NUMBER DENSITY AND SIZE OF PARTICLES OF A FINE AEROSOL FRACTION PRODUCED DURING EVAPORATION OF A HIGH–MELTING PARTICLE IN VACUUM

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Received June 24, 1991

During the gas–dynamic evaporation of high–melting aerosol particles in an intense optical field in vacuum the active condensation and formation of new particles takes place. The size distribution and number density of secondary particles are obtained using the thermodynamic method of estimating the vapor parameters.[5]

A number of authors[1,3] discussed the problem of propagation of powerful laser beams in the aerosol atmosphere, which contains high–melting particles (smog, soot, etc.). It is essential, that for carbon aerosol and for the intensity of the incident radiation of the order of 10^8 W/m^2 and higher the evaporation of particles occurs simultaneously with burning.[4] In the case of interaction of radiation with particles under conditions close to vacuum (for example, in the upper atmospheric layers) the burning process is absent and there takes place a gas–dynamic evaporation.[5] The results obtained using the model from Ref. 5 make it possible to raise a problem on the optical section of the system "particle + condensate".

The size distribution functions of secondary particles and the size of secondary particles as a function of different parameters of initial particles are found based on the use of the results from Ref. 5 and it is assumed that the temperature of the particle surfaces is preset.

As was shown in Ref. 6, there exists a relatively small spatial region, where the degree of supersaturation is noticeably different from unity. An intense nucleation processes take place in this region. Outside this region the expansion of vapor takes place according to adiabate of a two–phase system, the degree of supersaturation vanishes, and the excess of vapor condensates on the condensation nuclei generated earlier. Taking into account this fact, the secondary particle distribution function $f(t, a, r)$ will satisfy the equation

$$\frac{\partial f}{\partial t} = \dot{a}\,\frac{\partial f}{\partial a} + \frac{1}{r^2}\frac{\partial}{\partial r}\,(r^2 \upsilon f) = A(r_0, a_c)\,\delta(r - r_0)\,\delta(a - a_c)\,, \quad (1)$$

where t is time, a is the size of secondary particles, r is the distance to the particle center, $\dot{a}$ is the rate of the particle radius variation, υ is the mass velocity, and a_c is the minimum critical size, which the condensation nucleus must have in order to be the center of a new phase, called the k–phase. It is believed[7] that $a_c = 10^{-9}$ m. The values $a(t)$, $\upsilon(r)$ have been determined in Ref. 5.

The right part of Eq. (1) describes the birth of the secondary particles of the size a_c in the infinitely narrow spherical layer with r_0 in radius, $A(r_0, a_c)$ is the dimensional parameter whose form will be found below, and $\delta(r - r_0)$ and $\delta(a - a_c)$ are δ–functions.

A solution for $f(t, a, r)$ can be obtained analitically in the above–accepted approaches. Using the equation of continuity $\frac{\partial \mathrm{r}}{\partial t} + (\partial, \partial t)\,(r^2 \rho \upsilon) = 0$ and introducing the substitution of $f_0 = f/\rho(r)$, where $\rho(r)$ is the density of the system "vapor + secondary pacticles" found in Ref. 5, we obtain the equation for the function f_0

$$\frac{\partial f_0}{\partial t} + \dot{a}\,\frac{\partial f_0}{\partial a} + \upsilon\,\frac{\partial f_0}{\partial r} = \frac{A(r_0, a_c)}{\mathrm{r}}\,\delta(r - r_0)\,\delta(a - a_c)\,. \quad (2)$$

In order to solve Eq. (2) let us make the Laplacian transform with respect to the variables a and t with the following boundary conditions:

$$\bar{\bar{f}}_0(t, 0, r) = 0,\ \bar{\bar{f}}_0(0, q, r) = 0,\ \bar{\bar{f}}_0(p, q, r_0) = 0\,,$$

where $\bar{\bar{f}}_0(p, q, r)$ is the transform of the function $f_0(t, a, r)$.

