ON A TECHNIQUE FOR MEASURING THE SCATTERING PHASE FUNCTION USING THE LIGHT FIELD FROM A SOURCE WITH A WIDE DIRECTIONAL PATTERN

B.A. Tarashchanskii, O.N. Gaponenko, and V.I. Dobrynin

Scientific – Research Institute of Applied Physics at the State University, Irkutsk

Received July 25, 1994

A feasibility is discussed of reconstructing the scattering phase function from the light field formed in a homogeneous medium by isotropic emission from a source with a wide directional pattern. The influence of errors in light field brightness measurement on the accuracy of the phase function reconstruction is evaluated. Measurement data on angular distribution of brightness at a distance of 5 km from a Lambertian source are presented. The data are used to reconstruct the scattering phase function for water of the Lake Baykal.

One of the key problems in optics is the determination of primary optical properties of a medium (those which are independent of illumination or observation conditions). Among the primary hydrooptical characteristics are the absorption coefficient κ, the scattering coefficient σ, the extinction coefficient $\varepsilon = \kappa + \sigma$, and the scattering phase function χ (Ref. 1). Generally, measurements of these characteristics involve study of the structure of light fields from artificial sources of radiation. The point sources, such as isotropic, Lambertian (cosine), and unidirectional ones, have gained wide acceptance in optical recearches into ocean.[2]

The method for absorption coefficient measurements using an isotropically emitting light source coupled with the meter of half–space illumination has been proposed in Ref. 3. References 4 and 5 validate the calculations of κ and the single scattering albedo $\Lambda = \sigma/\varepsilon$ from the spatial and angular distribution of brightness of light field formed in sea water from a source of isotropic emission. The calculation of Λ by the method proposed in Ref. 5 requires an additional information on the scattering phase function.

The standard technique for measuring the phase function assumes photometric evaluation of scattered radiation at an angle γ with respect to the direction of an incident narrow beam.[2] The present paper studies the feasibility of reconstructing $\chi(\gamma)$ from the angular distribution of brightness field created by emission from a point source with a wide directional pattern. Similar problem with a source of isotropic radiation has been discussed in our previous work.[6]

Now consider the conditions of light propagation and assumptions used in deriving the formulas. To describe the spatial structure of the light field from a point isotropic source, we introduce the coordinate system shown in Fig. 1. The source of monochromatic radiation S is chosen to be located at the origin of coordinates. A photodetector D with the aperture $S_D \ll R_0^2$ and angle $\Omega_D \ll 4\pi$ is located at a distance R_0 from the source and oriented along the direction dΩ'. The maximum of radiation from the source is in the direction to photodetector; the source angular characteristic is described by known function: $F(\vartheta)$. A photon, emitted by the source in the direction dΩ, passes the distance R_1, undergoes the scattering at an angle γ, and passes an extra distance R_2 before reaching the photodetector.

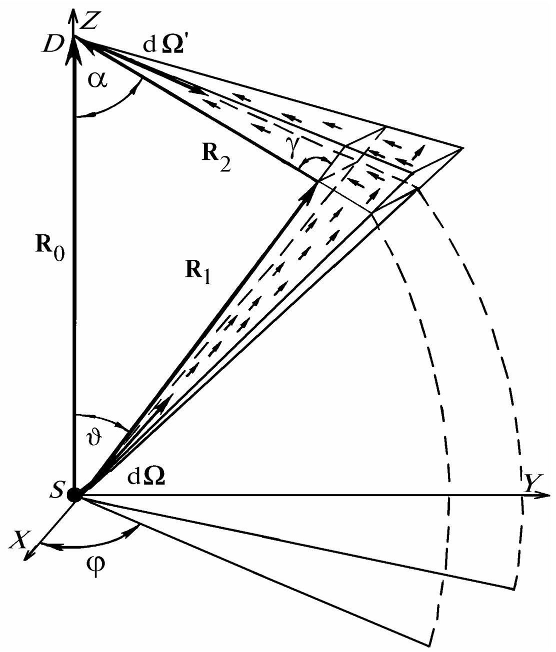

FIG. 1. The coordinate system. Here S is the source of light; D is the photodetector; dΩ *is the viewing direction;* ϑ, γ, *and* α *are the polar angles of emission, scattering, and view;* φ *is the azimuth angle.*

For a quantitative description we need to define the density of photon flux B_p through the surface element dS normal to the direction within an elementary solid angle dΩ at the moment t, that is

$$B_p = \mathrm{d}\, N_p/(\mathrm{d}\, t\, \mathrm{d}\, S\, \mathrm{d}\Omega), \qquad (1)$$

where N_p is the number of photons.

