DISTRIBUTION LAWS FOR THE ATMOSPHERIC TRANSMISSION IN THE VISIBLE AND IR REGIONS

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The distribution laws for the atmospheric transmission in the visible and IR regions have been found from the analysis of experimental data on CO_2*–laser radiation attenuation and meteorological visual range.*

The effeciency of atmospheric laser systems (ALS) is determined, to a considerable degree, by an actual state of a medium in which an optical radiation propagates. This aspect became one of the reasons for an extensive development of the methods for predicting the "optical weather"[1] the main component of which is the atmospheric transmission T – a key parameter for estimating the ALS operation reliability.[2]

The transmission is related to the medium's extinction coefficient for an optical wave at $\lambda = 0.55$ μm through the relation

$$T(0.55) = \exp\left[-\alpha\,(0.55)\,L\right] = \exp\left(-\frac{3.9}{S_M}\,L\right), \qquad (1)$$

where S_M is the meteorological visual range (MVR), L is the path length, and $\alpha\,(0.55)$ is the extinction coefficient.

For the waves in the near–IR region lying in the atmospheric transmission windows such a relationship primarily caused by aerosol scattering of radiation aerosol can be represented in the form [3]

$$T(\lambda_i) = \exp\left[-L\,\frac{3.9}{S_M}\left(\frac{0.55}{\lambda_i}\right)^{q}\right], \qquad (2)$$

where $q = 0.585 S_M^{1/3}$ for $S_M \le 6$ km and $q = 1.3$ for $S_M > 6$ km.

Relations (1) and (2) reveal that it is critically important to study statistical characteristics of the MVR. The bibliography in Refs. 4 and 5 reflects the efforts undertaken in this direction. However, the problem on the MVR distribution law and its dependence on a geographic region has not yet been solved.

This paper deals with study of temporal statistical regularities and the laws of the MVR distribution for the following regions of the European part of the USSR: Leningrad, Ul'yanovsk, Moscow, and Odessa (the names of the cities are given in chronological sequence of processing the observational data).[4–8] Many years of observations of the MVR from meteorological stations in the airports of the cities were used as the starting data. Total numbers of observations for these cities were $n_1 = 122736$, $n_2 = 18922$, $n_3 = 87219$, and $n_4 = 122567$, respectively. All the recorded MVR values were divided into 14 intervals: 0–0.1, 0.1–0.2, 0.2–0.4, 0.4–0.6, 0.6–0.8, 0.8–1.0, 1.0–1.2, 1.2–1.6, 1.6–2.0, 2.0–2.4, 2.4–3.2, 3.2–4.8, 4.8–8, and ≥ 8 km. Such gradations of S_M were chosen following the recommendations of the International Civil Aviation Organization (ICAO) in order to provide a possibility of using in our subsequent studies the data from meteorological stations of international airports which are more precise and frequent compared to the observations at ordinary meteorological stations conducted only several times a day. Moreover, this set of intervals is sufficientlly dense and, hence, allows one to construct a polygon of stored occurence frequencies with the accuracy sufficient for practical calculations of the effect of a propagation medium on the ALS parameters.

To find the analytical law of the MVR distribution we examined five theoretical distribution functions (truncated–normal, logarithmic–normal, Rayleigh, Reuss, and Johnson) including those used elsewhere[9–11] for studying statistical characteristics of the MVR.

The processing of observational data on S_M for the first two cities was carried out using the technique from Ref. 4. It was elucidated that in Leningrad for winter months and for annually mean experimental distribution of the MVR the best approximation is the Rayleigh distribution while for summer months it is the Reuss distribution. A modified Reuss distribution[4]

$$F(S_M) = \frac{1}{2\pi\sigma^2}\int\limits_0^{S_M} x\,\exp\left\{-\frac{x^2+(am)^2}{2\sigma^2}\right\} I_0\left(\frac{amx}{\sigma^2}\right) \mathrm{d}x \qquad (3)$$

was suggested as an approximation distribution for all months.

Here $\sigma = \frac{\sqrt{2}}{\pi}\,\bar{m}_{S_M}$ with $\bar{m}_{S_M} = \bar{m}$ is the mean value of S_M, $I_0(\cdot)$ is the Bessel function, and a is a regualating parameter. In this case $a = \begin{cases} 1 \text{ under } m_{S_M} \ge 9 \text{ km} \\ 0 \text{ under } m_{S_M} < 9 \text{ km} \end{cases}$, i.e., under complicated conditions for the ALS operation distribution (3) is reduced to the Rayleigh.

