COHERENCE RADIUS OF OPTICAL WAVE AT SLANT PATHS IN THE TURBULENT ATMOSPHERE

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The results of numerical calculations of coherence radius of plane and spherical waves propagating along slant paths in the turbulent atmosphere are presented for the model by M.E. Gracheva and A.S. Gurvich described the vertical dependence of structural parameter of atmospheric turbulence. In the case of spherical wave the optical radiation distribution in the atmosphere from the top downwards and from the bottom upwards and in the case of plane wave – from the top downwards are treated in the paper.

A coherence radius is the fundamental parameter of optical wave propagating through the randomly inhomogeneous medium.[1] All the main characteristics of optical systems operating in the atmosphere, for example, an integral resolution of telescope[2] depend on the coherence radius. The determining role of this parameter is shown in Ref. 3 for analyzing an efficiency of adaptive correction of images of objects observed through the turbulent atmosphere intended to improve the telescope image quality. There also the important role of the coherence radius as an image characteristic is noted in the general concept of "astroclimate". In particular, for the adaptive telescopes with correction of random slopes of wave front the coherence radius determines the size of isoplanicity area.[3] In this paper the results of calculations of optical wave coherence radii at the slant paths in the turbulent atmosphere are presented.

It is known[1] that when analyzing the formation of image of incoherent source observed through the randomly inhomogeneous medium the concept of optical transfer function (OTF) is introduced. The OTF is Fourier transform of intensity distribution over the space of optical system image which is created by the point source being in the space of object location. It was shown in Ref. 2 that the optical transfer function of the turbulent atmosphere $M(\mathbf{p})$ coincides with the second–order mutual coherence function of the optical field for "very long" exposures, i.e.,

$$M(\mathbf{p}) = \Gamma_2(x, \rho + \mathbf{p}) , \quad (1)$$

where $\Gamma_2(x, \rho+ \mathbf{p}) = \langle U(x, \rho_1)U^*(x, \rho_2)\rangle$ is the second–order mutual coherence function for the optical field $U(x, \rho)$ at the points of $\{x, \rho_1\}$ and $\{x, \rho_2\}$; $\mathbf{p}$ is the space scale; x is the longitudinal coordinate; and, $\rho = \{y, z\}$ is the transverse coordinate relatively to the direction of optical wave propagation.

The optical wave is spherical if it is emitted by the point source being inside the layer of the randomly inhomogeneous medium or near it. In that case when the point source is removed at the long distance from the layer of random inhomogeneities (for example, an observation of star by the ground–based telescope), the wave under study can be considered as the plane wave. That is why the calculations were carried out for two cases of limiting types of waves: plane and spherical.

According to Refs. 4 and 5 for the atmospheric turbulence with the Kolmogorov spectrum of air refractive index fluctuations when satisfying the condition $l_0 < p < L_0$ (where l_0 and L_0 are the inner and outer scale of atmospheric turbulence, respectively), the optical transfer function of randomly inhomogeneous atmosphere (1) can be represented in the form

$$M(\mathbf{p}) = M(0) \exp\left\{ -\left(\frac{p}{\rho_c}\right)^{5/3}\right\} , \quad (2)$$

where $M(0)$ is the optical transfer function of turbulent atmosphere for $p = 0$; $\rho_c = (1.45\, k^2 C_n^2 L)^{-3/5}$ is the coherence radius of optical wave in the turbulent atmosphere; $k = 2\pi/\lambda$, λ is the optical wavelength in vacuum; L is the propagation path length in the layer of the randomly inhomogeneous medium; $C_n^2 = \frac{1}{L}\int\limits_0^L dx\, C_n^2(h(x))\, \phi(x)$ is the effective value of structural parameter of atmospheric turbulence; $C_n^2(h(x))$ is the altitude profile of structural characteristic of atmospheric refractive index; $h(x) = [x^2 + (R_3 + h_0)^2 + 2x(R_E + h_0)\cos\theta]^{1/2} - R_3$ is the height of running point of propagation path[5]; R_E is the Earth's radius; θ is the zenith angle, $\theta \in [-\pi/2, \pi/2]$; h_0 is the minimum height of propagation path above the underlying surface; $\phi(x)$ is the spatial filtering function which determines the relative contributions of different sections of propagation path to the effective value of atmospheric structural parameter. For the plane wave $\phi(x) = 1$. In the case of spherical wave if the source is below the observation point $\phi(x) = (x/L)^{5/3}$ and if the source is above this point $\phi(x) = (1 - x/L)^{5/3}$. The altitude profile of the structural characteristic of air refractive index in the atmosphere is based on the model proposed in Ref. 6. This model stipulates two limiting dependencies:

