# FORMATION AND PROPAGATION OF NONDIFFRACTING LASER BEAMS

**A.G. Aristov, L.Ya. Margolin, L.Ya. Polonskii, L.N. Pyatnitskii**

*Institute of High Temperatures,*
*Academy of Sciences of the USSR, Moscow*
*Received August 23, 1989*

*Nondiffracting (Besselian) light beams are formed with the help of conical optics (mirrors, lenses), whose components have been termed axicons. Beam of this class differ substantially from the usually employed Gaussian beams by the fact that the diameter of the axial caustic is constant along the entire propagation path, by the fact that radiation is introduced into the focusing zone from the side at a constant angle, and by the fact that the intensity distribution in the transverse section is given by a Bessel function. The theoretical and experimental data on the structure of Besselian beams are compared and three regimes of optical breakdown for such beams are studied.*

When laser radiation is focused by a conical lens (axicon) a beam with a radial intensity distribution of the form $I \sim J_0^2\,(kr\,\sin\gamma)$, where $J_0$ is the zero order Bessel function of the first kind, $k$ is the wave vector of the focused radiation, and $\gamma$ is the focusing angle (Fig. 1), is formed.[1–3] Since the argument of the function $J_0$ does not contain the coordinate $z$ the form of the cross section of such a beam remains unchanged over the entire length of the propagation path $L \sim R/\gamma$, where $R$ is the aperture of the axicon $R_a$ or the starting beam $R_b$, whichever is smallest. For this reason Besselian beams are said to be nondiffracting. Their characteristic feature is that energy is introduced from the side at a constant angle $\gamma$ to the axis over the entire length of the propagation path. It is this energy that compensates the diffraction spreading-induced diminishment of the intensity near the axis, so that it would be more accurate to refer to these beams as diffraction-compensated.

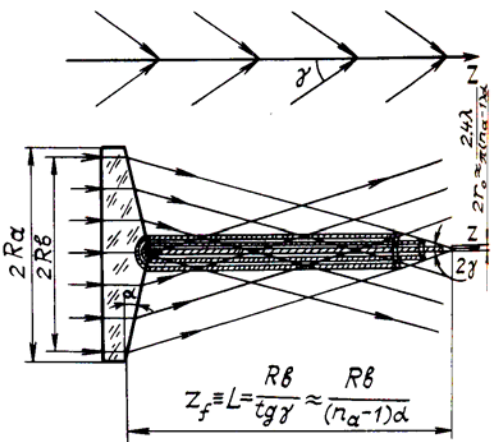



*FIG. 1. Formation of a nondiffractim (Besselian) light beam.*

The intensity on the axis can change as a function of the longitudinal coordinate $z$,[1,2] but the diameters of the central cylindrical zone of energy concentration and of the secondary hollow maxima do not change and they are determined by the properties of the function $J_0$.

The roots of the equation $J_0(x) = 0$ are given by the approximate expression[4]

$$x_0^{(s)} \approx \beta + \frac{1}{8\beta} - \frac{124}{3(8\beta)^3}, \tag{1}$$

where $s = 1, 2, 3, \ldots;\ \beta = (4s - 1)\pi/4$.

