MODAL REPRESENTATION OF ATMOSPHERIC INHOMOGENEITIES FOR NUMERICAL ANALYSIS OF STATISTICAL PARAMETERS OF LIGHT BEAMS

I.E. Tel'pukhovskii and S.S. Chesnokov

M.V. Lomonosov State University, Moscow
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A method is proposed for construction of spatial randomly inhomogeneous fields of the refractive index of the turbulent atmosphere. The problem of broadening and random jitter of a low–power beam is solved by way of example to show that energy parameters of the beam can be predicted with satisfactory accuracy without lengthy computations.

The problem of compensating for nonlinear and turbulent distortions of light beams propagating through the atmosphere is of great interest. The possibilities of analytical solution of problems of nonlinear statistical optics are relatively limited because of the limitations imposed on the statistics of the field of the refractive index, on the level of its fluctuations, and on the intensity of radiation. For this reason the requirements for the reliability of predicting the statistics of energy parameters of the light field in the image plane are stringent at present. In its turn, the adequacy of description of atmospheric inhomogeneities at the nodes of computational grid is largely determined by the model being used. We propose an update model applicable for wider spectrum of inhomogeneities including the low–frequency domain.

SPECTRAL MODELLING OF ATMOSPHERIC INHOMOGENEITIES

Propagation of the light beam through the randomly inhomogeneous nonlinear medium is described by a system of equations

$$2i\frac{\partial E}{\partial z} = \Delta_{\perp}E + \tilde{\varepsilon}E + R_0 TE,$$

$$\frac{\partial T}{\partial x} + (\tilde{\mathbf{V}}\Delta)T = EE^*,$$

where $\tilde{\varepsilon}$ are the natural fluctuations of dielectric constant of the medium with spectral density $\Phi_{\varepsilon} = \Phi_{\varepsilon}(\kappa)$, R_0 is the nonlinearity parameter which depends on the average velocity of the medium, and $\tilde{\mathbf{V}}$ is the random field of velocities. The principal technique used to solve such problems is numerical analysis, which opens the possibility to study the solution in the range of variations of parameters unavailable for analytical techniques.

Application of numerical techniques for the solution of differential equations implies that continuous fields are substituted by their discrete analog prescribed at the nodes of the computational grid. The available computer power limits the number of nodes of the grid and the corresponding size of computational grid L in general. Spatial and spectral discretization of the variables imposes limitations on the modeled physical fields E and $\tilde{\varepsilon}$.

The most significant difficulty in modeling the field of dielectric constant is associated with the wide rauge of size spectrum of the atmospheric inhomogeneities in the turbulent atmosphere. The smallest characteristic scale of change of spectral density of fluctuations is $\kappa_0 = \frac{2\pi}{L}$. The harmonics in a discrete spectrum are spaced out at $2\pi/L$, where L is the size of the domain in which the field is modeled. At least several harmonics are required to reproduce the change of spectrum with the scale κ_0. In the low–frequency domain this limitation results in a simulation of field with a smaller actual external scale at the nodes of the grid. The dependence of the effective outer scale $L_{\rm eff}$ reproduced at the nodes of the grid in the low–frequency domain on the number of nodes of the grid M and on the ratio L/a_0 calculated by the method of numerical simulation based on the variance of the displacements of the beam center of gravity, is a good illustration of this limitation in the low–frequency domain (see Fig. 1).

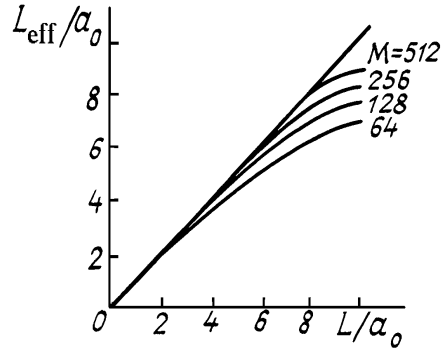

FIG. 1. Effective outer scale of turbulence as functions of the number of the nodes of computational grid and of the ratio L/a_0.

In the high–frequency domain the limitation imposed on the spatial spectrum can be estimated assuming that the modeled spectrum $\Phi^{\rm M}_{\varepsilon}(\kappa)$ of fluctuations $\tilde{\varepsilon}$ is truncated at the Nyquist frequency of the computational grid, i.e., at $\kappa_{\rm N} = \pi/\Delta x$, so that

$$\Phi^{\rm m}_{\varepsilon}(\kappa) = \begin{cases} 0.33C_{\varepsilon}^2 k^{-11/3}\exp(-x^2/x_{\rm M}^2) & \text{at } k \le k_{\rm N} \\ 0 & \text{at } k \gg k_{\rm N}. \end{cases}$$

Let us consider the variance of fluctuations of the level of a plane wave σ_x^2. For the modified Kolmogorov spectrum of turbulent fluctuations we have[1]

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$$\sigma_{\kappa}^{(m)2} = \frac{\pi^2 z^3}{12} \cdot 0.033 C_{\varepsilon}^2 \cdot 0.4639 \kappa_m^{7/3} \mu\left(\frac{k_N}{k_M}\right),$$

$$\mu = \mu\left(\frac{k_N}{k_M}\right) = \gamma(7/6,\ \kappa_N^2/\kappa_M^2)/\Gamma(7/6),$$

where $\gamma(\alpha, \beta)$ is the incomplete gamma-function. The effect of the finite frequency band is determined by the factor μ. For $\kappa_N/\kappa_M \geq 2$ we have $\mu \simeq 0.98$. Hence, making use of the relation $\kappa_M = 5.92/l_0$, we find $l_0/\Delta x \geq 4$. Thus the reproducible spectral range in the high—frequency domain is also limited and amounts to approximately half the Nyquist frequency.

