Spectra of strong scintillations caused by large-scale anisotropic inhomogeneities in the stratosphere in observing stars from onboard a satellite

V.V. Vorob'ev, D.A. Marakasov, and O.V. Fedorova

A.M. Obukhov Institute of Atmospheric Physics, Russian Academy of Sciences, Moscow
Institute of Atmospheric Optics, Siberian Branch of the Russian Academy of Sciences, Tomsk

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We present in this paper some results of numerical investigation into two-dimensional and one-dimensional spectra of strong scintillations on atmospheric inhomogeneities generated by the internal gravity waves. The calculations are based on the model of statistically homogeneous phase screen. Conditions for applicability of the perturbation theory in calculating spectra of weak scintillations are formulated. It is shown that the perturbation theory works well in describing large-scale scintillations even if the scintillations are strong. It is shown that, depending on the product of one-dimensional spectral density and the wave number, a plateau is formed, whose level is only determined by the value of β_0, provided that it does not exceed one third of the anisotropy factor squared. Conditions of normalization of a small-scale part of the spectrum have been found, under which the scintillation spectrum coincides with the spectrum of squared coherence function on the phase screen.

Introduction

Observations of stars through the Earth atmosphere from onboard an orbiting stations[1–4] have shown that scintillations become stronger as the observation sight line immerses into the atmosphere. The scintillation variance approaches unity at some altitude of the sight line perigee. This altitude depends on the radiation wavelength and the distance from the observer to the perigee point of the sight line. Usually, this altitude is from 25 to 30 km.[1]

As follows from the nowadays understanding of the fine structure of air density in the stratosphere and troposphere, the inhomogeneities of the refractive index are formed locally by the isotropic turbulence and internal waves. Analysis of the satellite observations of the stellar scintillation has also confirmed it.[3,4] These results are in a good agreement with a 3D-model of spatial spectrum for the refractive index inhomogeneities,[5,6] being the sum of two components, namely the isotropic (Kolmogorov) one and the component of anisotropic inhomogeneities strongly extended along the Earth surface.

The research presented in this paper aimed at studying spatial spectra of strong scintillations formed by anisotropic inhomogeneities in the observation plane. The investigation has been carried out numerically using a phase screen model. This model is widely applied in studying the inhomogeneities of interplanetary medium and the atmospheres of solar system planets including the Earth's atmosphere and ionosphere. The integrated relations are known for this model relating the fluctuation spectra of the electromagnetic wave intensity (scintillation spectra) in the observation point with the spectra of phase fluctuations on an effective phase screen. The phase fluctuations, in its turn, are determined by spectra of the medium refractive index fluctuations, through which the wave propagates.

Shishov[7,8] has formulated the integrated relations in the most general form. In his studies he also derived asymptotic formulas for two-dimensional scintillation spectra formed by the screens with piecewise power-law isotropic spectra of phase fluctuations. These asymptotics correspond either to small or large values of wave numbers. At present, description of the scintillation spectra in the intermediate range of wave numbers is only possible in terms of numerical integration of the initial equations. The one-dimensional spectra observed in practice are the integrals of two-dimensional spectra along the straight lines, which can cross the regions with unknown asymptotics. Therefore, there is no any alternative to numerical methods in describing one-dimensional spectra of strong scintillations.

There are many publications on numerical studies of strong scintillation spectra formed by phase screens. Some studies[9–11] are close to that presented in this paper. The two-dimensional scintillation spectra behind the isotropic phase screens were investigated in Refs. 9 and 10 and these were characterized by the small, compared to the Fresnel one, inner scale and different exponents in the power-law sections. The one-dimensional scintillation spectra were investigated in Ref. 11 behind two-dimensional anisotropic phase screens with the power-law inhomogeneity spectra, where the exponents of a power are more than three and less than four. Our statement of the problem differs in

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that we consider the models of the screen spectra allowing for both the anisotropy and large, compared to the Fresnel one, external and minimum scales.

Equations for scintillation spectra behind the statistically homogeneous phase screen and their asymptotics

Let us assume that a plane light wave of unity intensity is incident on a phase screen (plane (z, y)). Having in mind that the results obtained could be applied to the scintillation observations through the Earth's atmosphere, we shall choose a vertical in the beam perigee plane as z-axis. The scintillation observations are carried out in the plane that is parallel to the phase screen at the distance L from it. The two-dimensional scintillation spectrum or the spectrum of relative fluctuations of the light intensity in the observation plane $\Delta I(z, y, L) = I(z,y,L)/<I> -1$, where $<I>$ denotes the average value of light intensity (over an array of random realizations), is determined by the equations[7,8]:

$$F_I(p,q) = \frac{1}{4\pi^2}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\left[\exp(-k_0^2\Psi(z,y,\frac{L}{k_0}\kappa_z,\frac{L}{k_0}\kappa_y) - 1\right]\times \exp(-i\kappa_z z - i\kappa_y y)\mathrm{d}z\mathrm{d}y, \quad (1)$$

$$\Psi(z,y,z',y') = \left[D_S(z,y) + D_S(z',y') - \frac{1}{2}[D_S(z+z',y+y') + D_S(z-z',y-y')\right], \quad (2)$$

$$D_S(z,y) = 2\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}[1-\cos(\kappa_z z + \kappa_y y)]F_S(\kappa_z,\kappa_y)\mathrm{d}\kappa_z\mathrm{d}\kappa_y, \quad (3)$$

where $k_0 = 2\pi/\lambda$, λ is the light wavelength; L is the distance from the screen to the observation plane. The functions D_S and F_S are, correspondingly, the structure function and the spectrum of eikonal fluctuations on the phase screen.