Bellow, B_p will be called brightness because for the monochromatic radiation it is related to the energy brightness B_e simply as $B_p = B_e/h\nu$ (Ref. 2), where $h\nu$ is the photon energy.

0235-6880/94/11–12 819-04 $02.00

It can be readily shown that the angular distribution of brightness of scattered radiation in the single scattering approximation is

$$B_{p_1}(R_0, \alpha) = (I_0/4\pi R_0^2)(\sigma R_0)(1/\sin\alpha) \times$$
$$\times \int\limits_0^{\pi-\alpha} \chi(\alpha + \vartheta) \exp[-\varepsilon R(\alpha, \vartheta)]\, F(\vartheta)\, d\vartheta. \qquad (2)$$

Here I_0 is the intensity of emission from the source;

$$R(\alpha, \vartheta) = R_1 + R_2 = R_0(\sin\alpha + \sin\vartheta)/\sin(\alpha + \vartheta)$$

is the path traversed by a photon from the source to the receiver. This result is independent of the azimuth angle φ due to the axial symmetry.

Now we give the expressions for angular characteristics for a source of isotropic radiation

$$F(\vartheta) = 1/4\pi$$

and the Lambertian one

$$F(\vartheta) = \begin{cases} \cos\vartheta/\pi & \text{for } \vartheta \le \pi/2 \\ 0 & \text{for } \vartheta > \pi/2\ . \end{cases}$$

Equation (2) has a simple physical meaning: for single scattering the radiative field in the direction $d\Omega'$ is formed by photons scattered at angles $\alpha \le \gamma = (\alpha + \vartheta) \le \pi$ $(0 \le \vartheta \le \pi - \alpha)$; the probability of scattering per unit path is determined by the value of scattering coefficient for a given direction, i.e., $\sigma(\gamma) = \sigma\, \chi(\gamma)/4\pi$; due to attenuation by medium particular scattering contributions have weights $\exp[-\varepsilon\, R(\alpha, \vartheta)]$; the factor $(\sin\alpha)^{-1}$ comes when changing from the flux of scattered photons to the brightness, i.e., the flux density within a unit solid angle $d\Omega' = \sin\alpha\, d\alpha\, d\phi$.

The photon flux at the photodetector is[1]

$$\dot{N}_p = \frac{d N_p}{d t} = \int\limits_{S_D}\int\limits_{\Omega_D} B_p(R_0, \alpha)\, d\Omega'\, d S. \qquad (4)$$

For angles α such that $\sin\alpha \gg \Delta\alpha$ ($2\Delta\alpha$ being the receiver aperture angle), we can restrict ourselves by simple evaluation of integrals (4) assuming the integrand as a constant over narrow intervals: $\Omega_D/4\pi \ll 1$ and $S_D/R_0^2 \ll 1$. Let the factors independent of the angular coordinate α be introduced into the factor $\dot{N}_0 = I_0(\sigma R_0)(\Omega_D/4\pi)(S_D/R_0^2)$. Then, taking into account Eqs. (2) and (4), we obtain

$$\dot{N}_p(\alpha) \sin\alpha = \dot{N}_0 \int\limits_0^{\pi-\alpha} \chi(\alpha + \vartheta) \exp[-\varepsilon\, R(\alpha, \vartheta)]\, F(\vartheta)\, d\vartheta. \quad (5)$$

From this formula it is seen that the scattering phase function $\chi(\gamma = \alpha + \vartheta)$ can be found from the experimental data on $\dot{N}_p(\alpha)$ by solving relevant integral equation. The constant $\dot{N}_0$ can hardly be determined directly from the experimental data. Fortunately, it does not enter into the final result since the phase function satisfies the normalization condition

$$\int\limits_{4\pi} \chi(\Omega)\, d\Omega = 2\pi \int\limits_0^{\pi} \chi(\gamma) \sin(\gamma)\, d\gamma = 1.$$