Observations in Ul'yanovsk[7] confirm the choice of such an approximation for the month distributions of the MVR. At the same time distribution (3) turned out to be useless for approximating the annually mean and the majority of monthly empirical polygons in Moscow and Odessa. Other mentioned–above distribution functions were also inadequate. Therefore, an additional statistical data processing based on the use of the system of the Pearson distributions for the approximation to be made was accomplished. A comprehensive treatment[8] showed that the best approximation of the empirical polygons for winter months in all regions is the modified beta–distribution

$$F(S_M) = \int\limits_0^{S_M/12} x^{\,c-1}\,(1-x)^{\,b-1}\,\mathrm{d}x\,, \qquad (4)$$

where

$$c = \frac{\bar{m}_{S_M}}{12}\left[\frac{\bar{m}_{S_M}\left(12 - \bar{m}_{S_M}\right)}{\bar{D}_{S_M}}\right] \text{ and}$$

$$b = \frac{c\left(12 - \bar{m}_{S_M}\right)}{\bar{m}_{S_M}}$$

are the distribution parameters, $\bar{D}_{S_M}$ is the S_M variance, and $S_M = 12$ km is the accepted upper limit of the last interval of the S_M values. The beta–distribution is also the best approximation of the annually mean polygons for Moscow and Odessa. The annually mean polygons for Leningrad and Ul'yanovsk as well as the polygons for summer months in all of these cities are approximated through a modified Reuss distribution.

The type of the distribution to be used was determined depending on the coefficient of asymmetry

$$\bar{\gamma}_1 = \bar{\mu}_3 \bar{D}_{S_M}^{-3/2},$$

where $\bar{\mu}_3$ is the third central sampling moment, i.e., for $\bar{\gamma}_1 < 0$ the best approximation is beta–distribution and for $\bar{\gamma}_1 > 0$ it is a modified Reuss distribution, when $\bar{\gamma}_1 = 0$ the accuracy of approximation by both of these distribution is roughly equal.

After passing over to the law of the atmospheric transmission distribution let us limit our discussion by the situation which is most unfavorable for the ALS, i.e., when an actual distribution of the MVR is represented by the Rayleigh distribution law (Eq. (3)). By making a transformation of the Rayleigh distribution according to Eq. (1) we obtain for $\lambda = 0.55$ μm the probability density function for the value T

$$\omega(T) = \frac{\gamma}{T \ln T} \exp\left(-\frac{\gamma}{2}\right), \tag{5}$$

where $\gamma = \dfrac{(3.9L)^2}{\sigma^2 \ln^2 T}$ and σ is the parameter of the initial Rayleigh distribution.

Since the value T has a physical meaning only in the interval $0 \le T \le 1$, it is necessary to use in calculations truncated distribution (5) in the form

$$F(T) = \frac{d_1 \gamma}{T \ln T} \exp\left(-\frac{\gamma}{2}\right), \tag{6}$$

$$\text{where } d_1 = \left[\int_0^1 \frac{\gamma}{T \ln T} \exp\left(-\frac{\gamma}{2}\right) \mathrm{d}T\right]^{-1}.$$

The form of the distribution T in the entire visible range will be slightly different from Eq. (6) because the variable $\left(\dfrac{0.55}{\lambda_i}\right)^q$ are close to unity in this range.

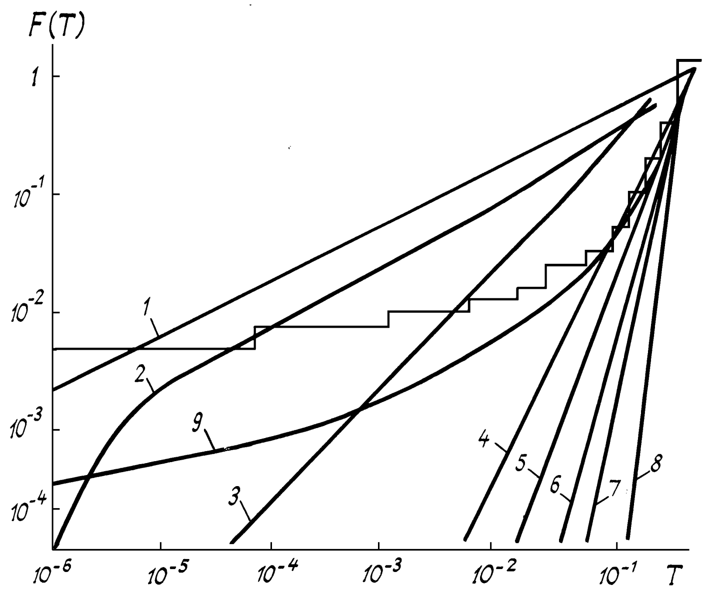

FIG. 1. Empirical (stepwise curve) and theoretical distribution functions for T ($\lambda = 0.55$ μm and $L = 1$ km). 1 – 8) numbers of curves corresponding to the numbers of distributions in the text and 9) distribution (6).