(1) for the minimum level of turbulence (the "best" conditions)

$$\lg\,(C_n^2(h) - 5.19\cdot 10^{-16-0.00086h}) = -18.34 + 0.29\cdot 10^{-3}\, h - 2.84\cdot 10^{-8}\, h^2 + 7.43\cdot 10^{-13}\, h^2 ; \quad (3)$$

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(2) for the maximum level of turbulence (the "worst" conditions)

$$\lg \left(C_n^2(h) - 9.5 \cdot 10^{-14-0.00209h}\right) = -14.39 + 0.17 \cdot 10^{-3}\, h - $$

$$- 3.48 \cdot 10^{-8}\, h^2 + 9.59 \cdot 10^{-13}\, h^3 ; \qquad (4)$$

when $h \in [10\text{ m}, 20\text{ km}]$. The height h is in units of meters in Eqs. (3) and (4), therefore $C_n^2(h)$ is in units of $m^{-2/3}$. Formulas (3) and (4) correspond to the model of altitude profile of structural characteristic of air refractive index for the wavelength $\lambda = 5 \cdot 10^{-7}$ m. For the arbitrary optical wavelength it is recommended[6] to use the estimation

$$C_n^2(h, 1) = a(\lambda)\, C_n^2(h, \lambda = 5 \cdot 10^{-7}\text{ m}) , \qquad (5)$$

where $a(\lambda)$ is the multiplicative factor allowing for the spectral dependence of C_n^2 for the whole profile (the dependence of $a(\lambda)$ on h is assumed to be a less order of smallness). In the spectral interval, which has no the resonant frequencies, the Cauchy formula can be used within the good accuracy, i.e., one can consider that

$$a(\lambda) = N^2(\lambda) \,/\, N^2(\lambda = 5 \cdot 10^{-7}\text{ m}) , \qquad (6)$$

where $N(\lambda) = A\, [1 + B(\lambda_0/\lambda)^2]$; $A = 273 \cdot 10^{-6}$; $B = 3 \cdot 10^{10}$; $\lambda_0 = 5 \cdot 10^{-7}$ m.

Thus, to calculate the coherence radius of optical wave at the slant paths it is necessary to use Eqs. (2) – (6). An analysis of these formulas shows that the coherence radius of optical wave ρ_c depends on initial geometry of the wave, zenith angle (θ), path length (L), propagation path orientation, level of turbulence, and parameters h_0, λ, and R_E. In accordance with the condition of the problem, as was above–discussed, the plane wave is assumed to be propagated from the top downwards, and the spherical wave is considered for two variants of path orientation: from the top downwards and from the bottom upwards. The path length for the case of the plane wave equals to length of the entire atmospheric column up to the position of receiver of optical radiation. For the spherical wave the value of L is chosen within the interval from 100 m to 100 km. The zenith angle θ varies from 0° (vertical paths) to 90° (horizontal paths) for the both cases. The only limiting levels of turbulence are considered: minimum and maximum. It is assumed that by this way the boundaries of variation range of values sought (the coherence radius of optical radiation) can be assigned. The minimum height of propagation path h_0 is the height of receiver location when the optical wave (plane or spherical) propagates from the top downwards and the height of source location when the spherical wave propagates from the bottom upwards.

Let us prescribe the values of h_0 from 10 m to 10 km, and perform the calculations at three wavelengths of optical radiation for the spherical wave: $\lambda_1 = 0.5$ (the area of maximum sensitivity of eye), $\lambda_2 = 1.06$, and $\lambda_3 = 10.6$ μm (the wavelengths of most frequently applied IR lasers) and for the plane wave at one wavelength: $\lambda_1 = 0.5$ μm. The mean Earth's radius $R = 6370$ km is taken as R_E.

