.

In this study we have considered two types of particle orientation. In the one the long axes of cylinders were taken to be randomly oriented in a horizontal plane, other parameters used in calculations being as mentioned above. In the second case the axes were taken to be grouped about some preferred direction in a horizontal plane. The parameters used in calculations in the latter case are as follows. The modal radius of cylinders $a_{\mathrm{m}} = 1$ μm, $\sigma = 0.5$, $l = 9a$, $\lambda = 1.06$ μm, and $m = 1.299 - i2{\cdot}10^{-4}$. The method of calculating the amplitudes of light scattering by an individual particle and their averaging over an ensemble may be found in our earlier publications, see for instance, Ref. 9. The number concentration, N, of particles was taken to be 10^3 per liter, in all the cases considered. As an example, Figs. 1 and 2 show the extinction matrix elements calculated for ensembles of particles randomly oriented in a horizontal plane. The polar angles γ and φ, describe the position of the normal to the plane that involves, in the coordinate system $\mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3 = \mathbf{k}/k$, where the long axes of cylinders lay. In other words these angles determine the position of the ensemble symmetry axis with respect to the polarization basis in which the Stokes parameters of the incident and passed radiation are being determined.

The monotonic fall off of the element ε_{11} that may be seen in Fig. 1*b* with the increasing incidence angle γ has quite a simple physical meaning. The element ε_{11} as the meaning of the extinction coefficient of the medium for unpolarized light. Light extinction on nonabsorbing particles occurs due to scattering. If particles are large a significant contribution appears from the diffraction part of the scattering phase function. This contribution is determined by the area of the particle cross section onto the plane perpendicular to the direction of incident light propagation. The area of this projection decreases with increasing angle of incidence thus resulting in the fall off of the extinction coefficient of such a medium. In the case of smaller particles the diffraction plays a less significant role in the light scattering process, and there may occur resonance in the dependence of ε_{11} on γ. One can see this situation with resonance in Fig. 1*a* in the vicinity of the angle $\gamma = 45°$. No dependence on the angle φ is observed because the natural light is unpolarized.

The lengths of the long cylinder axis projection onto the plane perpendicular to the light wave vector are different for cylinders that are parallel to the incidence plane and perpendicular to it. As a result, the projections of cylinder long axes form an ellipse compressed along the symmetry axis of the particle ensemble. This, in its turn, makes different conditions for propagation of light beams polarized in the incidence plane and perpendicular to it. From the standpoint of the description in terms of the extinction matrix there appears a dependence of the propagation conditions on how is the polarization basis, in which the Stokes parameters are determined, oriented with respect to this ellipse. In the particular case we consider here this will result in a dependence of the off-diagonal elements of the extinction matrix on the angle φ.