The scintillation observations from onboard a spaceborne platform are carried out along its flight trajectory. In interpreting data of such observations one-dimensional spectral densities, $V_I(\kappa, \varphi)$, of scintillations observed in the plane $x = L$ along the straight line inclined with respect to the z—axis at an angle of φ are important, where κ is the wave number along the chosen straight line. In this paper, we shall restrict ourselves to the critical angles $\varphi = 0$ and $\varphi = \pi/2$. The one-dimensional spectra corresponding to these angles will be called vertical V_{ver} and horizontal V_{hor} spectra. These are determined as

$$V_{\text{ver}}(\kappa_z) = 2\int\limits_0^{\infty}F_I(\kappa_z,\kappa_y)\mathrm{d}\kappa_y,$$
$$V_{\text{hor}}(\kappa_y) = 2\int\limits_0^{\infty}F_I(\kappa_z,\kappa_y)\mathrm{d}\kappa_z. \quad (4)$$

Models of the phase fluctuation spectra

In specific calculations, the two-dimensional spectra of eikonal fluctuations F_S were set as follows

$$F_{SA}(K) = \frac{C\eta^2L_0^4}{K(1+L_0^4K^4)}\exp(-l_0^2K^2), \quad (5)$$

$$F_{SB}(K) = \frac{C\eta^2L_0^3}{K(1+L_0^3K^3)}\exp(-l_0^2K^2), \quad (6)$$

$$F_{SC}(K) = \frac{C\eta^2}{4(1-l_0^2/L_0^2)^3}\times \left[\frac{L_0^5(3-7l_0^2/L_0^2-4l_0^2K^2)}{(1+L_0^2K^2)^{5/2}} + \frac{l_0^5(7-3l_0^2/L_0^2+4l_0^2K^2)}{(1+l_0^2K^2)^{5/2}}\right], \quad (7)$$

where $K = \sqrt{\kappa_z^2 + \eta^2\kappa_y^2}$, η is the anisotropy factor; C is the parameter characterizing the fluctuation intensity, L_0 and l_0 parameters are the outer and minimum scales, respectively. Spectra (5) and (7) are the generalization of power-law spectrum $\sim K^{-5}$ typical for the inhomogeneities generated by the internal gravity waves in the stratosphere. The spectrum model (6) is characterized by the critical value of the exponent $\alpha = 4$ in the spectrum power-law portion $F_S \sim K^{-\alpha}$ in the range $(1/L_0 \ll K \ll 1/l_0)$. For the spectrum $F_S(K) \sim K^{-\alpha}$ at $\alpha \ge 4$, no D_S structure function exists. Model (7) differs from the model (5) by the spectrum behavior in the region of small and large wave numbers: $F_{SC} \to \text{const}$ at $|K|\,L_0 \ll 1$ and $F_{SC} \to |K|^{-9}$ at $|K|\,l_0 \gg 1$. It is convenient to use model (7) in calculations, since the corresponding one-dimensional spectrum and the structure function are set by the analytical expressions:

$$V_{SC}(\kappa_z) = 2\int\limits_0^{\infty}F_{SC}(\kappa_z,\kappa_y)\mathrm{d}\kappa_y = \frac{C\eta L_0^4}{(1+L_0^2\kappa_z^2)^2(1+l_0^2\kappa_z^2)^2}; \quad (8)$$

$$D_{SC}(r) = \frac{C\eta\pi}{(1-l_0^2/L_0^2)^2}\left\{L_0^3d_2\left(\frac{r}{L_0}\right) - \frac{4l_0^2}{(1-l_0^2/L_0^2)}\left[L_0d_1\left(\frac{r}{L_0}\right) - l_0d_1\left(\frac{r}{l_0}\right)\right] + l_0^3d_2\left(\frac{r}{l_0}\right)\right\}, \quad (9)$$

where

$$r(z,y) = \sqrt{z^2 + y^2/\eta^2};\quad d_1(\xi) = 1-\exp(-|\xi|),$$
$$d_2(\xi) = 1-[1+|\xi|]\exp(-|\xi|).$$

Simple asymptotic relations are known[8] for the scintillation spectra at small and large wave numbers.

At $(\kappa_z,\kappa_y \to 0)$, $\Psi \to 0$. By expanding the exponential term of Eq. (1) into a series, we obtain

$$F_I(\kappa_z,\kappa_y) = -\frac{k_0^2}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Psi(z,y,\kappa_z,\kappa_y)\times$$

$$\times\left(1-\frac{k_0^2\Psi(z,y,\kappa_z,\kappa_y)}{2}+\ldots\right)\exp(-i\kappa_z z - i\kappa_y y)\mathrm{d}z\mathrm{d}y. \quad (10)$$

If taking only one term of a series, Eq. (10) yields the formula of perturbation method of the geometric optics method

$$F_I^{(1)}(\kappa_z,\kappa_y) = L^2F_\mathrm{S}(\kappa_z,\kappa_y)(\kappa_z^2+\kappa_y^2)^2. \quad (11)$$

The spectrum (11) determines the variance β_0^2 of weak scintillations in the geometric optics:

$$\beta_0^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F_I^{(1)}(\kappa_z,\kappa_y)\mathrm{d}\kappa_z\mathrm{d}\kappa_y. \quad (12)$$