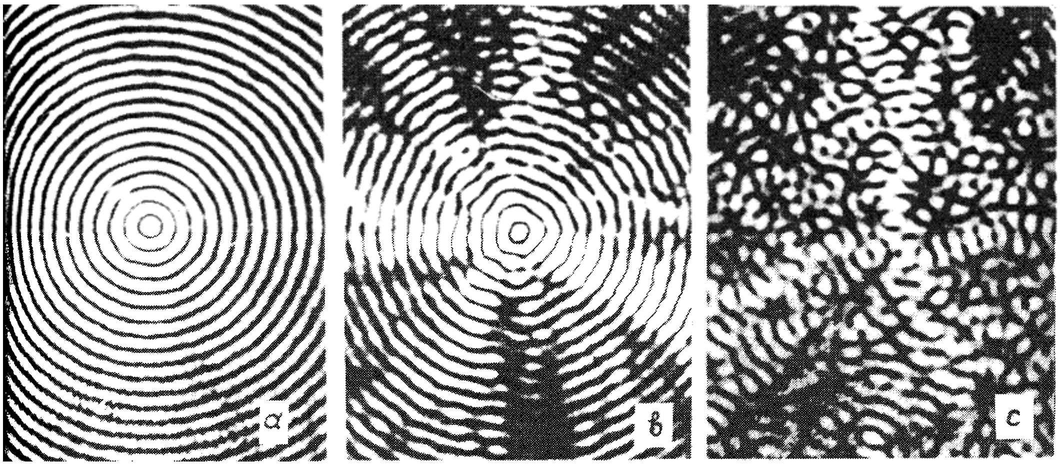
Thus the diameter of the rings of zero intensity $d_0^{(s)}$ and the diameter of rings of maximum intensity $d_m^{(s)}$ can be expressed in the form

$$d_0^{(s)} = \frac{2x_0^{(s)}}{k\,\sin\gamma} = \frac{x_0^{(s)}\lambda}{\pi\,\sin\gamma};$$

$$d_m^{(s)} = \frac{d_0^{(s)} + d_0^{(s+1)}}{2} \tag{2}$$

where $\lambda$ is the wavelength of the focused radiation.

We formed nondiffracting light beams with the help of glass axicons for which $\gamma = 7.5$, 5, and 2.5°. We used the radiation from a single-mode monopulse neodymium-glass laser ($\lambda = 1.06$ μm and the divergence $\sim 10^{-4}$ rad).[5] We recorded the intensity distribution by making microphotographs of the zone of the beam near the axis with a magnification of $10^3$ using 1–1060 infrared film (Fig. 2), after which we performed opacity density measurements with a microdensitometer (Fig. 3). We inserted phase plates, which increased the spreading of the radiation, into the starting beam in order to vary its divergence.




—— 20 μm

*FIG. 2. The transverse cross section of a nondiffracting laser beam with $\gamma = 5°$ (the same along the focal segment ($0 < z < L$)) with different divergence $\theta$ of the starting radiation $\lambda = 1.06$ μm: a) – $\theta \sim 10^{-4}$ rad, b) – $\theta \sim 58 \cdot 10^{-4}$ rad, c) – $\theta \sim 3 \cdot 10^{-2}$ rad.*

The photographs of the transverse cross sections of nondiffracting beams in Fig. 2. correspond to divergences of the starting laser beam of $10^{-4}$ (*a*), $5 \cdot 10^{-3}$ (*b*), and $3 \cdot 10^{-2}$ (*c*) rad. Transverse structure in the form of alternating rings of equidistant maxima and minima can be seen clearly in the first two photographs. As the quality of the initial beam decreased, the diameters of the axial and secondary maxima remained unchanged within certain limits; only local distortions in the shape of the rings were observed (Fig. 2*b*). Above the admissible limit of the distortions of the wavefront of the starting radiation the nondiffracting beam was simply not formed (Fig. 2*c*). This limit is associated with the overlapping of neighboring ring maxima.

The microdensitometer traces of the transverse cross section are shown in Fig. 3 (for $\gamma = 5$ and 2.5°). The figure also shows, for comparison, the function $J_0^2$ ($kr$ sin$\gamma$) (dashed lines). One can see that distortions in the shape of the wavefront result in the appearance of a scattered component of the radiation, as a result of which on the microdensitometer traces the intensity does not drop to zero at $r = x_0^{(s)}$.

From a comparison of the theoretical values of the diameters of the maxima $d_m^{(s)}$ and minima $d_0^{(s)}$ of the intensity in a nondiffracting beam in Fig. 3 and Table I we can conclude that the theory is in good agreement with the experiment.