First let us consider the plane wave propagation through the turbulent atmosphere (this case corresponds to the observation of star by ground–based telescope), in this case from Eq. (2) the following formula for the coherence radius can be derived:

$$\rho_{c,\, p} = \left[1.45\, k^2 \int\limits_0^\infty dx\, C_n^2(h(x))\right]^{-3/5} . \qquad (7)$$

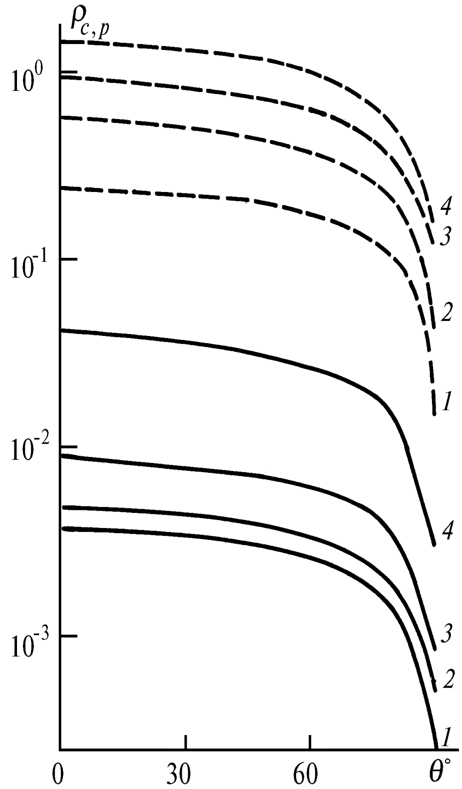

FIG. 1. Plane wave coherence radius in the turbulent atmosphere at the slant path for the different heights of the observation point h_0: h_0 = 10 m (1), 1 km (2), 5 km (3), and 10 km (4). The "worst" propagation conditions (solid curves) and the "best" ones (dashed curves).

Figure 1 shows the results of calculations of the plane wave coherence radius by Eq. (7) as a function of the zenith angle θ for the different heights h_0 of a source above the underlying surface. Here h_0 = 10 m, 1, 5, and 10 km. The results for the "worst" conditions (maximum level of turbulence), are shown by solid curves and for the "best" conditions – by dashed curves (minimum level of turbulence). As is seen from Fig. 1, the dependence of $\rho_{c,\, p}$ on the zenith angle θ is monotonic and the greatest variations in the coherence radius are observed at $\theta > 80°$. The layer of the randomly inhomogeneous medium decreases as the height of receiver position above the ground increases that leads to the increase in the optical wave coherence radius. The plane wave coherence radius increases by an order of magnitude as the height h_0 increases from 10 m to 10 km, i.e., we can say that the principle influence on $\rho_{c,\, p}$ is from the random inhomogeneities being near the Earth's surface in the layer of 3–5 km thickness.

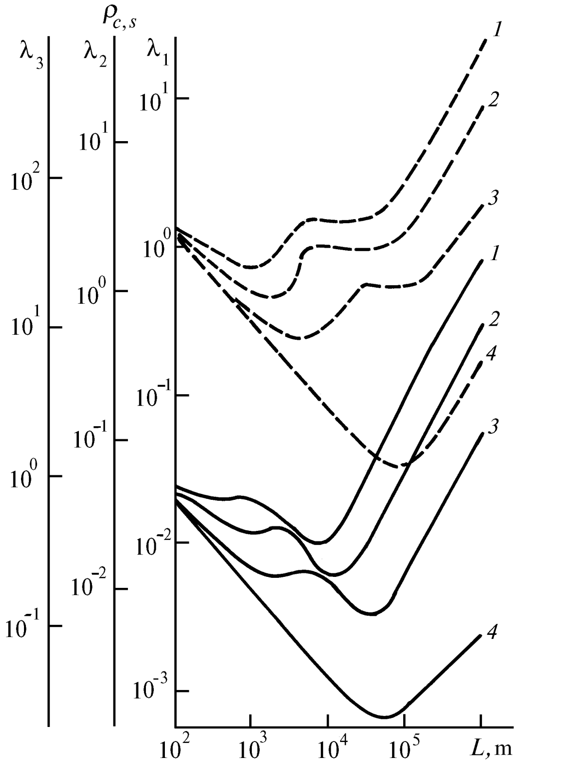

FIG. 2. Spherical wave coherence radius in the turbulent atmosphere at the slant path for the different zenith angles θ: θ = 0° *(1),* 60° *(2),* 80° *(3), and* 90° *(4). The "worst" propagation conditions (solid curves) and the "best" ones (dashed curves). The radiation source is located below the observation point.*