The quantity of β_0^2 is one of the basic parameters determining the scintillation spectra at a given distance from the screen. Allowing for Eqs. (11) and (3), β_0^2 can be presented as

$$\beta_0^2 = L^2\sigma_\mathrm{c}^2 = (1/2)L^2(1/F_z^2+1/F_y^2), \quad (13)$$

where

$$\sigma_\mathrm{c}^2 = \frac{1}{2}(\partial^2/\partial z^2+\partial^2/\partial y^2)^2 D_\mathrm{S}(z,y)\Big|_{z,y=0}$$

is the variance of fluctuations of the sum of principal curvatures of the eikonal distribution over the phase screen $c_z = \partial^2S/\partial z^2$ and $c_y = \partial^2S/\partial y^2$. The quantities

$$F_z = \left[\frac{\partial^4D_\mathrm{S}(z,y)}{\partial z^4}\Bigg|_{z,y=0}\right]^{-1/2},\quad F_y = \left[\frac{\partial^4D_\mathrm{S}(z,y)}{\partial y^4}\Bigg|_{z,y=0}\right]^{-1/2} \quad (14)$$

are the characteristic distances, at which the radiation is focused by the phase screen along vertical and horizontal directions, respectively. The variance σ_θ^2 of the refraction angle fluctuations is significant in the formation of strong scintillation spectra together with the screen parameter σ_c^2. It is determined as

$$\sigma_\theta^2 = \frac{1}{2}(\partial^2/\partial z^2+\partial^2/\partial y^2)D_\mathrm{S}(z,y)\Big|_{z,y=0}. \quad (15)$$

Another asymptotic equation following from Eq. (1) at large wave numbers[8] $KL/k_0 \gg L_0$ is the equation

$$F_I(\kappa_z,\kappa_y) = \frac{1}{4\pi^2}\times$$

$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp(-k_0^2D_\mathrm{S}(z,y)\exp(-i\kappa_z z - i\kappa_y y)\mathrm{d}z\mathrm{d}y, \quad (16)$$

since for the spectra (5)–(7) the structure function

$$D_\mathrm{S}(z,y)\to\text{const},\quad \Psi(\kappa_z,\kappa_y,z,y) = D_\mathrm{S}(z,y)$$

at $|z|\gg L_0$, $|y|\gg L_0\eta$. The scintillation spectrum (16) is the spectrum of the squared coherence function

$$\Gamma_2(z,y)^2 = \exp[-k_0^2D_\mathrm{S}(z,y)]$$

for the light field on the phase screen.

In this paper we propose a generalization of the asymptotic formulas (11) and (16) applicable to calculation of scintillation spectra behind the screen with the large-scale inhomogeneities in a wider region. In order to formulate these in a more compact form, it is convenient to pass to the dimensionless variables: $Z=z/R_\mathrm{F}$, $Y=y/R_\mathrm{F}$, $p=\kappa_zR_\mathrm{F}$, $q=\kappa_yR_\mathrm{F}$, where $R_\mathrm{F}=\sqrt{L/k_0}$ is the Fresnel scale. Equation (1) is written, using these variables, as follows

$$F_I(p,q) = \frac{1}{4\pi^2}\times$$

$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\exp(-\Psi(Z,Y,p,q)-1\right]\exp(-ipZ-iqY)\mathrm{d}Z\mathrm{d}Y, \quad (17)$$

and Eq. (2) remains the same. Spectral parameters of the screen inhomogeneities F_S and the corresponding structure function are normalized in the way as the variables do.

Let l_{0Z} and l_{0Y} be the dimensionless minimum scales of the structure function $D_\mathrm{S}(Z,Y)$. In the vicinity of the point with Z and Y coordinates, the function $D_\mathrm{S}(Z+z', Y+y')$ can be approximately presented as

$$D_\mathrm{S}(Z+z',Y+y') = D_\mathrm{S}(Z,Y)+(A_Zz'+A_Yy')+$$

$$+\frac{1}{2}[A_{Z,Z}z'^2+2A_{Z,Y}z'y'+A_{Y,Y}y'^2], \quad (18)$$

if $|z'|\ll l_{0Z}$, $|y'|\ll l_{0Y}$. Here A_Z, A_Y, $A_{Z,Z}$, $A_{Z,Y}$, and $A_{Y,Y}$ are the first and the second derivatives of the D_S function with respect to the coordinates indicated in the subscripts. Then for the asymptotics of the spectrum (1), allowing for the definitions by Eqs. (2) and (3), we obtain

$$F_I(p,q) = \frac{1}{4\pi^2}\times$$

$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\exp[-\frac{k_0^2}{2}(B_{Z,Z}p^2+2B_{Z,Y}pq+B_{Y,Y}q^2)]-1\right]\times$$

$$\times\exp(-ipZ-iqY)\mathrm{d}Z\mathrm{d}Y \quad (19)$$

at $|p|\ll l_{0,Z}$, $|q|\ll l_{0,Y}$ and

$$F_I(p,q) = \frac{1}{4\pi^2}\times$$

$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left[-\frac{k_0^2}{2}(B_{p,p}Z^2+2B_{p,q}ZY+B_{q,q}Y^2)\right]\times$$

$$\times\exp(-ipZ-iqY)\mathrm{d}Z\mathrm{d}Y =$$

$$= \frac{1}{2\pi k_0^2\sqrt{B_{p,p}B_{q,q}-B_{p,q}^2}}\exp\left[-\frac{B_{p,p}p^2-2B_{p,q}pq+B_{q,q}q^2}{2k_0^2(B_{p,p}B_{q,q}-B_{p,q}^2)}\right] \quad (20)$$

at $|p| \gg l_{0,Z}$, $|q| \gg l_{0,Y}$. Here

$$B_{\xi_1,\xi_2} = A_{\xi_1,\xi_2}(0,0) - A_{\xi_1,\xi_2}(\xi_1,\xi_2). \tag{21}$$

Asymptotics (19) for the scintillation spectrum is a refractive one (or geometric optics one). It represents the generalization of the formula (11) in the first approximation of the perturbation method of the geometric optics in the case of strong scintillations. The asymptotics (20) we shall call the diffraction one. Let us note that terms "refraction and diffraction scintillations" are widely used in astrophysics in order to denote the large-scale and small-scale ranges of scintillation spectra.[9,10] For description of these spectra, Eqs. (11) and (16) are applicable, respectively. The applicability of Eqs. (19) and (20) at scintillations caused by the large-scale inhomogeneities is essentially wider than that of Eqs. (11) and (16). This fact is confirmed by the comparison of the results on the scintillation spectra calculated by formulas (1)–(3), and by the approximate formulas.