*TABLE I.*

| $s$ | $d_0^{(s)}$, μm | | $d_0^{(s)}$, μm | | $I_0^{(s)}$, rel. un. | | $I_0^{(s)}$, rel. un. | |
|---|---|---|---|---|---|---|---|---|
| | theor. | exper. | theor. | exper. | theor. | exper. | theor. | exper. |
| 1 | 9.3 | 9.3 | 14 | 14.2 | 0 | 0.8 | 0.16 | 0.9 |
| 2 | 21.4 | 21.6 | 27.4 | 27.5 | 0 | 0.5 | 0.09 | 0.8 |
| 3 | 33.5 | 33.4 | 39.6 | 38 | 0 | 0.4 | 0.062 | 0.7 |
| 4 | 45.6 | 46 | 51.6 | 50.8 | 0 | 0.3 | 0.048 | 0.66 |
| 5 | 57.7 | 58 | 63.7 | 63.3 | 0 | 0.25 | 0.038 | 0.62 |
| 6 | 69.9 | 70 | 76 | 76 | 0 | 0.2 | 0.032 | 0.5 |
| 7 | 82.1 | 82 | 88.1 | 87.5 | 0 | 0.14 | 0.028 | 0.5 |
| 8 | 94.2 | 94 | 100.3 | 100 | 0 | 0.13 | 0.024 | 0.6 |
| 9 | 106.4 | 106 | 112.4 | 112 | 0 | 0.08 | 0.022 | 0.5 |
| 10 | 118.4 | 118 | 124.4 | 124 | 0 | 0.05 | 0.02 | 0.5 |
| 11 | 130.4 | 130 | 136.6 | 136.5 | 0 | 0 | 0.018 | 0.44 |
| 12 | 142.8 | 142 | 148.3 | 149 | 0 | 0 | 0.016 | 0.44 |

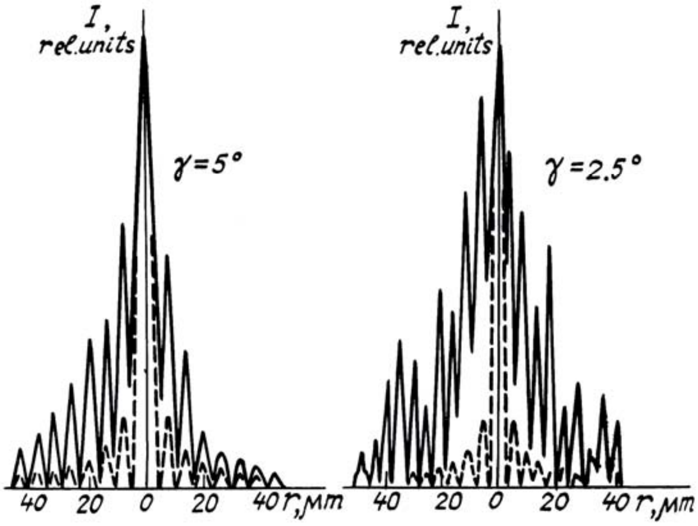



*FIG. 3. The intensity distribution in nondiffracting laser beams ($\lambda$ = 1.06 μm) with $\gamma$ = 5 and 2.5° (solid lines) and $J_0^2$ (kr sin$\gamma$) (dashed lines).*

The agreement between the experimental and theoretical radial distributions of the intensity $I$ is much worse. This is connected with the finite divergence of a laser beam, which does not change the geometric dimensions of the maxima and minima but does result in the appearance of a scattered component of the radiation. It has a diffuse character and results in attenuation of the central maximum and some secondary maxima closets to it. As a consequence the contrast of the picture decreases. But the calculations were performed for initial beams whose wavefront was ideal.