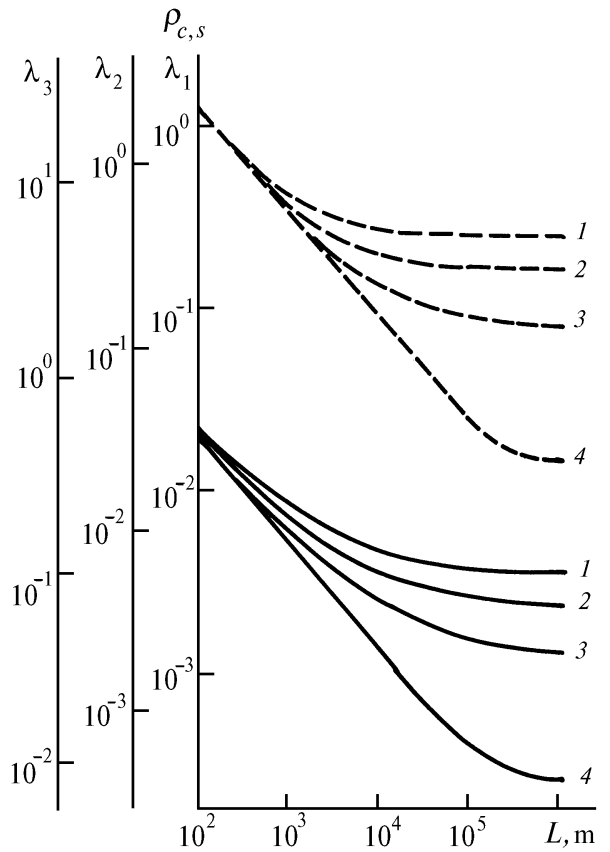

FIG. 3. The same as in Fig. 2 but the radiation source is located above the observation point.

Figures 2–5 present the results of calculations of the coherence radius for the spherical wave propagating through the turbulent atmosphere at the slant paths. The same as for the case of plane wave, the results for the "worst" conditions are shown by solid curves and for the "best" ones – by dashed curves. Three values for the coherence radius $\rho_{c,\,s}$ at $\lambda_1 = 0.5$, $\lambda_2 = 1.06$, and $\lambda_3 = 10.6$ μm, respectively, are given in Figs. 2–5. Figures 2 and 4 correspond to the case when the source is positioned below the observation point, while Figs. 3 and 5 –– above it. The coherence radius $\rho_{c,\,s}$ as a function of propagation path length L is presented in Figs. 2 and 3 for the different zenith angles θ: curves *1–4* are plotted at θ = 0°, 60°, 80°, and 90°, respectively. Here h_0 equals to 10 m, in addition, for the case presented in Fig. 2 h_0 is the height of location of optical radiation source, but in Fig. 3 h_0 is the height of observation point. Figures 4 and 5 show the values of $\rho_{c,\,s}$ at θ = 0° (vertical paths) for the different heights of source location (Fig. 4) or observation point (Fig. 5) relatively to the underlying surface of the Earth: curves *1–4* are for h_0 = 10 m, 1, 5, and 10 km, respectively.

An analysis of the data presented in Fig. 2 shows that at λ = 10.6 μm when the source is located on the Earth's surface, $\rho_{c,\,s}$ equals to about 1–20 m for L = 100 m; 20 cm – 10 m for L = 1 km, and 10 cm – 5 m for L = 5 km. It is easy to note that decreasing in the optical wavelength leads to decreasing in the coherence radius. The values of $\rho_{c,\,s}$ increase as the height of source location above the ground increases (Fig. 4). An analogous effect is observed in the case when the source is located above the observation point (Fig. 5).

For the radiating source positioned high above the ground (Figs. 3 and 5), the spherical wave coherence radius $\rho_{c,\,s}$ depends on the propagation path length L monotonically, $\rho_{c\,,s}$ decreases with increase of L. Moreover, this dependence has a tendency to be saturated. Thus, at θ = 0° (vertical paths) the saturation occurs at $L > 2\cdot10^4$ m, at the same time at θ = 90° (horizontal paths) – at $L \simeq 10^6$ m. In the scheme of a lower height of location of spherical wave source the dependence of $\rho_{c,s}$ on L has one or two minima (Figs. 2 and 4). The global minimum at θ = 0° under the "worst" conditions can be found at $L = 6\cdot10^3$ m while at θ = 90° – at $L = 5\cdot10^4$ m (Fig. 2).