Two-dimensional scintillation spectra

Figure 1 gives a general idea of the properties of two-dimensional weak and strong scintillation spectra generated by isotropic and anisotropic inhomogeneities.

It represents the lines of scintillation spectrum levels $G_I(\kappa_z, \kappa_y)$, the products of the spectral density $F_I(\kappa_z, \kappa_y)$, and the squared module of the wave number $(\kappa_z^2 + \kappa_y^2)$ normalized to the maximum value of this product, i.e.,

$$G_I(\kappa_z,\kappa_y)=(\kappa_z^2+\kappa_y^2)F_I(\kappa_z,\kappa_y)/\max[(\kappa_z^2+\kappa_y^2)F_I(\kappa_z,\kappa_y)]. \tag{22}$$

These quantities are calculated using the phase screen model (7). In all calculations we used the following values, $\lambda = 5\cdot 10^{-7}$ m, $L = 2200$ km, $R_F = \sqrt{\lambda L/2\pi} = 0.42$ m, $L_0 = 200$ m, and $l_0 = 10$ m as constants. The left column in Fig. 1 presents the two-dimensional weak scintillation spectra at $\eta = 1$, 2, and 10. Calculations of the weak scintillation spectra were also made for other values of η. These calculations have shown that even at $\eta = 1.015$, there is an apparent difference in scintillation spectra behind the isotropic and anisotropic screens in the vicinity of spectral maxima. The rearrangement at $\eta = 2$ effects the entire spectrum, while at the further increase in η, the spectral pattern does not change qualitatively.

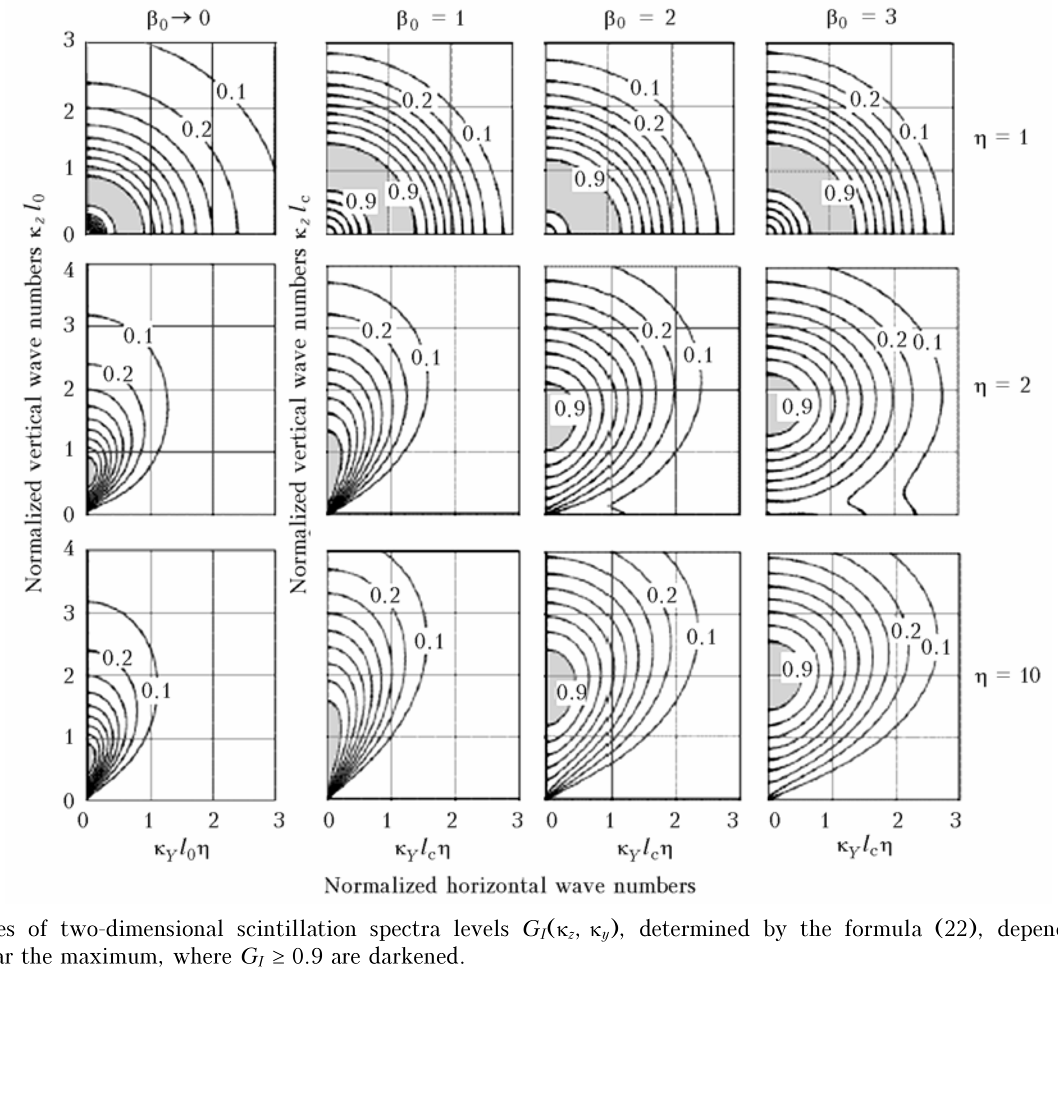

Fig. 1. Lines of two-dimensional scintillation spectra levels $G_I(\kappa_z, \kappa_y)$, determined by the formula (22), depending on η. Regions near the maximum, where $G_I \geq 0.9$ are darkened.