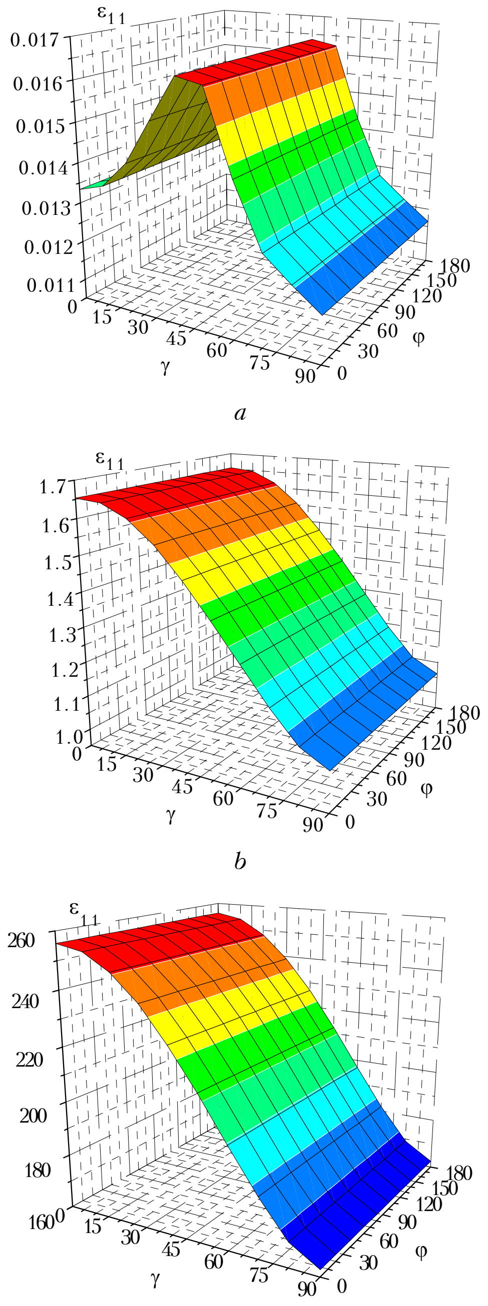

FIG. 1. Dependence of the extinction matrix element, ε_{11} (km^{-1})*, of the ensemble of ice cylinders randomly oriented in a horizontal plane on the polar,* γ*, and azimuth,* φ*, angles of the position of symmetry axis of the ensemble with respect to the polarization basis where the Stokes vectors of incident and passed radiation are determined. Calculations were made for ensembles with the modal radius* $r_{\rm m}$ *of* 1 μm *(a);* 10 μm *(b); and* 125 μm *(c).*

Figure 2 shows the element ε_{12} calculated for these same ensembles as in the case presented in Fig. 1. Other elements of the matrix have also been calculated in our study, but their graphical presentation in this same form would take a lot of place while hardly adding much new information. For that reason we shall present below the view of an extinction matrix in one, specially chosen, coordinate system and show using it some properties that do not depend on the choice of a coordinate system.

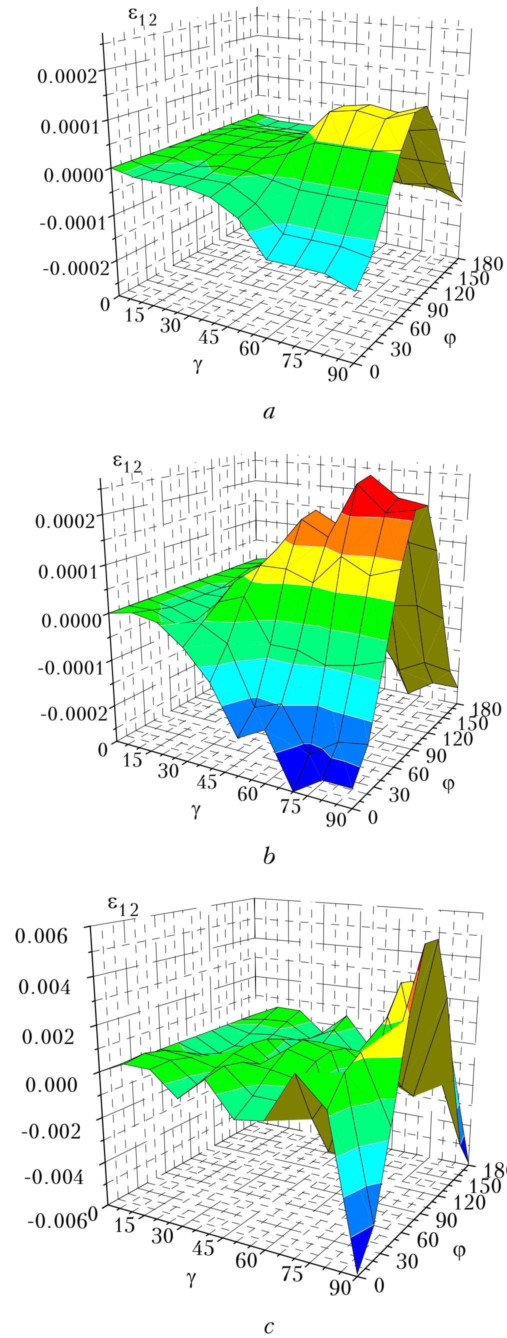

FIG. 2. Dependence of the extinction matrix element, ε_{12} (km^{-1})*, analogous to those in Fig. 1 and for these same ensembles as in Figure 1.*

Let us define the polarization basis so that the vector $\mathbf{e}_1$ be in the incidence plane. That corresponds to the case of $\varphi = 0$. From Figure 2 one may see that the section by the plane $\varphi = 0$ enables obtaining the view of the dependence ε_{12} $(\gamma, \varphi = 0)$ from which it is seen that at small γ values this element has the magnitude close to zero while taking negative values at increasing γ. Let us write this tendency using the following expression $\varepsilon_{12} = -|\varepsilon_{12}|$. If we assume this rule to be valid for other elements of the matrix, we may write the view of the extinction matrix for ensembles with all the three size-distribution functions used in calculations

$$\boldsymbol{\varepsilon} = \varepsilon_{11} \begin{pmatrix} 1 & -|\bar{\varepsilon}_{12}| & 0 & 0 \\ -|\bar{\varepsilon}_{12}| & 1 & 0 & 0 \\ 0 & 0 & 1 & |\bar{\varepsilon}_{34}| \\ 0 & 0 & -|\bar{\varepsilon}_{34}| & 1 \end{pmatrix}, \qquad (12)$$

where $\bar{\varepsilon}_{ij} = \varepsilon_{ij}/\varepsilon_{11}$, $\varepsilon_{11} > 0$.