The results of calculations by formula (6) are shown in Fig. 1. In the same figure, as well as in Figs. 2 and 3, are depicted using the data from Ref. 4 and Bouguer's law the annually mean empirical polygons of stored occurrence frequencies for the transmission.

As can be seen from Figs. 2 and 3, an increase of the length of hypothetically chosen paths makes the form of polygons simpler and their linearization is observed. This makes it possible to assume the presence of a simpler approximation than relation (6). To elucidate this, we have analyzed eight distributions (truncated Weibull,[1] modified arcsin,[2] truncated exponential,[3] truncated Rayleigh,[4] truncated Maxwell,[5] truncated normal,[6] beta–distribution,[7] and truncated logarithmically normal[8]) which are shown in Fig. 1.

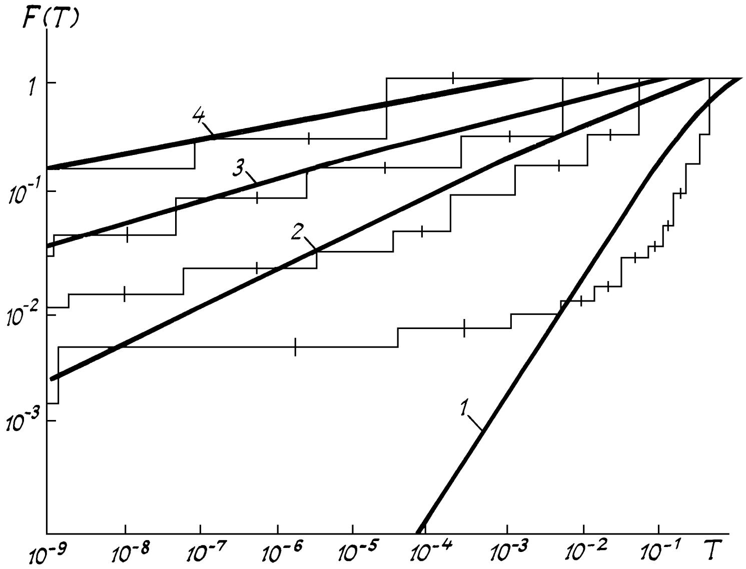

FIG. 2. Probabilistic distribution of T for $\lambda = 0.53$ μm *and* $L = 1$, 5, 10, *and* 20 km (*curves 1, 2, 3, and 4, respectively*). *1) exponential distribution* and *2, 3, and 4) distribution (7) with the values of the parameter* $r(\lambda, L) = 1/2$, 1/3, 1/5, *and* 1/7, *respectively.*

As the analysis has shown distribution (6) approximates the polygon T (Fig. 1) more accurately according to the Kolmogorov criterion. Satisfactory results for $L \geq 5$ km are also provided by a modified truncated Weibull distribution (Figs. 2 and 3)

$$F(T) = \frac{1 - \exp\left[-\left(\frac{T}{m_T}\right)^{r(\lambda\,,\,L)} \right]}{1 - \exp\left[-m_T^{-r(\lambda\,,\,L)} \right]}, \tag{7}$$

where $r(1\,,\,L) = L^{1.38\exp(-\sqrt{\lambda})}$ is the distribution parameter.

The results are valid for the visible and the near–IR. However, relation (2) used for the middle IR leads to a significant error, it is particularly true for $\lambda = 10.6$ μm which is promising for the ALS. This error at $\lambda = 10.6$ μm is mainly due to the effect of continuous absorption of radiation by water vapor. Therefore, to determine statistical characteristics of the transmission at this wavelength we used the measurements of the attenuation of a CO_2–laser radiation performed in Voeikovo (Leningrad region) during three years and different seasons.[13] The total number of measurements amounted 19062, 18387 of which were made in hazes of different density. The procedure for processing the measurement data with the extraction of the aerosol component of attenuation was described in detail in Ref. 14. The statistical characteristics of transmission were computed and the distribution functions were constructed (Fig. 4) for the same hypothetical path lengths. Figure 4 shows that the truncated Weibull distribution (7) with the regulating parameter $r = 2\frac{\lambda}{L}$ describes best of all the distribution of experimental data starting with $L \simeq 5$ km both in the visible and the near–IR. This can be explained by different ratio of the radiation wavelengths to the aerosol particle size in these regions.