Weak scintillation spectra behind an isotropic screen have the maximum at $k = \sqrt{\kappa_z^2 + \kappa_y^2} \approx 0.7/l_0$; the spectrum width (at 1/3 of the maximum value) equals approximately to $2/l_0$. Almost the same coordinates have the maximum and spectral width behind the anisotropic screen in vertical sections ($\kappa_y = 0$), irrespective of η. The principle difference between the scintillation spectra behind the isotropic and highly anisotropic screens ($\eta > 2$) is that behind the anisotropic screen the spectrum is concentrated in the sector $|\kappa_z|/|\kappa_y| \geq \eta$. Thus, at larger values of the anisotropy coefficient η, scattering takes place only within a narrow sector near the vertical.

Strong scintillation spectra (see Fig. 1) are calculated by the formulas for diffraction asymptotics Eq. (20) for $\beta_0 = 1$, 2, and 3 and $\eta = 1$, 2, and 10. Let us note that wave numbers — both vertical and horizontal are normalized not to the minimum scale l_0 at strong fluctuations, but to the radius of coherence l_c, determined by the equation

$$k_0^2 D_S(l_c, 0) = 2. \tag{23}$$

The values of l_c for the spectrum model (7) at the calculated values of β_0 are equal to $3.9 \cdot 10^{-3}$ m (at $\beta_0 = 1$), $1.9 \cdot 10^{-3}$ m ($\beta_0 = 2$), and $1.3 \cdot 10^{-3}$ m (at $\beta_0 = 3$). Therefore, l_c is approximately 10^3–10^4 times less than l_0. The spectrum of strong scintillations spreads as compared with that of weak scintillations by this same factor. The small-scale component of the scintillation spectrum appears due to the radiation focusing by the large-scale random lenses of phase screen (with sizes on the order of L_0). The radius of a light spot a when focused by the perfect lens at a distance L can be estimated by the formula $a \approx L/(k_0 L_0)$. The characteristic scale of the interference pattern formed by inhomogeneities separated by the L_0 distance in the phase screen plane has the same order of magnitude. The parameters chosen for calculation are $a \approx 2 \times 10^{-3}$ m, i.e., the same as the values of coherence radii.

One-dimensional vertical scintillation spectra

As was mentioned, the one-dimensional scintillation spectra, in particular, vertical and horizontal, determined by formulas (4) are of a special interest for practical applications. Calculations for the vertical spectra were carried out by asymptotic formulas for a model of one-dimensional phase screen. The assumption of the applicability of the one-dimensional phase screen model to calculation of the vertical spectra formed by two-dimensional anisotropic screens with rather high η, has been formulated earlier.[12] The one-dimensional asymptotic formulas following from Eqs. (19) and (20), if D_S does not depend on y coordinate in dimensional variables, have the following form:

refraction asymptotics

$$V_I(\kappa_z) = \frac{1}{\pi}\int_0^\infty \cos(\kappa_z z)\left[\exp(-\frac{1}{2}L^2 D_\theta(z)\kappa_z^2) - 1\right]\mathrm{d}z, \tag{24}$$

where

$$D_\theta(\xi) = \partial^2 D_S(\xi)/\partial\xi^2\Big|_{\xi=0} - \partial^2 D_S(\xi)/\partial\xi^2 =$$

$$= 4\int_0^\infty p^2[1 - \cos(pz)]V_S(p)\mathrm{d}p \tag{25}$$

is the structure function of the refraction angle fluctuations on the phase screen;

diffraction asymptotics in one-dimensional case is reduced to the algebraic expression

$$V_I(\kappa_z) = \frac{1}{k_0\sqrt{2\pi D_\theta(\kappa_z L/k_0)}}\exp\left(-\frac{\kappa_z^2}{2k_0^2 D_\theta(\kappa_z L/k_0)}\right). \tag{26}$$

Note that both, refraction and diffraction asymptotics, are determined by one function, D_θ.

The numerical calculations of one-dimensional vertical and horizontal spectra were carried out for the models of eikonal fluctuation spectra (5) and (6) at $\eta = 10$. The calculated vertical spectra are shown in Figs. 2 and 3.

Figure 2*a* presents the large-scale parts of one-dimensional vertical scintillation spectra for the model (5), Fig. 2*b* presents that for the model (6). These spectra are normalized to the value of β_0^2. Such a normalization allows one to estimate the applicability of the perturbation theory to calculation of a large-scale part of the one-dimensional scintillation spectra. If the perturbation theory were applicable to the description of scintillation spectra, all the curves in Fig. 2 would coincide with the dashed lines.

With increasing β_0^2, the applicability of the perturbation theory gets narrower. In particular, when $\beta_0^2 = 100$, the perturbation theory is applicable to the description of spectrum interval $\kappa_z R_F \leq 10^{-3}$ or $\kappa_z \leq 1/L_0$.

As follows from the data presented in Figs. 2*a* and 3, the one-dimensional refraction asymptotics is applicable to calculation of the scintillation spectra behind two-dimensional screen over the range of wave numbers $\kappa_z R_F \geq 2 \cdot 10^{-4}$ or ($\kappa_z \geq 1/\eta L_0$) at $0.1 \lesssim \beta_0^2 \lesssim 100$.

Unlike the applicability of the perturbation theory, the applicability of the refraction asymptotics (24) extends with increasing β_0 into the region of large wave numbers. At $\beta_0 \geq 1$, it is applicable over the range of $\kappa_z R_F \leq (l_0/R_F)$, whereas in the region of weak scintillations at $\beta_0^2 \to 0$, the applicability of the refraction asymptotics is limited by the wave number from above by the well-known condition $\kappa_z R_F \leq 1$.