Let us now determine, by substituting this matrix into Eq. (11), the polarization state of radiation that has propagated through the path $z - z_0$ in the layer, the polarization state of light incident on the layer being described by the Stokes vector $\mathbf{S}_0$.

Let the incident beam be unpolarized, that is

$$\mathbf{S}_0 = I_0(1,0,0,0)^{\mathrm{T}}, \qquad (13)$$

where the superscript T denotes the operation of transposition.

By substituting Eqs. (12) and (13) into the expression (11) we obtain

$$\begin{aligned} I(z) &= \left[1 - (z - z_0)\,\varepsilon_{11}\right] I_0 \\ Q(z) &= (z - z_0)\,\varepsilon_{11} \left|\varepsilon_{12}\right| I_0, \quad U = V = 0. \end{aligned} \qquad (14)$$

Since $Q(z) > 0$, the result obtained means that the radiation propagated through the $z - z_0$ layer takes partial polarization with the polarization plane being the plane of incidence.

If the radiation incident on the layer is linearly polarized with the polarization plane being the plane of incidence, we have

$$\mathbf{S}_0 = I_0(1,1,0,0)^{\mathrm{T}}.$$

By substituting this expression into the Eq. (11) we obtain

$$\begin{aligned} I(z) &= \left[1 - (z - z_0)\,\varepsilon_{11}(1 - \left|\varepsilon_{12}\right|)\right] I_0 \\ Q(z) &= \left[1 - (z - z_0)\,\varepsilon_{11}(1 - \left|\varepsilon_{12}\right|)\right] I_0, \quad U = V = 0. \end{aligned} \qquad (15)$$

From this one can see that the second normalized Stokes parameter $Q(z)/I(z) = 1$ did not change. That, in its turn, means that the radiation propagated through the layer $z - z_0$ has the same polarization as the incident one.

By comparing the values $I(z)$ in expressions (14) and (15) one may readily come to a conclusion that linearly polarized light with the polarization plane being the plane of incidence undergoes weaker attenuation in such a medium as compared to the attenuation experienced by natural (unpolarized) light. One may show, in a similar way, that linearly polarized light with the polarization plane being perpendicular to the plane of incidence undergoes stronger extinction as compared to that experienced by natural light, while the radiation passed through the layer $z - z_0$ has the same polarization state as the incident beam. In the cases when incident light is polarized linearly at ±45° angles with respect to the incidence plane the radiation propagated through the layer takes the clockwise and counterclockwise ellipticity in its polarization state. If the incident light is circularly polarized the propagated radiation is elliptically polarized.

The absolute values of the off-diagonal elements of the extinction matrix make only small fractions of per cent from the values of the diagonal elements of this matrix and only in an extreme case of the exact orientation of cylinders that have size in the resonance region ($r_{\mathrm{m}} \approx \lambda$) their values can reach approximately 10% of the diagonal elements' magnitude. Certain ideas on how the degree of the polarization state transformation due to propagation through the anisotropic media depends on the optical depth τ of the media may by gotten from the curves depicted in Fig. 3. This figure illustrates the dependence on τ of the parameter Q or the degree of polarization, what is the same in this case, that appears in the radiation propagated through the layer of cylinder particles having two orientations when natural light is incident on it at a slant angle. As one can see from this figure, the degree of polarization could reach only few per cent at the optical depth characteristic of real cirrus clouds even in the extreme case when all particles of the ensemble are oriented exactly along a preferred direction. Furthermore, if we take into account the facts that only partial orientation of particles normally occurs in cirrus clouds and the particle size far exceeds the wavelength (see Fig. 2*c*) then we may surely neglect the transformations of the polarization state of the radiation propagated through the layer. However, this conclusion may be revised if calculations are made that allow for the birefringence of ice, which is quite significant. The account for this factor is yet to be done while from the results already obtained and discussed in this paper it follows that the anisotropy of a scattering medium due to particle orientation only manifests itself in a strong dependence of the extinction matrix diagonal elements on the incidence angle γ for the ensembles of the first kind and, in addition, on the angle between the direction of preferred particle orientation and the incidence plane, as is seen from Fig. 4.