To solve Eq. (5), let us use discrete variables: $\alpha_\kappa = \kappa h$ and $\vartheta_l = l h$ $(\kappa, l = 0, \ldots, n-1;\ h = \pi/n)$ instead of continuous ones α and ϑ. Subsequent conversion of integral into a finite sum (the method of rectangles) and moving the term with $\vartheta_{l=0}$ to the left–hand side yield

$$\chi(\alpha_\kappa) = \Big\{ f(\alpha_\kappa)/h -$$
$$- \sum_{l=1}^{n-(\kappa+1)} \chi(\alpha_\kappa + \vartheta_l) \exp\big[-\varepsilon R(\alpha_\kappa, \vartheta_l)\big] F(\vartheta_l) \Big\} \exp(\varepsilon R_0)/F(0), \quad (6)$$

where $f(\alpha) = (\dot{N}_p(\alpha)/\dot{N}_0) \sin\alpha$.

As follows from Eq. (6), at a point α_κ the phase function can be determined experimentally by measuring photon flux $\dot{N}_p(\alpha)$ at $\alpha = \alpha_\kappa$ as well as from values of the scattering phase function at points $\alpha_{\kappa+1}, \ldots, \alpha_{n-1} = \pi - h \cong \pi$. For the recurrence to start, the value of the scattering phase function at the point $\alpha = \pi$ is to be known. Making use of Eq. (5) we can find

$$\chi(\pi) = -\exp(\varepsilon\, R_0) \lim_{\alpha \to \pi - 0} (\partial f/\partial\alpha) \,/\, F(0),$$

thus completing the solution.

It should be emphasized that the equation (5) falls into the class of ill–posed integral equations, that is, slight changes in the function $f(\alpha)$ can have a drastic effect on the solution $\chi(\gamma)$. The accuracy of calculations by formula (6) increases for smaller h. At the same time, if h is too small, the error in $\dot{N}(\alpha)$ measured becomes too large, so it is necessary to select an optimal step. The present method of determining $\chi(\gamma)$ is stable. Of course, slight deviations occurring in one value of $\chi(\alpha_m)$ (for $\kappa < m < n$) are incapable to affect noticeably the value of the scattering phase function at a point α_κ since the latter, as seen from formula (6), is a sum of many contributions of the same order of magnitude.

We choose the data from Ref. 4 to illustrate the method proposed for the scattering phase function reconstruction. In this reference one finds angular distributions of brightness $B_e(R, \alpha)$ at distances $10 \le R \le 115$ m from the source of isotropic radiation. The measurements were carried out in the North – West of the Black Sea at 100–m depth. Similar measurements of the scattering phase function for a number of angles γ are described in Ref. 4 for the same sea waters.

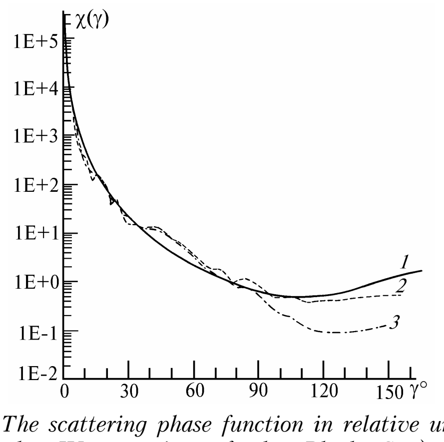

FIG. 2. The scattering phase function in relative units (for the North – West region of the Black Sea): in situ *measurements of the scattering phase function[4] (1), scattering phase function calculated from the angular distribution of brightness of radiation field formed by isotropic emission in sea water (the brightness measurements are borrowed from Ref. 4) (2), and the scattering phase function calculated using the same data by the method from Ref. 7 (3).*

Figure 2 shows the measured scattering phase function χ_{bs} and that calculated by formula (6) $\chi_2(\gamma)$ (for $R_0 = 10$ m and $\varepsilon = 0.12\ \text{m}^{-1}$).

