INVESTIGATION OF TURBULENCE SPECTRUM ANISOTROPY IN THE GROUND ATMOSPHERIC LAYER. PRELIMINARY RESULTS

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Some properties of the ground atmospheric layer are considered from the point of view of the influence of its turbulent inhomogeneities on the characteristics of optical radiation propagating in this layer.

Turbulence in the ground atmospheric layer is the most variable and rather complex dynamic structure.

It is known that from the viewpoint of description of optical wave intensity fluctuations the spectral density of fluctuations of the index of air refraction is an isotropic function. However, in recent years, especially in connection with the development of adaptive optics, investigations of fluctuations of optical wave phase evoke great interest. Correct description of phase fluctuations of optical waves already demands consideration of deviation of the atmospheric turbulence spectrum from the power Kolmogorov-Obukhov law in the low spatial frequency range.[1–7] At the same time, just this portion of turbulence spectrum range is the most poorly investigated. Undoubtedly, one of the important properties of the spectrum in low-frequency range is its anisotropy.

In this paper we consider some properties of this medium from the point of view of its influence on the characteristics of optical radiation propagating in the ground atmospheric layer.

If we are within the limits of a model description of the spectral density of fluctuations of the air refractive index when we calculate the phase fluctuations of optical waves, the most acceptable[12] is the following model:

$$\phi_n(\kappa_2, \kappa_3, x) = 0.033 C_n^2(x)\ (\kappa_2^2 + \kappa_3^3)^{-11/6} \times$$
$$\times \{1 - \exp[-\kappa_2^2/\kappa_{02}^2 - \kappa_3^2/\kappa_{03}^2]\}, \quad (1)$$

where x is the coordinate, along which the optical wave propagates; κ_2 and κ_3 are orthogonal components of spatial wave numbers. Model (1) is isotropic in the inertial interval of wave numbers (for $\kappa_2 \gg \kappa_{02}$, $\kappa_3 \gg \kappa_{03}$), and it may describe the anisotropy of spectrum at inequality of components of the outer scale ($\kappa_{02} = \kappa_{03}$) out of the inertial interval.

Among the characteristics of optical waves, propagating in turbulent atmosphere, the most sensitive to turbulent spectrum anisotropy are variances of orthogonal components of random displacements of the center of gravity of an optical source image formed (σ_y^2, σ_z^2).

When making calculations, we shall use the expression for phase fluctuations of plane optical wave propagating in turbulent atmosphere, calculated in approximation of smooth perturbation method, as the initial approach:

$$S(\rho, L) = k\int_0^L d\xi \int\int d^2 n(\boldsymbol{\kappa}, x) \times$$
$$\times \cos[\boldsymbol{\kappa}^2 (L - x)/2k] \exp(i\kappa\rho), \quad (2)$$

where k is the radiation wave number; (ρ, L) is the observation point; $n_1(\rho, x) = \int\int d^2 n(\boldsymbol{\kappa}, x) \exp(i\boldsymbol{\kappa}\rho)$ are fluctuations of the refractive index.

By definition the angle of slope of the wave front (under condition of small intensity fluctuations) is:

$$\boldsymbol{\alpha} = -(k\,\Sigma)^{-1} \int\int_\Sigma d^2\rho\ \nabla_\rho\, S(\rho, L). \quad (3)$$

Here Σ is the receiving aperture of a measurer of wave front slope. Using Eq. (2), we write down expressions for gradient of phase fluctuations $\nabla_\rho S(\rho, L)$ and its components $\partial S(\rho, L)/\partial y, \dots, \partial S(\rho, L)/\partial z$.