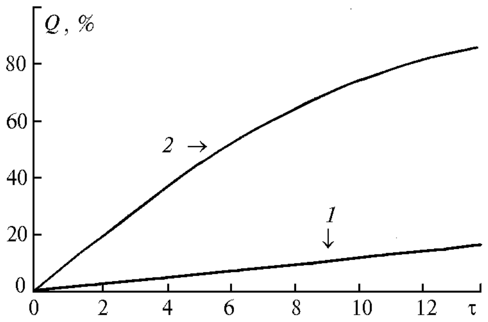

FIG. 3. The degree of linear polarization that appears in natural light propagated through a layer of cylinder particles, when incident at a slant angle, as a function of the layer optical depth τ in the case when cylinders are randomly oriented in a horizontal plane (curve 1) and oriented exactly along the direction that lays in the incidence plane and this horizontal plane simultaneously (curve 2).

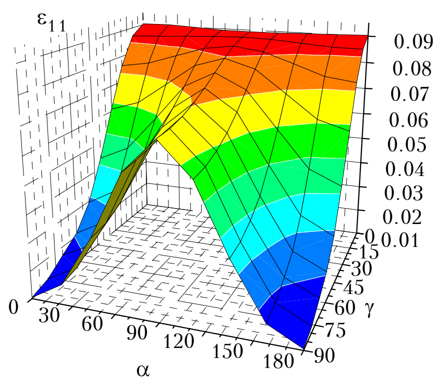

FIG. 4. Dependence of the extinction matrix element, ε_{11} (km^{-1})*, of the ensemble of ice cylinders oriented along a preferred direction in a horizontal plane on the incidence angle γ and angle α, in the horizontal plane, between the direction of preferred orientation of the particle axes and the plane of incidence.*

In the latter case and if the cylinder axes are oriented along a preferred direction that lies in the incidence plane the extinction coefficient may vary within an order of magnitude at the variation of the incidence angle from 0 to 90 degrees. This is primarily caused by particle orientation and elongated shape of columns. These factors may manifest themselves, for instance, in the dependence of the extinction coefficient of such a medium for solar radiation on the azimuth and elevation angles of the Sun. This may happen already at the degree of particle orientation that have been found experimentally.[10] Thus we may state that the account for angular behavior of the extinction coefficient should make the calculations of the transmitted and scattered radiation, as well as of conditions for laser radiation propagation through crystal clouds more accurate.

REFERENCES

1. O.P. Volkovitskii, L.P. Pavlov, and A.G. Petrushin, *Optical Properties of Crystal Clouds* (Gidrometeoizdat, Leningrad, 1984), 198 pp.
2. Y. Takano and K. Jayaweera, Appl. Opt. **24**, No. 19, 3254–3263 (1985).
3. K.D. Rokcwitz, Appl. Opt. **28**, No. 19, 4103–4110 (1989).
4. Y. Takano and K.N. Liou, J. Atm. Sci. **46**, No. 1, 3–19 (1989).
5. H.C. Van de Hulst, *Light Scattering by Small Particles* (Wiley, New York, reprinted by Dover: New York, 1981).
6. A.P. Prishivalko, Yu.N. Babenko, and V.N. Kuz'min, *Light Absorption and Scattering by Inhomogeneous and Anisotropic Spherical Particles* (Nauka i Tekhnika, Minsk, 1984), 263 pp.
7. A.Z. Dolginov, Yu.N. Gnedin, and N.A. Silantiev, *Propagation and Polarization of Radiation in Space* (Nauka, Moscow, 1979), 423 pp.
8. A.D. Myshkis, *Mathematics. Special Course* (Nauka, Moscow, 1971), 632 pp.
9. D.N. Romashov, Atmos. Oceanic. Opt. **10**, No. 1, 4–10 (1997).
10. B.V. Kaul', O.A. Krasnov, A.L. Kuznetsov, E.P. Polovtseva, I.V. Samokhvalov, and A.P. Stykon, Atmos. Oceanic. Opt. **10**, No. 2, 119–125 (1997).