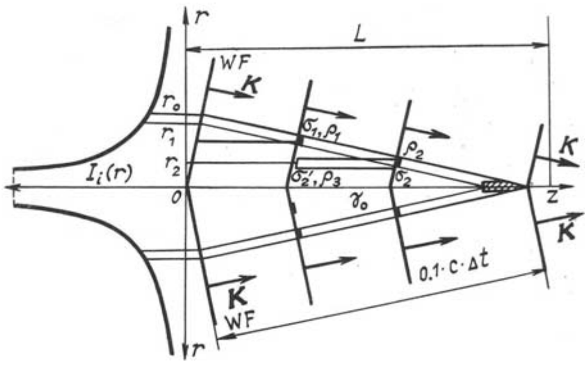
The intensity distribution along the axis of propagation of a nondiffracting beam depends on the profile of the focused beam. In principle the profile of the initial beam can be matched to that the axicon so that the intensity on the axis is constant along the entire focal segment.[2] There are three possible regimes of optical breakdown of the gas in which a nondiffracting beam propagates: 1) the quasistationary regime $\Delta t \gg L / c$; 2) the traveling-focus regime $\Delta t \ll L / c$; and, the intermediate regime $\Delta t \sim L / c$; here $\Delta t$ is the width of the front of the laser pulse and $L / c$ is the propagation time of the light along the length of the spark $L$. We shall study these regimes in succession.

1. $\Delta t \gg L / c$; In this case the intensity on the axis of the nondiffracting beam increases with time practically simultaneously along the entire breakdown zone $L$. Let a beam with a conical wavefront (WF) and a hyperbolic radial distribution of the intensity ($I = I_0 R_a / r$, where $r$ is the radial coordinate) propagate along the $z$ axis (Fig. 4). Then a ray tube with the cross section $\delta = 2\pi r\, dr$ contains radiation with the power

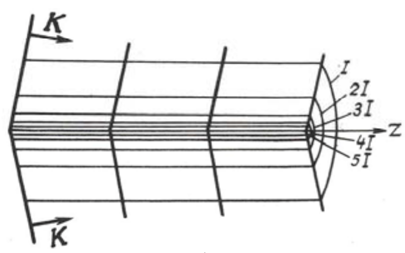
$$dW = 2\pi R_a I_0 dr, \quad (3)$$

i.e., $dW / dr$ = const. As the nondiffracting light beam propagates the ring $\delta_1$ is converted into the ring $\delta_2$:

$$\sigma_1 = 2\pi r_1 dr, \quad \sigma_2 = 2\pi r_2 dr. \quad (4)$$




*a*




*b*

*FIG. 4. The transformation of a Light beam with a hyperbolic radial distribution of the intensity into a nondiffracting beam (a) and transformation of the isophot of the nondiffracting beam in the quasistationary regime $\Delta t \gg L / c$) (b).*

Thus the radiation intensity at the points 1, 2, and 3 will be equal to

$$I(p_1) = \frac{dW}{\sigma_1} = \frac{I_0 R_a}{r_1}; \quad (5)$$

$$I(p_2) = \frac{dW}{\sigma_2} = \frac{I_0 R_a}{r_2}; \tag{6}$$

$$I(p_3) = \frac{dW}{\sigma'_2} = I(p_2). \tag{7}$$

Therefore the lines of constant radiation intensity (and constant electric field strength) have the form of coaxial cylinders, and, in addition, the intensity is inversely proportional to the distance from the axis. For this reason the threshold intensity for optical breakdown is reached simultaneously along the entire focal segment (irrespective of the length of the focal segment).

The cylindrical surface with a fixed value of the radiation intensity expands in an axisymmetric manner as the power of the initial laser pulse increases.

Theoretically an extended continuous laser spark can be formed in such a nondiffracting laser beam instantaneously. In practice, however, the formation time will be determined by the statistical spread in the rate of development of the avalanche at different points of the focal segment and by the time it takes for the neighboring breakdown zones to merge. These times are characteristically of the order of ~ 1 ns. For this reason in the quasi stationary regime an extended continuous laser spark can be produced in a nondiffracting beam over the same period of time.