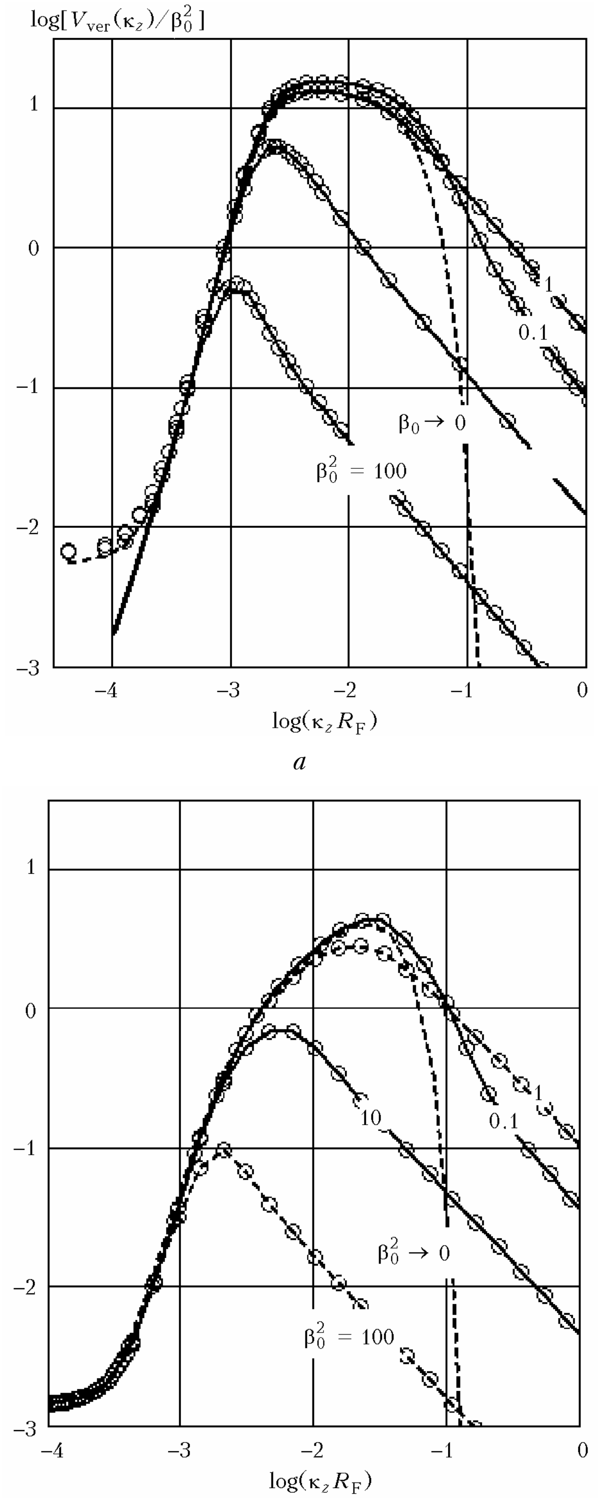

Fig. 2. The one-dimensional vertical scintillation spectra in the region of small wave numbers: (*a*) behind the screen with spectrum (5), (*b*) behind the screen with spectrum (6) at η = 10. Continuous lines: (*a*) denote the data calculated by the refraction asymptotics for one-dimensional screen model by the formulas (24), (25); dots show data calculated by formulas (1)–(4). No calculations approximate formulas were made for the model (6).

Figure 3 presents the small-scale parts of one-dimensional vertical scintillation spectra behind the screens with the spectra (5) and (6). The wave numbers are normalized to R_F, as in Fig. 2. One can see the difference in scintillation spectra behind the screens with various eikonal fluctuation spectra. This difference is almost insignificant, if the wave numbers are normalized to the coherence scale l_c. There is a plateau clearly distinguished in spectra with the functions $\kappa_z V_I(\kappa_z) \approx \text{const}$, $\kappa_z V_{\text{ver}}(\kappa_z) \approx \text{const}$.

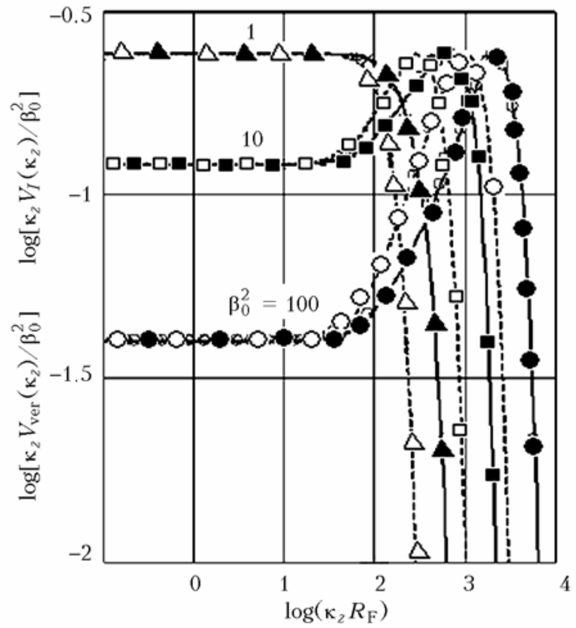

Fig. 3. The one-dimensional vertical spectra in the region of large wave numbers normalized to the reverse Fresnel scale R_F. Lines are the calculations on diffraction asymptotics (26) for the one-dimensional screen model, dark tags denote the calculation data by the formulas (1)–(4) for model (5), light tags denote the calculation data for model (6).