To find the transmission distribution law for the entire depth of the atmosphere along the slant paths we used the data of actinometric observations for which it is convenient to write the integral transmission in the form

$$T = \frac{I_S}{I_0} = \frac{\int\limits_0^\infty I_0(\lambda)\, T(\lambda)\, d\lambda}{\int\limits_0^\infty I_0(\lambda)\, d\lambda}, \tag{8}$$

where I_s and I_0 are the solar radiance near Earth's surface and in the free space, respectively; $I_0 = 1370$ W/m^2 is the solar constant.

For the dependence of the measured value I_{s_0} on the solar height (angle of elevation Ω) to be taken into account we introduced a relative transmission

$$T_S = \frac{I_{s_0}}{I_0}, \tag{9}$$

where I_{s_0} is the integral solar radiance near Earth's surface for an ideal atmosphere, the extinction of solar radiation in which is reduced only to scattering and, resulting thus in $S_{\text{м}} = S_{\text{мmax}}$.

The value I_{s_0} is related to the solar constant I_0 at the sea level with the Forbs effect[15] taken into account through

$$I_{s_0} = I_0 \left(1.04 - 0.160 \sqrt{l} \right), \tag{10}$$

where l is the relative length of a slant path (the number of optical masses)

$$l = \frac{1}{H}\left(\sqrt{H^2 + 2RH + R^2 \sin^2\Omega} - R \sin \Omega \right), \tag{11}$$

where $R = 6384$ km is Earth's radius, $H = 8.0$ km is the depth of the homogeneous atmosphere.

Using Eqs. (8) and (9) we obtain

$$T = T_s\, T_0\,, \tag{12}$$

where $T_0 = \dfrac{I_{s_0}}{I_0}$.

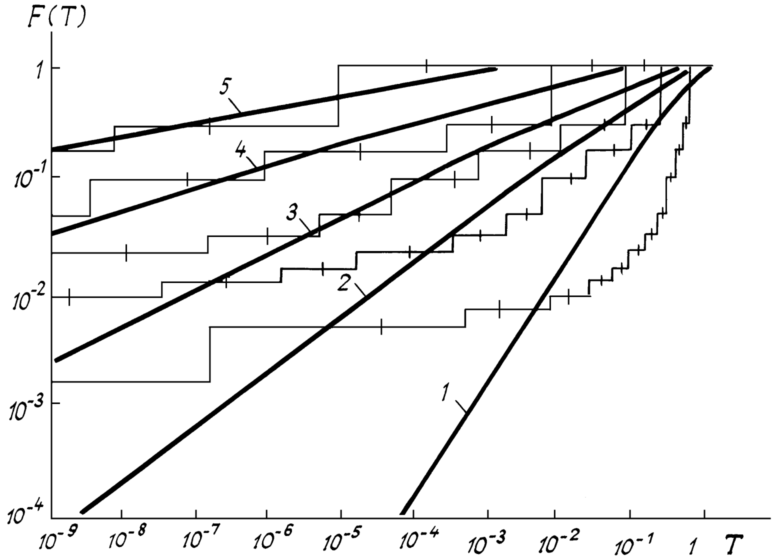

FIG. 3 Probabilistic distributions of T for $\lambda = 1.06$ μm *and* $L = 1$, 5, 10, 20, *and* 50 km (*curves 1, 2, 3, 4, and 5, respectively*). *1) exponential distribution, 2, 3, 4, and 5) distribution (7) with values of the parameter* $r(\lambda, L) = 1/2$, 1/3, 1/5, *and* 1/7, *respectively.*