2. The second breakdown regime is realized with a short pulse: $\Delta t \ll L / c$;. In this case the zone of high radiation intensity moves along the 3 axis with the velocity $c \cdot \cos\gamma$. This corresponds to the traveling-focus regime; the velocity of the focus is determined by the velocity of propagation of the spark along the axis. By changing the angle $\gamma$ the velocity can be varied within wide limits.

The formation time of the channel under these conditions will be given by

$$t_f = \frac{L}{c\cdot\cos\gamma}. \tag{8}$$

For example, for $L = 1$ m and $\gamma = 5$ we obtain $t \approx 3$ ns.

3. In the intermediate case $\Delta t \sim \lambda / c$ the delay in the arrival of the light at the distant end of the focal segment will be important, since the rate of change of the intensity of the laser radiation is comparable to the velocity of light. The .surfaces of equal intensity under such conditions will no longer be cylindrical; they will contract toward the distant end of the focal segment. The shape of these surfaces depends on the behavior of the intensity of the laser pulse as a function of time.

In the simplest case of linearly increasing intensity the surface of constant intensity will be a cone. We shall approximate the spatiotemporal profile of the laser pulse by a function of the type

$$I_1(r,\ t) = I_0 \frac{a}{b + r}\cdot\frac{t}{\Delta t}, \tag{9}$$

where $a = \dfrac{I_m \cdot I_0}{I_m - I_0}(R_b - r_0)$, $b = \dfrac{R_b I_0 - r_0 I_0}{I_m - I_0}$ are constants describing the spatial profile of the beam ($I_0$ is the intensity at $r = R_b$ and $I_m$ is the intensity at $r = r_0$).

To determine the shape of the surface of constant intensity we shall study two points with the coordinates $(r_1,\ 0)$ and $(r,\ z)$ at the time $t$. At the former point the radiation intensity is equal to

$$I_1 = I_0 \frac{a}{b + r_1}\cdot\frac{t}{\Delta t}, \tag{10}$$

while at the latter point

$$I_2 = I_0 \frac{a}{b + r}\cdot\frac{t - z/(c\cdot\cos\gamma)}{\Delta t}, \tag{11}$$

since the wavefront that passed through the surface $z = 0$ at a time earlier than $t$ by the amount $z / (c \cdot \cos\gamma)$ is located at this point.

In order for the chosen points lie on a surface of constant intensity the condition $I = I_1$ must be satisfied. This gives $r = r_1 - (r_1 + b)\ z / z_m$, where $z_m = c \cdot t \cdot \cos\gamma$ is the propagation distance of the front of the nondiffracting beam up to the moment $t$.

Thus the properties of nondiffracting laser beams are radically different from those of Gaussian beams, so that nondiffracting beams should be segregated into a special class. Optical breakdown in such beams results in the formation of continuous extended laser sparks,[5] which cannot be obtained in any other manner. The nondiffracting (Besselian) laser beams could also be useful for other applications.

## REFERENCES


1. N.A. Volkova, V.V. Korobkin, et al., *Focusing of Laser Radiation with Axicons*, Preprint No. 5–126, Institute of High Temperatures of the Academy of Sciences of the USSR, Moscow (1984).

2. V.V. Korobkin, L.Ya. Polonskii, V.P. Poponin, and L.N. Pyatnitskii, Kvant. Elektron., **13**, No. 2, 256 (1986).

3. J. Durnin, J. Opt. Soc. Amer., **4**, No. 4 (1987).

4. A. Gray and G. Mathews, *A Treatise on Bessel Functions and their Applications to Physics*, MacMillan, London (1922). [Russian translation, Inostr. Lit., Moscow (1949)].

5. L.Ya. Polonskii and L.N. Pyatnitskii, Atm, Opt., **1**, No. 7, 86 (1988).