The data calculated by formula (26) and by the exact formulas for two-dimensional screen at η = 10 coincide accurate to 1% over the ranges of wave numbers $\kappa_z \geq (2\text{–}3)/(l_0\beta_0^{3/2})$ at $0.1 \leq \beta_0^2 \lesssim 100$. These results point out the correctness of both one-dimensional screen model and refraction and diffraction asymptotics for the description of one-dimensional vertical spectra within the range of the parameters variation used in this study. The applicability of the refraction and diffraction asymptotics can overlap within a significant interval of wave numbers, where the spectrum plateau is observed $\kappa_z V_I(\kappa_z)$. As follows from formula (24) at $\kappa_z L\sigma_\theta \gg 1$ and formula (26) at $\kappa_z \ll l_0 k_0/L$, the spectrum level on the plateau is determined by the relation

$$\kappa_z V_I^{(\text{p})}(\kappa_z) = \frac{1}{\sqrt{2\pi}\beta_0}\exp\left(-\frac{1}{2\beta_0^2}\right). \quad (27)$$

It depends on the single parameter of scintillation problem, namely, on β_0, equal, according to Eq. (13), to the ratio of the distance L to the effective radius of curvature of the eikonal F_z on the phase screen.

As follows from the asymptotic formula (26), the spectrum $\kappa_z V_I(\kappa_z)$ reaches its maximum value $\max[\kappa_z V_I(\kappa_z)] = 1/\sqrt{2\pi e} \approx 0.242$ at $\kappa_z^2 = D_\theta(\kappa_z L/k_0)$. Formula (26) allows estimating the wave number range where the scintillation spectrum is normalized behind the one-dimensional screen, i.e., represents the spectrum of squared coherence function of the light field. In one-dimensional case, behind the screen with large-scale inhomogeneities, the spectrum of squared coherence function is presented as

$$V_I(\kappa_z) = \frac{1}{k_0\sqrt{2\pi D_\theta(\infty)}} \exp\left(-\frac{\kappa_z^2}{2k_0^2 D_\theta(\infty)}\right), \quad (28)$$

i.e., it presents a limiting expression following from Eq. (26) at $\kappa_z L/k_0 \gg L_0$, when $D_\theta(z) = D_\theta(\infty)$. Figure 4 illustrates the type of structure functions $D_\theta(z)$ for the considered models of the spectra.

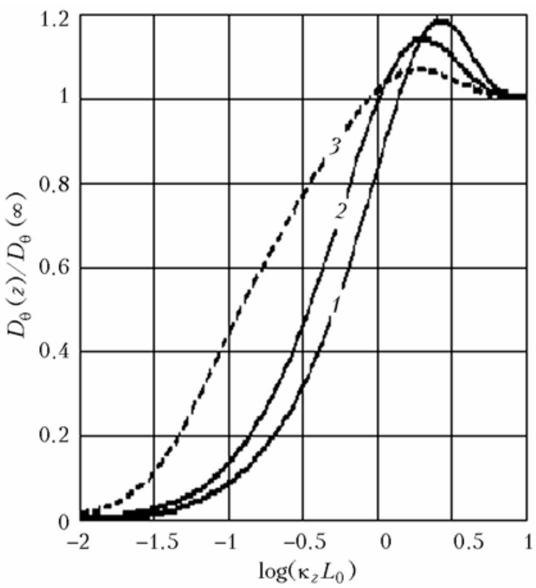

Fig. 4. Structure functions of the refraction angle fluctuations for models (5) (curve *1*), (6) (curve *2*) and (7) (curve *3*).

In the region of $z \le l_0$, there is a function $D_\theta(z) \approx \beta_0^2 z^2/L^2$ for any eikonal spectra. The dependences of $D_\theta(z)$ have maximum at $z \approx (2\text{–}3)L_0$ and reach the fixed level of $D_\theta(\infty)$ at $z \gg L_0$. It is an important feature of the structure functions $D_\theta(z)$ and $D_\theta(\infty)$ that their difference is insignificant at $z \ge L_0$. The value $\Delta = \max[D_\theta(z)]/D_\theta(\infty) - 1$ does not exceed 20% at $z \ge L_0$ over the range of the ratio $0.01 \le l_0/L_0 \le 0.1$ for all the three models. Having this in mind, one can accept $\kappa_z L/k_0 \ge L_0$ as the applicability condition for formula (27). This condition has a clear physical meaning in the angular representation $\xi = \kappa_z/k_0$ and written as $|\xi| \ge L_0/L$. It means that normalization can be observed in the angular range where scintillations are formed by the screen inhomogeneities separated by distances longer or equal to the outer scale of inhomogeneities.

The conclusions about the applicability of one-dimensional screen model to calculation of the vertical spectra were drawn based on their comparison with the calculated data for the region behind the screen at $\eta = 10$ and $\beta_0 \le 10$. The calculations made using large values of β_0 have shown that the one-dimensional screen model becomes incorrect. Strong focusing by screen inhomogeneities on the horizontal axis is not taken into account in one-dimensional model. The effect of this focusing can be significant, under condition that $F_y \approx \eta^2 F_z \lesssim L$, where F_y and F_z are the effective radii of curvature along the horizontal and vertical directions. This condition can be written as $\beta_0\eta^2 \gtrsim 1$ allowing for $\beta_0 = L/F_z$. Incorrectness of the one-dimensional screen model in the case of a one-dimensional vertical spectrum manifests itself, in particular, in that the level of the spectrum plateau $\kappa_z V_I(\kappa_z)$ behind a two-dimensional screen can differ from the level determined by formula (27). The quantitative applicability conditions for one-dimensional screen model can be formulated based on calculations by the formulas for the two-dimensional diffraction asymptotics (20). Calculations for model (5) with $\eta = 5$, 10, and 15 have shown that error of calculations of one-dimensional vertical scintillation spectra behind an anisotropic screen by formulas for one-dimensional screen does not exceed 10%, if $\beta_0 \le 0.3\eta^2$.