Assuming that υ and a are fixed along the integration lines[8], the equation for the transform $\bar{\bar{f}}_0(p, q, r)$ will be written in the form

$$\frac{\partial \bar{\bar{f}}_0}{\partial r} + \bar{\bar{f}}_0\,\frac{(p + \dot{a}q)}{\upsilon} = \frac{A(r_0, a_c)}{p \upsilon \rho}\,\mathrm{d}(r - r_0)\,\mathrm{e}^{-qa_c}. \quad (3)$$

Solution of Eq. (3) has the form

$$\bar{\bar{f}}_0(p, q, r) = N(r_0, a_c)\,\exp\left(\int\limits_{R_0}^{r_0} \frac{p + \dot{a}q}{\upsilon}\,dr\right) \times$$

$$\times \exp\left(-\int\limits_{R_0}^{r} \frac{p + \dot{a}q}{\upsilon}\,dr\right)\chi(r - r_0)\,, \quad (4)$$

where

$$N(r_0, a_c) = \frac{A(r_0, a_c)}{\mathrm{r}(r_0)\,\upsilon(r_0)\,p}\,\exp\,(-qa_c)\,,$$

$$\chi(r - r_0) = \begin{cases} 0, & \text{if } r \le r_0, \\ 1, & \text{if } r \ge r_0; \end{cases}$$

$\rho(r_0)$ and $\upsilon(r_0)$ are the boundary conditions for the velocity and density of the mixture set in Ref. 5.

By introducing the following notation:

$$\Psi(r) - \Psi(r_0) = \int\limits_{r_0}^{r} \frac{a}{\upsilon}\,\mathrm{d}r\,,\ \varphi(r) - \varphi(r_0) = p\int\limits_{r_0}^{r} \frac{\mathrm{d}r}{\upsilon}\,, \quad (5)$$

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and appling the inverse Laplacian transform with respect to the variables q and p, we can find the expression for the function f_0

$$\bar{\bar{f}}_0 = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} \frac{N}{p} \exp\left[-q(a_c + \Psi(r) + \Psi(r_0))\right] \times$$

$$\times \exp(qa)\,\mathrm{d}q \left\{ \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} \exp\left[-p(\varphi(r) - \varphi(r_0))\right] \exp(pt)\,\mathrm{d}p \right\}. \quad (6)$$

By integrating over the complex plane and taking into account the fact that the moment of the birth of the secondary particles is selected as an origin moment of time, scale, i.e., $\Psi(r_0) = 0$, the solution of Eq. (1) will be written after transformations in the form

$$f(t, a, r) = \frac{\rho(r)\, A(r_0, a_c)}{\upsilon(r_0),\ \rho(r_0)}\, \delta(a - \Psi(r)) \,. \quad (7)$$

It can be seen from Eq. (7) that the size distribution function of the secondary particles is independent of time in the spherical layer r_0, but it is dependent on the boundary conditions of the problem and on the medium parameters.

The value $\Psi(r)$ in the δ–function is the size of the secondary particles at the distance r from the primary particle. It can be seen from Eq. (5) that in order to find the value $\Psi(r)$ it is necessary to know the rate of variations of the secondary particle radius. However, one can assume that the vapor is always in the thermodynamic equilibrium with the condensate, and, therefore, all excess of the vapor condenses on the secondary particles practically immediately. In this case the size of the secondary particles at any distance r from the primary particle can be found as follows.

Since the mass of the evaporated substance in the volume dV at the distance r from the particle is

$$\mathrm{d}m = x\rho m_e\, \mathrm{d}V,$$

and the mass of a secondary particle is $\frac{4}{3}\pi\rho_\kappa a^3(r)$, the expression for the concentration of the secondary particles has the form

$$n(r) = \frac{3}{4\pi a^3(r)\rho_k} \frac{\mathrm{d}m}{\mathrm{d}V} = \frac{3x(r)\,\rho(r)\, m_e}{4\pi a^3(r)\rho_k} \,, \quad (8)$$

where m_e and ρ_κ are the molecule mass and the density of evaporating substance, respectively.

In the region, in which the condensation nuclei are formed, $a(r_0)$ is equal to a_c, i.e., to the minimum critical size. Therefore, the initial concentration $n(r_0)$ can be calculated from Eq. (8).

The value of $n(r)$ is found from the condition

$$4\pi\, r_0^2 \upsilon_0 dt\, n_0 = 4\pi\, r^2 \upsilon\, dt\, n(r) \,.$$

Using the well–known relation

$$n(t, r) = \int_0^\infty f(t, a, r)\, \mathrm{d}a$$

and the specific view of function (7), we find

$$n(r) = \frac{\rho(r)\, A(r_0, a_c)}{\upsilon(r_0)\, \rho(r_0)} \quad (9)$$

and

$$A(r_0, a_c) = n_0\, \upsilon(r_0) \,.$$

Thus, finally the distribution function takes the form

$$f(a, r) = \frac{n_0\, \rho(r)}{\rho(r_0)}\, \delta(a - a(r)) \,. \quad (10)$$

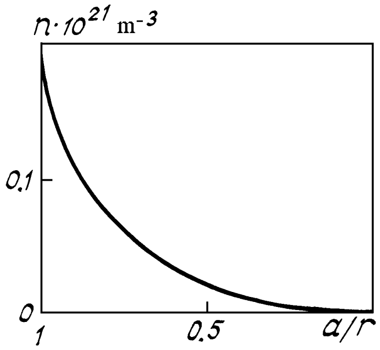

FIG. 1. Concentration of the secondary particles as a function of the inverse distance from the primary particle with a size of 100 μm.