Also shown is the phase function obtained by taking the derivative of brightness field from source of isotropic radiation with respect to the angle α (Ref. 7)

$$\chi_1(\gamma) = -\left[\frac{\partial\{B_e(R_0,\ \alpha)\ \sin\alpha\}}{\partial\alpha}\right]_{\alpha=\gamma}. \qquad (7)$$

Reference 7 uses the common property of scattering phase functions in natural waters, i.e., their strong forward peaked shape. In this case, photon trajectories differ slightly ($R(\alpha,\ \vartheta) \cong R_0$), and the formula (7) can be derived (accurate to a constant factor) directly from Eq. (5). The domain of applicability of the method (7) is restricted to angles $\gamma < 30°$ (Ref. 7). However, to compare both calculation techniques, we present the results over the entire range of $\chi_1(\gamma)$.

The measured and calculated scattering phase functions were rescaled to avoid their different normalizations. For the calculated scattering phase functions, this was done by multiplying them by the coefficients $m_{1,2}$ given by the expression

$$m_{1,2} = (1/n) \sum_{i=1}^{n} \chi_{bs}(\gamma_i)/\chi_{1,2}(\gamma_i),$$

where the summation is done over the range $4.5° \le \gamma_i \le 95°$.

As seen from Fig. 2, $m_{1,2}\chi_{1,2}(\gamma)$ and $\chi_{bs}(\gamma)$ agree quite well. For example, while the scattering phase function varies by about four orders of magnitude ($\chi_{bs}(4.5°)/\chi_{bs}(95°) \simeq 6\cdot 10^3$), relative discrepancies between calculations and measurements averaged over the interval 4.5÷95°

$$\delta_{1,2} = \frac{1}{m_{1,2}} \sqrt{(1/n) \sum_{i=1}^{n} \left[\chi_{bs}(\gamma_i)/\chi_{1,2}(\gamma_i) - m_{1,2}\right]^2},$$

do not exceed 30%.

Since the brightness is measured with a quite large error ($\cong 20\%$), while the scattering phase function is measured with the error no less than 10% (Ref. 4), such an agreement between the experimentally measured scattering phase function and that calculated from brightness field can be recognized as satisfactory.

Comparison of the functions $\chi_1(\gamma)$, $\chi_2(\gamma)$, and $\chi_{bs}(\gamma)$ reveals that the first method and the second one provide the same accuracy of reconstruction for $\gamma \le 90°$ (the extension of domain of applicability of formula (7) for χ_1 up to 90° is, seemingly, due to strong forward elongation of the scattering phase function in the Black Sea water. The method (χ_2), proposed in the present paper, describes the behavior of $\chi_{bs}(\gamma)$ for angles $\gamma > 90°$ much better.

We applied the above approach to study the scattering phase functions in the water of the Lake Baykal.

In situ optical measurements in the Lake Baykal were carried out within the frameworks of the study of feasibility of deep–water recording of elementary particles.[7–9] In particular, the angular distribution of brightness at a distance of 5 m from Lambertian source was investigated in one experiment. The measurements carried out at a 1–km depth in the South–Baikal basin, 3.5 km away from the coast (in the location of neutrino detector), where water was optically homogeneous.

A description of the device and light source are given in Ref. 8 in detail. For brightness measurements, the light scattering collector, located in front of the device's window and used in measurement of κ, was replaced by the system of two plane mirrors, one of which rotatable to provide scanning over α with a step of 0.5°. The rotation was performed through the ratchet gear with electromagnetic drive. The photoreceiver aperture angle was bounded within $\Delta\alpha = 0.5°$ using a collimator.

The linear dynamic range of the photodetector (PMT–130 operating in the photon counting mode) and of recording electronics is much narrower than the range of variations of scattered radiation brightness. So, source brightness was increased stepwisely for two α values: 2.5° and 9°, for which we calculated rescaling coefficients required to obtain a smooth function $B(\alpha)$.

In the experiment we measured the number $N_{pe}(\alpha)$ of single–photoelectron pulses on the PMT anode during time $\tau = 10$ s. The photoelectron count rate $\dot{N}_{pe}(\alpha) = N_{pe}(\alpha)/\tau$ is proportional to the photon flux through the water in a given direction, that is,

$$\dot{N}_{pe}(\alpha) = W\ \eta\ K\ \dot{N}_p(\alpha), \qquad (8)$$

where K is the product of the mirrors reflection coefficients and the window and light filter transmittances, η is the quantum efficiency of a PMT's photocathode at a given wavelength, W is the probability of photoelectron detection.