Then we calculate the overall variance of the fluctuations of angular position fluctuations σ_α^2, as well as its orthogonal components σ_y^2, σ_z^2. First of all we will write down the expression for general variance:

$$\sigma_\alpha^2 = 2\pi\,\Sigma^2 \int_0^x dx \int\int d^2\kappa\,\kappa^2\,\phi_n(\kappa, x) \cos^2(\kappa^2 (L - x)/2k) \times$$
$$\times \int\int_\Sigma d^2\rho_1\, d^2\rho_2 \exp(i\kappa\rho). \quad (4)$$

Calculations by Eq. (4) use Gaussian aperture of a size a and spectrum given by Eq. (1). We obtain

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$$\sigma_\alpha^2 = 2\pi\, 0.033 \int_0^L \mathrm{d}x\, C_n^2(x) \times$$

$$\int_0^\infty \mathrm{d}\kappa\,\kappa^3 \cos^2[\kappa^2 (L-x)/2k] \exp(-\kappa^2 a^2)(\kappa_2^2 + \kappa_3^2)^{-11/6} \times$$

$$\times \int_0^{2\pi} \mathrm{d}\varphi \{1 - \exp[-\kappa^2(\cos^2\varphi/\kappa_{02}^2 + \sin^2\varphi/\kappa_{03}^2)]\}.$$

By introducing polar coordinates

$\mathrm{d}^2\kappa = \mathrm{d}\kappa\,\kappa\, \mathrm{d}\varphi$, ..., $\kappa_2 = \kappa \cos\varphi$, ..., $\kappa_3 = \kappa \sin\varphi$.

After computation the value of an integral we have

$$\sigma_\alpha^2 = \pi\, 0.033\, \Gamma(1/6)\, a^{-1/3} \int_0^L \mathrm{d}x\, C_n^2(x) \times$$

$$\times \int_0^{2\pi} \mathrm{d}\varphi \{1 - (1 + a^{-2} b^2)^{-1/6}\}, \quad (5)$$

where $b^2 = \cos^2\varphi/\kappa_{02}^2 + \sin^2\varphi/\kappa_{03}^2$. For isotropic spectrum $(\kappa_{02} = \kappa_{03})$ $a^{-2}b^2 = (a^2\kappa_{02}^2)^{-1}$ and Eq. (5) takes the form

$$\sigma_\alpha^2 = 2\pi^2\, 0.033\, \Gamma(1/6)\, a^{-1/3} \times$$

$$\times \int_0^L \mathrm{d}x\, C_n^2(x) \{1 - (1 + \kappa_{02}^{-2} a^{-2})^{-1/6}\}, \quad (6)$$

which, under the condition $\kappa_{02} a \ll 1$, reduces to the known expression:

$$\sigma_\alpha^2 = 3.62\, a^{-1/3} \int_0^L \mathrm{d}x\, C_n^2(x) \{1 - (\kappa_{02}\, a)^{1/3}\}.$$

Further we introduce the parameter δ, characterizing the spectrum (1) anisotropy in the low-frequency range, in such a way that

$$\kappa_{02}^{-2} = \kappa_{03}^{-2} (1 + \delta). \quad (7)$$

If $\delta = 0$ we obtain isotropy, otherwise we have the anisotropic spectrum. Then taking into account Eq. (7) we have

$$b^2/a^2 = \cos^2\varphi/a^2\kappa_{02}^2 + \sin^2\varphi/a^2\kappa_{03}^2 =$$
$$= (1+\delta)\cos^2\varphi/\kappa_{03}^2 a^2 + \sin^2\varphi/\kappa_{03}^2 a^2 =$$
$$= \kappa_{03}^{-2} a^{-2} (1 + \delta \cos^2\varphi).$$

In further calculations we shall make use of the fact that the corresponding variances of components σ_y^2 and σ_z^2 can be expressed in the integral form by analogy with Eq. (4). Anisotropic spectrum 1 is described by three parameters C_n^2, κ_{03}, δ, therefore three equations, relating these parameters to data of variance measurements $(\sigma_\alpha^2, \sigma_y^2, \sigma_z^2)$, are necessary.

It can be shown that results of calculations of the corresponding variances are expressed by means of a universal integral

$$\int_0^{2\pi} \mathrm{d}\varphi \begin{Bmatrix} 1 \\ \cos^2\varphi \\ \sin^2\varphi \end{Bmatrix} \{1 - \left(1 + \kappa_{03}^{-2} a^{-2} (1+\delta) \cos^2\varphi\right)^{-1/6}\}. \quad (8)$$

Let us calculate numerically just those characteristics, which are measured in optical experiments, namely, σ_y^2 and σ_z^2. Then use them as the initial data and calculate the following quantities:

$$K = \left(\sigma_y^2 - \sigma_z^2\right)/\left(\sigma_y^2 + \sigma_z^2\right), \qquad K_1 = \sigma_y^2/\sigma_z^2. \quad (9)$$

It is not difficult to show that for homogeneous optical paths the measured values K and K_1 are independent of $\int_0^L \mathrm{d}x C_n^2(x)$, as a result, no absolute measurements are needed.