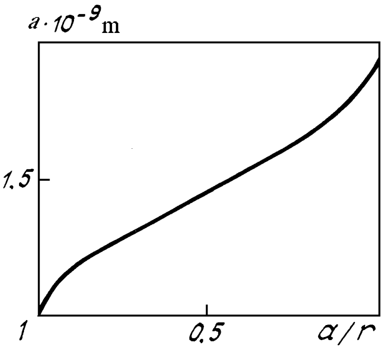

FIG. 2. The size of the secondary particles as a function of the inverse distance from the primary particle with a size of 100 μm.

The solution for the distribution function of the secondary particles (10) is the result of the taken–above approaches: the δ–function describes the appearence of particles of a certain size a at the distance r from the primary particle.

The unknown value $a(r)$, which characterizes the growth of particles, can be found from Eqs. (8) and (9). The result is

$$a(r) = \sqrt[3]{\frac{3x(r)\, m_e\, \mathrm{r}(r)\, \mathrm{t}(r)\, r^2}{4\mathrm{r}_k \mathrm{t}_0\, n_0\, r_0^2}} \,. \quad (11)$$

Since the values $n_0 \upsilon_0 r_0^2$ and $\rho\, \upsilon\, r^2$ are constant, relation (11) can be rewriten in the form

$$a(r) = \sqrt[3]{\frac{3x(r)\, m_e}{4\rho_\kappa c}} \,, \quad (12)$$

where

$$c = \frac{n_0\, \upsilon_0\, r_0^2}{\rho(r)\, \upsilon(r)\, r^2} .$$

If the experimental values of the size of the secondary particles $a(r)$ is known one can find from Eq. (12) the degree of the vapor condensation in the vicinity of an evaporating particle as well as calculate the fields of temperature, density, and pressure by the model proposed in Ref. 5. The value $x(r)$ depends on the substance of the primary particle and on the boundary conditions of the problem.

The concentration of the secondary particle $n(r)$ and the function of growth of the secondary particles size $a(r)$ as a function of the inverse distance from the primary particle, are shown in Figs. 1 and 2, respectively. Calculations were made for the primary particle with a size of 100 μm.

REFERENCES

1. V.I. Bukatyi, *Interaction of Powerful Laser Radiation with Solid Combustible Aerosol*, Doctoral Dissertation in Physical–Mathematical Sciences, Insittute of Atmospheric Optics, Siberian Branch of the Academy of Sciences of the USSR, Tomsk (1986).
2. G.M. Strelkov, *Propagation of Intense Laser Beams in the Troposphere*, Doctoral Dissertation in Physical Mathematical Sciences, Institute of Radioelectronics, the Academy of Sciences of the USSR, Moscow (1982).
3. D.S. Bobuchenko and V.K. Pustovalov, in: *Abstracts of Papers at the 4th All–Union Conference, on the Radiation Propagation in a Disperse Medium,* Obninsk–Barnaul, (1988), Pt. 2, p. 242.
4. V.I. Bukatyi, V.N. Krasnopevtsev, and A.M. Shaiduk, Izv. Vyssh. Uchebn. Zaved, Fiz., No. 10, 110–113 (1986).
5. V.I. Bukatyi, G.V. Lyamkina, and A.M. Shaiduk, in: *Nonlinear Interaction of Powerful Laser Radiation with Solid Aerosol*, Barnaul (1989).
6. Ya.B. Zeldovich and Yu.P. Raizer, *Physics of Shock Waves and High–Temperature Hydrodynamic Phenomena* (Nauka, Moscow, 1966).
7. V.T. Borukhov, N.V. Pavlyukevich, and S.P. Fisenko, in: *Abstracts of Papers at the 20th Conference on Modern Problems of Aerodisperse Systems,* Odessa (1989), Pt., 1, p. 9.
8. A.N. Filatov and L.B. Sharova, *Integral Inequalities and Theory of Linear Oscillations* (Nauka, Moscow, 1976), 152 pp.