The errors in measuring $\dot{N}_{pe}(\alpha)$ are caused by the temporal instability in the recording channel and radiation source, statistical fluctuations of N_{pe}, optical system misalignment, and errors in determining rescaling coefficients. The relative error in $\dot{N}_p(\alpha)$ measurements due to the above–mentioned causes is estimated to be no more than 5% for $\alpha < 100°$, giving the error of $\chi(\gamma)$ reconstruction not exceeding 20%.

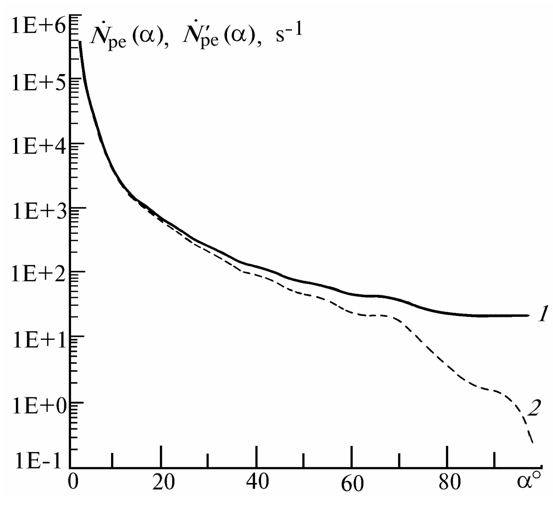

FIG. 3. The angular distribution of brightness at a distance of 5 m from a Lambertian source of light (the Lake Baykal 1–km *depth): counting rate of photon pulses $\dot{N}_{pe}(\alpha)$ measured with a* PMT–130 *(1), calculations of $\dot{N}'_{pe}(\alpha)$ (2).*

Measurements of $\dot{N}_{pe}(\alpha)$ are shown in Fig. 3. For the given distribution we used the formula (6) to calculate the scattering phase function for water of the Baykal, $\chi_{Bw}(\gamma)$. Then these values were substituted into equation (5) to yield the flux of scattered radiation $\dot{N}'_p(\alpha)$. From Eq. (8) we obtain the expected rate of a PMT count rate, $\dot{N}'_{pe}(\alpha)$.

The discrepancy between the calculated $\dot{N}'_{pe}$ and measured $\dot{N}_{pe}$ gives an estimate of the error in reconstruction of the scattering phase function. As is seen from Fig. 3, $\dot{N}_{pe}(\alpha)$ and $\dot{N}'_{pe}(\alpha)$ are close within a wide range of angles $\alpha \lesssim 70°$.

For $\alpha > 100°$, the total count rate $\dot{N}_t$ of a PMT becomes comparable to the count rate of a PMT dark current pulses $\dot{N}_{dc}$, even for maximum of the source brightness. As a result, the relative error in determining $\dot{N}_{pe}$ increases sharply, because $\dot{N}'_{pe} = \dot{N}_t - \dot{N}_{dc}$. In experiment the values of $\dot{N}'_{pe}(\alpha)$ were measured with a required accuracy only for α up to 100°. In calculations of the scattering phase function, we set $f(\alpha_\kappa) = 0$ for $\alpha_\kappa > 100°$. This resulted in the discontinuity of the function $f(\alpha)$, whose edge anomaly affected the result of a specific numerical procedure (6), explaining the existing discrepancy between $\dot{N}_{pe}$ and $\dot{N}'_{pe}$ for $\alpha \geq 70°$. Actually, the difference $(\dot{N}_{pe} - \dot{N}'_{pe})$ is vanishing when moving away from the point of discontinuity, what illustrates the solution stability mentioned above.

Table I presents the values of scattering phase function of Baykal water calculated from the angular distribution of brightness at a 5–m distance from a Lambertian light source. The scattering phase function is scaled to unity within the interval $2° \leq \gamma \leq 100°$.

TABLE I. The scattering phase function of Baykal water. In situ *measurements on March 26, 1988* (H = 1000 m, λ = 497 nm, $\varepsilon = 0.08\ m^{-1}$).