We return to numerical calculations on the basis of a model. Using the universal integral (8), we derive:

$$K = \int_0^{2\pi} \mathrm{d}\varphi \{\cos^2\varphi - \sin^2\varphi\} \left(1 + \kappa_{03}^{-2} a^{-2} \times\right.$$

$$\left. \times (1+\delta\cos^2\varphi)\right)^{-1/6} / \int_0^{2\pi} \mathrm{d}\varphi \left(1 - \kappa_{03}^{-2} a^{-2} (1+\delta\cos^2\varphi)\right)^{-1/6}, \quad (10)$$

$$K_1 = \int_0^{2\pi} \mathrm{d}\varphi \cos^2\varphi \{1 - \left(1 + \kappa_{03}^{-2} a^{-2} \times\right.$$

$$\left. \times (1+\delta\cos^2\varphi)\right)^{-1/6}\} / \int_0^{2\pi} \mathrm{d}\varphi \sin^2\varphi \{1 - (1 + \kappa_{03}^{-2} a^{-2} \times$$

$$\times (1 + \delta\cos^2\varphi))^{-1/6}\}. \quad (11)$$

It is easy to check using Eqs. (10) and (11) that under isotropic conditions ($\delta = 0$) $K = 0$, $K_1 = 1$.

Let us now make the change of variables in the integrals in formulas (10) and (11):

$$\int_0^{2\pi} \mathrm{d}\varphi \begin{Bmatrix} \cos^2\varphi \\ \sin^2\varphi \end{Bmatrix} (1 + \kappa_{03}^{-2} a^{-2} (1 + \delta\cos^2\varphi))^{-1/6} =$$

$$= 2 \int_0^1 \mathrm{d}t\, t^{-1/2} (1-t)^{-1/2} \begin{Bmatrix} t \\ 1-t \end{Bmatrix} \left[1 + a^{-2} \kappa_{03}^{-2} (1+\delta t)\right]^{-1/6}. \quad (12)$$

Let us write I for a first integral in Eq. (12) and II for the second, then we derive:

$$K = (\mathrm{II} - \mathrm{I})/[(\mathrm{II} + \mathrm{I}) - \pi/2],$$
$$K_1 = [2\mathrm{I} - \pi/2]/[2\mathrm{II} - \pi/2]. \quad (13)$$

Unfortunately, the integrals in Eq. (12) cannot be reduced to a simple analytical form, therefore their numerical calculation is required. As a result of numerical calculations of integrals in Eq. (12), we derive the tables of values of K and K_1 as functions of two parameters δ and $\kappa_{03}a$. The former parameter characterizes the magnitude of the spectrum (1) anisotropy, and the latter determines the relation between the outer scale of turbulence and the size of the receiving aperture a.

The calculations for values $\delta = \kappa_{03}^2/\kappa_{02}^2 - 1$ in the interval from 0 to 100 and for parameter $b = \kappa_{03}a$ correspondingly in the interval from 0.01 to 1.00 make it possible to cover all possible atmospheric variations of the components of the outer scale of turbulence.

Optical measurements were accompanied by the corresponding meteorological measurements, which were carried out at the heights of 1.25, 2.5, and 5.0 m. Thus obtained results were used for calculating the following characteristics at the height of the optical radiation propagation (h = 2.5 m):

– Obukhov parameter of stability

$$B = gh\,(T_{2h} - T_{h/2})/T(h)\bar{v}^2(h), \quad (14)$$

– outer scale of turbulence (from meteorological data)

$$L_0^{\text{met}} = [C_T^2/(2.8\,(\Delta T/\Delta h)^2)\,(p/1000)^{0.572}]^{3/4}, \quad (15)$$

where g is the acceleration of gravity; p is the pressure at the height of 2.5 m, C_T^2 has the dimensionality of $\deg^2/\mathrm{cm}^{2/3}$. In the latter formula a multiplier $(p/1000)^{0.572}$ reduces the value of pressure to the level at 1000 mb.