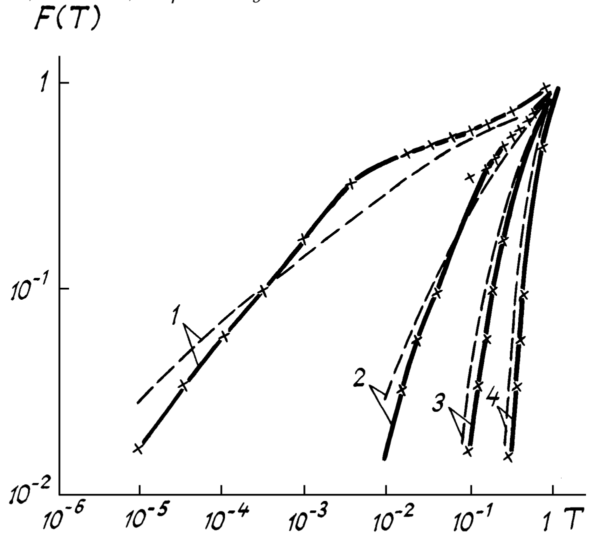

FIG. 4. Empirical (stepwise) curve and theoretical function of the distribution of the atmospheric transmission for $\lambda = 10.6$ *μm: solid lines – experiment and dashed lines – theory. 1)* $L = 50$, *2)* $L = 20$, *3)* $L = 10$, *and 4)* $L = 5$ km.

In Eq. (12) T_0 is a determined value depending on Ω alone while T_s is a random value determining variations of the transmission T.

A computerized processing was carried out of the data of daily standard observations of I_s conducted during actinometric monthly measurements and of the corresponding values Ω found during the same measurements made in Leningrad region for three years (1980-1982). The range of T_s values calculated using I_s and Ω was divided into 19 intervals from 0 to 1. The subsequent processing of calculational data was performed by the procedure described in Ref. 8.

The analysis of the general set of data revealed two characteristic situations that occurred during actinometric observations. The first situation includes the cases where the sun is trunslucent through the clouds and the atmospheric transmission is mainly determined by absorption and scattering in the clouds, the values of T_s varying from 0 to 0.1. The second one is related to the aerosol absorption and scattering in hazes when the solar disk is not covered with clouds and in this case the values T_s are in the range from 0.1 to 1.0. For the first situation the best chi–square approximation for empirical transmission distributions is the truncated Rayleigh distribution (Fig. 5), and for the second one it is the modified beta–distribution (Fig. 6).

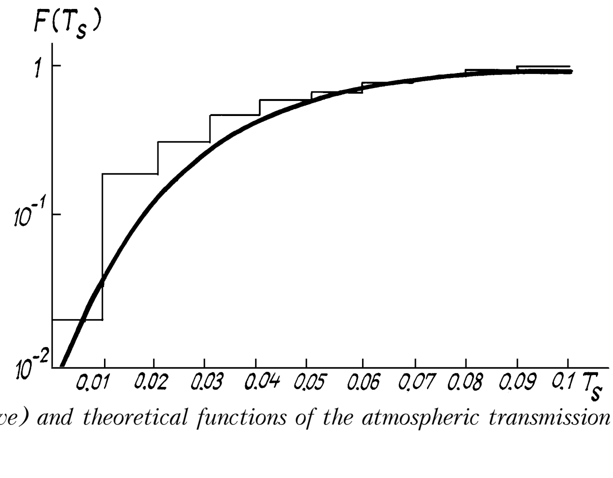

FIG. 5. Empirical (stepwise curve) and theoretical functions of the atmospheric transmission distribution along the slant paths for the first situation.

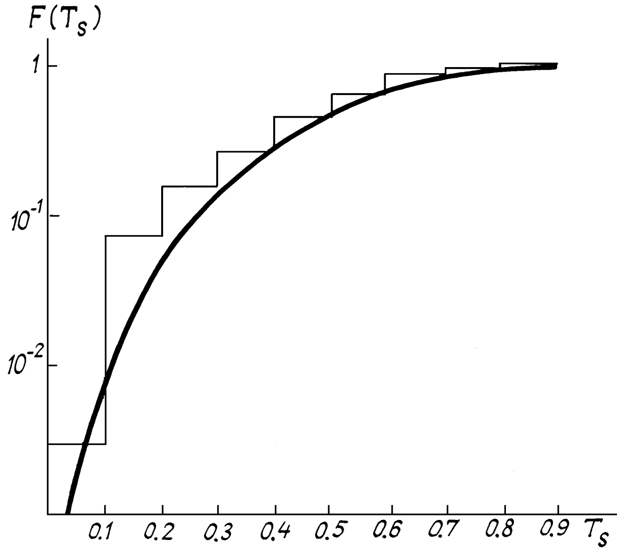

FIG. 6. Empirical (stepwise curve) and theoretical functions of the atmospheric transmission distribution along the slant paths for the second situation.

CONCLUSIONS

1. The truncated Weibull distribution was found to be a good approximation for the empirical distribution of the atmocpheric transmission along horizontal paths both in the visible and IR regions.

2. Along slant paths the transmission is better approximated by the truncated Rayleigh and modified beta–distributions.

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