γ, °	$\chi(\gamma)$, sr^{-1}	γ, °	$\chi(\gamma)$, sr^{-1}	γ, °	$\chi(\gamma)$, sr^{-1}
2	$1.5\cdot10^{2}$	10	$8.9\cdot10^{-1}$	50	$1.6\cdot10^{-2}$
3	$3.6\cdot10^{1}$	15	$1.9\cdot10^{-1}$	60	$9.2\cdot10^{-3}$
4	$1.2\cdot10^{1}$	20	$1.2\cdot10^{-1}$	70	$6.5\cdot10^{-3}$
5	$7.5\cdot10^{0}$	25	$7.6\cdot10^{-2}$	80	$4.7\cdot10^{-3}$
6	$5.4\cdot10^{0}$	30	$5.3\cdot10^{-2}$	90	$3.4\cdot10^{-3}$
7	$3.6\cdot10^{0}$	35	$3.8\cdot10^{-2}$	100	$2.8\cdot10^{-3}$
8	$2.3\cdot10^{0}$	40	$2.8\cdot10^{-2}$	–	–
9	$1.5\cdot10^{0}$	45	$2.2\cdot10^{-2}$	–	–

To calculate $\chi(\gamma)$ by Eq. (6), one should know the extinction coefficient ε in addition to $N_p(\alpha)$. Reference 8 gives the typical Baykal water extinction coefficient being about $0.08\ m^{-1}$ at 1–km depth. Numerical simulation reveals weak sensitivity of the scattering phase function to slight variations in ε in the vicinity of this value (changes of $\chi(\gamma)$ in Eq. (6) due to small ε variations are eliminated by normalizing). So, replacement of ε by any of its estimate $\tilde{e}$, with their difference of, e.g., 30% causes only 10% change in the scattering phase function at $\gamma \leq 100°$.

The results discussed in the present paper have been obtained in the single scattering approximation. Generally, equation (2) can be considered as the first term in expansion of the brightness into a series over the number of successive single scatterers. The first order approximation is valid only for small expansion parameter, $\sigma R_0 \ll 1$. In the experiment, this parameter is estimated to be $\sigma R_0 \leq 0.2$. Then, according to Ref. 7 the correction for multiple scattering, calculated by Monte Carlo method, is within 10% for γ from 0 to 30°.

The account of the effect of multiple scattering on the behavior of $\chi(\gamma)$ at large γ and a more correct estimation of ε (from the spatial – angular distribution of brightness of scattered radiation) will be the subject of a separate study.

REFERENCES

1. *Hychooptical Characteristics* (*Terms and Definitions*), GOST 19210–73 (Gosstandart, Moscow, 1974), 10 pp.
2. A.S. Monin, ed., *Oceanic Optics. Vol. 1. Physical Optics of Ocean* (Nauka, Moscow, 1983), 372 pp.
3. D. Bauer, C. Brun–Cattan, and A. Saliot, Cah. Oceanogr. **23**, No. 9, 841–858 (1971).
4. V.N. Pelevin and T.M. Prokudina, in: *Atmospheric and Oceanic Optics* (Nauka, Moscow, 1972), pp. 148–157.
5. T.M. Prokudina and V.N. Pelevin, *ibid.*, pp. 157–168.
6. O.N. Gaponenko, V.I. Dobrynin, R.R. Mirgazov, K.A. Pocheikin, and B.A. Tarashchanskii, in: *Proceedings of 1st Inter–Republic Symposium on Atmospheric and Oceanic Optics,* Tomsk (1994), P. I, pp. 90–91.
7. L.B. Bezrukov, N.M. Budnev, and B.A. Tarashchanskii, *Marine and Atmospheric Optics* (*Abstracts of Reports*), Institute of Physics, Siberian Branch of USSR Academy of Sciences, Krasnoyarsk, (1990), P. 2, pp. 10–11.
8. L.B. Bezrukov, N.M. Budnev, and B.A. Tarashchanskii, Oceanology **30**, No. 6, 1022–1026 (1990).
9. L.B. Bezrukov, N.M. Budnev, V.I. Dobrynin, et al., Dokl. Akad. Nauk SSSR **277**, No. 5, 1240–1244 (1984).
10. A.P. Vasil'eva and N.M. Tikhonov, *Integral Equations* (Moscow University Publishing House, Moscow, 1989), 156 pp.
11. A.F. Verlan' and V.S. Sizikov, *Integral Equations: Methods, Algorithms, Computer Programs* (*Reference Manual*) (Naukova Dumka, Kiev, 1986), 542 pp.