Besides, the reduction of optical measurements was carried out for elimination of the influence of atmospheric refraction. It is known that in a stratified medium, such as the atmospheric ground layer, there are stable temperature gradients. However the presence of temperature gradients brings up to the deflection of optical beams. If substantial changes of mean temperature gradient within the beam size occur, this leads to the nonuniform broadening of optical beam, especially, in the ground layer this must lead to characteristic beam elongation in vertical direction.

The angle of refraction r is calculated by formula:

$$r = 1.25\,L(n - 1)\,\mathrm{d}T/\mathrm{d}h/T,$$

correspondingly, relative increase of vertical displacements is related to the increment of refraction angle within the beam, i.e.

$$\mathrm{d}r = 1.25\,L(n - 1)\,\mathrm{d}^2T/\mathrm{d}h^2(a/T)\ ,$$

the relative increase of vertical displacement is connected, in its turn, with the change in refraction angle at the change in optical beam size, i.e., with $\mathrm{d}^2T/\mathrm{d}h^2$.

Estimation of the second derivative for temperature is carried out taking into account atmospheric instability by the following formulas:

$$\mathrm{d}^2T/\,\mathrm{d}h^2 = -\frac{4}{3}\,h\,\mathrm{d}T/\mathrm{d}h\ \ldots(B \le 0),$$

$$\mathrm{d}^2T/\mathrm{d}h^2 = -\frac{1}{h}\,\mathrm{d}T/\mathrm{d}h\ \ldots(B \to 0).$$

Thus, the amplitude of additional contribution from the refraction to jitter of the image, formed in the stratified medium, because of a change in the mean temperature at unstable stratification can be estimated using the following formula:

$$\sigma_r = 2L(n - 1)\,(\Delta T/\Delta h)\,(a/Th). \quad (16)$$

As one can see, the influence of refraction should be expected for very wide beams, when the scale of temperature gradient variations is compared with the beam size.

For elimination of the influence of refraction on the experimental data, it is necessary to calculate σ_r by formula (16) using meteorological measurements and to decrease rms deviations for random vertical shifts.

Experiments have been carried out in the form of cycles of observations for many hours in the atmosphere under conditions of a uniform underlying surface, in both day and night time. The results of processing of experimental (optical and meteorological) measurements form data files for the following quantities: K, K_1, C_T^2 (or C_n^2 for the wavelength λ = 0.6328 μm), B, L_0^{met}. Data files in comparison with the tables of numerical calculations of this quantities on the basis of formulas (12) and (13) allow us to obtain values of the anisotropy and the outer scale of turbulence L_0^{met} (size of receiving aperture and of optical beam were measured experimentally).

The values L_0^{met} and a allow us to determine approximate value of the parameter $b = a/L_0^{\text{met}}$. The more precise determination of quantities δ and b simultaneously is made by retrieving of numerical calculations based on current values of K and K_1 from data files.

Analysis of the results has shown that the greatest values of anisotropy δ are reached at unstable stratification of the atmosphere (the great positive values of B), with the increase of stability the values δ decrease. For the height of 2.5 above the underlying surface, the high anisotropy has been revealed (from 5.5 to 8.6 for $\sqrt{\delta - 1}$). This means that the greatest (at this height) inhomogeneities of turbulent atmosphere are in horizontal direction with the sizes five to eight times as large as the vertical. The meteorological data allow one to calculate values of the outer scale L_0^{met} by formula (15). As one can see from our data, most probable values of this parameter lie in the interval between 0.3 and 3 m.

Thus, these results convincingly confirm that in the ground atmospheric layer the largest turbulent inhomogeneities, influencing the phases of optical waves, which propagate in the atmosphere, have a pronounced anisotropy of properties. The level of this anisotropy depends on the parameter of instability of the atmosphere (at this height), and the height above the underlying surface. Second part of this statement requires the additional investigations, although, as our laboratory measurements[8-11] show, anisotropy of properties in the low-frequency region of turbulent spectrum depends substantially on the height above the underlying surface.

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