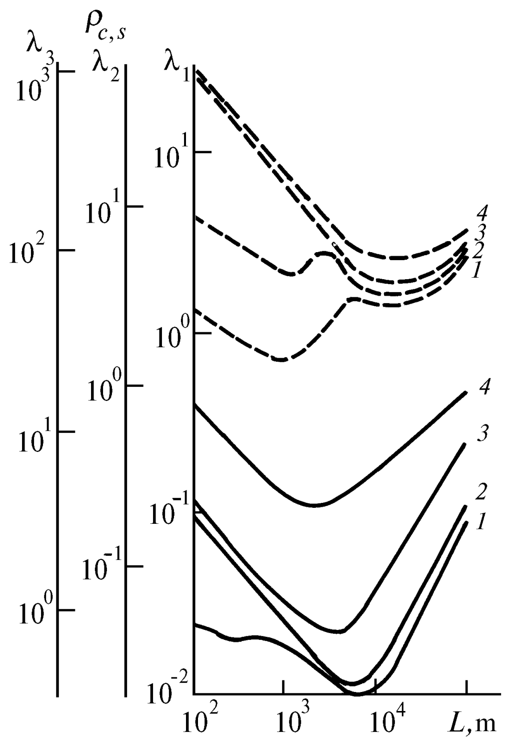

FIG. 4. Spherical wave coherence radius in the turbulent atmosphere at the vertical path ($\theta = 0°$) for the different heights h_0: 10 m *(1),* 1 km *(2),* 5 km *(3), and* 10 km *(4). The "worst" propagation conditions (solid curves) and the "best" ones (dashed curves). The radiation source is located below the observation point. The value h_0 is the height of source location above the underlying surface.*

The presence of this minimum is caused by the following causes: increasing in the propagation path length, on the one hand, leads to decreasing in the coherence radius due to increase in the optical thickness of the turbulent atmospheric column. On the other hand, the presence of filtering function $\phi(x) = (x/L)^{5/3}$ decreases the contribution from ground layers having the maximum turbulence level into the value of the spherical wave coherence radius. The existence of two opposite tendencies leads to that the dependence $\rho_{c,s}$ on L is nonmonotonic. Since the profiles of $C_n^2(h)$ (3) and (4) obey the different laws of decreasing with the height variation, the minima of the curves $\rho_{c,s}(L)$ are for the different values of L. The presence of the second (local) minimum is explained by variations in the rate of $C_n^2(h)$ decreasing with increase of height in models (3) and (4).

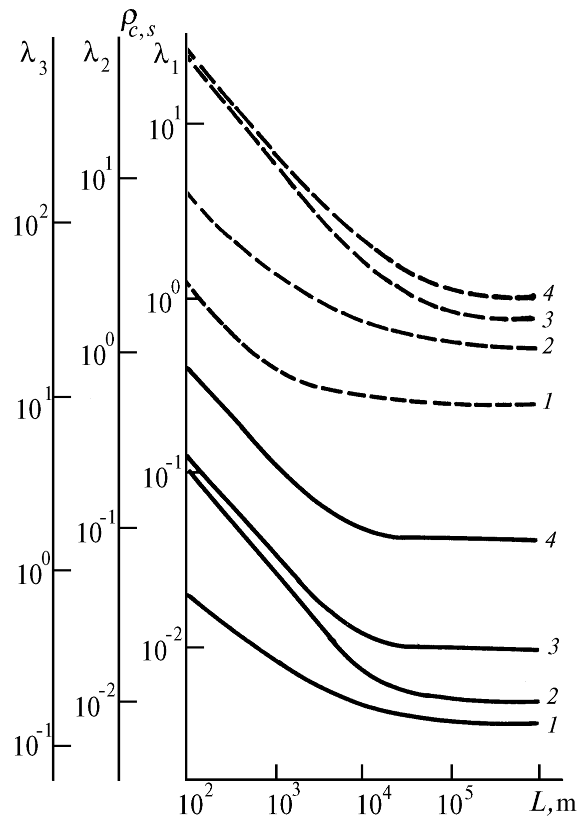

FIG. 5. The same as in Fig. 4 but the radiation source is located above the observation point and h_0 is the observation point height above the underlying surface.

In conclusion we note that the estimation of the optical radiation coherence radius for the arbitrary wavelength can be obtained with the use of formula (6) and the results presented in Figs. 1–5.

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