Horizontal scintillation spectra

The results calculated for horizontal spectra are presented in Fig. 5. Figure 5*a* presents the spectra at small wave numbers. They were calculated only by exact formulas (1)–(4). Let us note that for the horizontal spectra calculated at $\kappa_y \to 0$, the perturbation theory is not applicable already at $\beta_0^2 = 1$. Figure 5*b* presents the horizontal spectra at large wave numbers. They were calculated both by exact formulas (marked by asterisks), and by the formulas for two-dimensional diffraction asymptotics (20) (solid lines) with subsequent calculation of the second integral (4). As follows from the comparison of results by exact and approximated formulas, they almost do not differ over the range of wave numbers $\kappa_y \ge 0.01/r_F \approx 2.5/(l_0\eta)$ for all calculated values of β_0^2.

Let us note that in contrast to vertical spectra (see Fig. 3), where the plateau of the product of $\kappa_z V_I(\kappa_z)$ and $\kappa_z V_{\text{ver}}(\kappa_z)$ is occurs over a wide range, in the horizontal spectra the plateau of $\kappa_y V_{\mathrm{hor}}(\kappa_y)$, strictly speaking, does not forms. Roughly, accurate within 10%, this plateau is formed over a narrow range of the wave numbers $2.5/(l_0\eta) \le \kappa_y r_F \le 1$. It also should be noted that dependence of the function maximum $\kappa_y V_{\mathrm{hor}}(\kappa_y)$ on β_0^2 at $1 \le \beta_0^2 \le 1000$ is nonmonotonic, i.e., horizontal spectrum is not normalized even at $\beta_0^2 = 1000$.

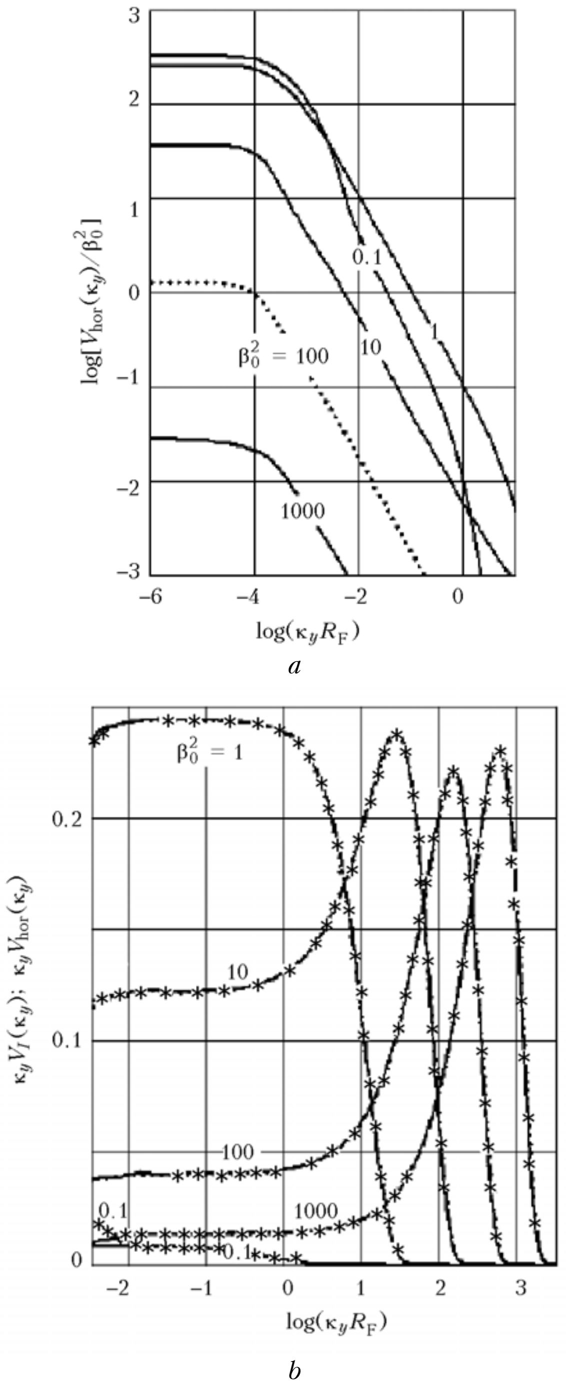

Fig. 5. Horizontal scintillation spectra behind the phase screen with spectrum (5) at small (*a*) and large (*b*) wave numbers.

Conclusion

The calculations for strong scintillation spectra formed by atmospheric inhomogeneities and generated by internal gravity waves have been made using the model of statistically homogeneous phase screen. Inhomogeneities are characterized by the anisotropy of their spectra; moreover, their dimensions exceed the Fresnel's zone scale in the observation plane.

The applicability limits of the perturbation theory in calculating the scintillation spectra have been investigated. The large-scale part of the vertical scintillation spectra is shown to be described by the perturbation theory even when scintillations are strong, the root-mean-square value of the relative fluctuation of intensity β_0 calculated according to the perturbation theory, being about 10. At the same time, the perturbation theory is inapplicable to the description of the small-scale part of the vertical scintillation spectra already at $\beta_0 > 0.3$. For description of large-scale range of the horizontal scintillation spectra, the perturbation theory is applicable, if $\beta_0 \le 1$.

The asymptotic formulas for description of large-scale and small-scale scintillation spectra have been proposed, called the refraction and diffraction asymptotics. Their applicability and applicability of the one-dimensional screen model have been investigated in calculation of vertical spectra behind the two-dimensional anisotropic screen. The one-dimensional model is shown to be applicable for calculation of small-scale spectral range under condition that $\beta_0 \ge 0.3\eta^2$. If this condition is satisfied, the product of the one-dimensional spectral density and the wave number has a plateau, whose level is determined only by the value of β_0. In the region of large-scale wave numbers the maximum in the scintillation spectrum is formed with the value equal to 0.242.

The normalization conditions for the small-scale part of the spectrum have been formulated. Under these conditions, the scintillation spectrum is the spectrum of squared coherence function on the phase screen. Normalization is possible over the range of scattering angles larger than the ratio of the outer scale to the distance between the phase screen and the observation point.

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