MODAL REPRESENTATION OF ATMOSPHERIC INHOMOGENEITIES

Below we propose an efficient technique for construction of the random fields based on modal representation of atmospheric inhomogeneities. This representation may be constructed most accurately using the Karuhnen—Loeve functions, which have uncorellated coefficients in their expansion. However, the tabulated form of these functions as well as difficulties of their calculation result in the fact that they are usually substituted by normalized Zernike polynomials.

To model random phase perturbation $\tilde{\psi}(r, \vartheta)$ on the section of the path Δz, we use its expansion in a system of the polynomials $Z_j(\rho, \vartheta)$ within given aperture of radius R:

$$\tilde{\psi}(\rho, \vartheta) = \sum_{j=0}^{\infty} \tilde{\alpha}_j Z_j(\rho, \vartheta), \quad \rho = r/R,$$

where the random coefficients $\tilde{\alpha}_j$ are assumed to be normally distributed with zero mean and variance determined by atmospheric conditions on the path. For the Kolmogorov spectrum of turbulent fluctuations the values $\tilde{\alpha}_j$ are related to the structure constant of the refractive index fluctuations C_n^2 in terms of the Fried's correlation radius of phase fluctuations r_0 (see Ref. 2)

$$<\tilde{\alpha}_j> = a_j^2\left(\frac{2R}{r_0}\right)^{5/3},$$

where $r_0 = 1.68(C_n^2 \kappa^2 \Delta z)^{-3/5}$ and a_j are the weighting factors.

To find the radius of the aperture R in the expansion we naturally assume that it is sufficient to restict ourselves to the first few polynomials (j = 2, 3) in the expansion of phase to describe random jitter of the beam. Then

$$\tilde{\psi} = \tilde{\alpha}_2 x + \tilde{\alpha}_3 y. \tag{1}$$

The variances of the random numbers $\tilde{\alpha}_2$ and $\tilde{\alpha}_3$ are given by the relation:

$$\sigma_{2,3}^2 = 13.5 \kappa^3 C_n^2 a_0^4 R^{-1/3} \Delta z.$$

Let the light beam successively pass N equidistantly spaced statistically independent phase screens located on the path. According to Eq. (1), the random tilt of the wavefront at the ith screen is equal to $\beta_i = 2\tilde{\alpha}_2^{(i)}/\kappa R$. Its average value is $<\beta_i> = 0$ and its variance is $<\beta_i^2> = \dfrac{4<\alpha_2^2>}{\kappa^2 R^2}$. For homogeneous turbulence the variance of the tilt can be represented in the form $<\beta_i^2> = A\,\dfrac{z}{N}$, where $A = 5.35 a_2^2 C_n^2 R^{-1/3}$ is the constant independent of either spacing or the number of screens.

The displacement ρ_N in the plane $N\Delta z$ is

$$\rho_N = \Delta z(\beta_1 + (\beta_1 + \beta_2) + \ldots + (\beta_1 + \beta_2 + \ldots + \beta_N)),$$

and its rms deviation is

$$\sigma^2 = Az^3 S_N, \text{ where } S_N = \frac{1}{N^3} + \frac{2^2}{N^3} + \ldots + \frac{N^2}{N^3}.$$

As $N \to \infty$, which corresponds to the continuously stratified turbulent medium, $S_N = \dfrac{1}{3}$ or

$$\sigma^2 = \frac{\Delta z^3}{3} A. \tag{2}$$

Comparing Eq. (2) with the relation for the variance of the displacement of the beam center of gravity obtained in Ref. 4 for the Kolmogorov spectrum of turbulent fluctuations, we can find that $R_0 = a_0/2$.

RESULTS OF NUMERICAL SIMULATION

The proposed model was applied for the solution of the problem of broadening and random jitter of the beam in the propagating through the turbulent atmosphere. Results of calculations are shown in Fig. 2.

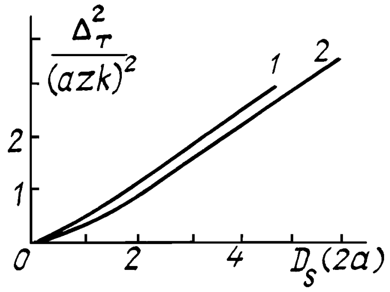

FIG. 2. Turbulent broadening as a of function the structural phase function of the spherical wave: 1) analytical estimate and 2) method of numerical simulation.

Here $\Delta_T^2 = a_{\mathrm{eff}}^2 - a_d^2$, where a_{eff}^2 is the squared effective beamwidth and a_d^2 is the squared diffraction width of the collimated beam. The relative turbulent broadening is shown as a function of the parameter $D_S(2a)$ which characterizes the atmospheric turbulence on the path. The deviation of the results of numerical experiments can be explained by the fact that modeling the random field of the refractive index based on the modal representation, we consider only the first 5 terms of the expansion.

Thus, the model is proposed for construction of the spatial randomly inhomogeneous fields of the refractive index of the turbulent atmosphere, which makes it possible to significantly expand their spectrum toward the low—frequency domain. Its realization is reduced to choosing a random set of numbers with prescribed correlation coefficient instead of modeling the random field with a prescribed correlation function over the entire range, so that we can significantly reduce the volume of computation.

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