PHASE CONJUGATION AND INSTABILITY IN AN INTERACTION OF GAUSSIAN DIFFRACTING LIGHT BEAMS PROPAGATING IN OPPOSITE DIRECTIONS THROUGH MEDIA WITH THE KERR NONLINEARITY. I. CONDITIONS OF THE CONVECTIVE INSTABILITY DEVELOPMENT

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Development of perturbations is analyzed for non-stationary interaction of two counterpropagating noncollinear light beams. Invariants of interaction are written down. Dependences of amplification increment on the perturbation frequency are derived. It is shown that noncollinearity of interaction enriches a range of spatial frequencies, at which the convective instability develops. The range of extended frequency spectrum is determined by the sign of self-action.

1. INTRODUCTION

The problem of phase conjugation in a four-wave interaction (PC-FWI) is widely studied in laser physics.[1,2] It is connected, first of all, with the facts that PC is one of the most effective methods of compensation in real time for small-scale distortions of light beams caused by nonlinearity of the medium, and that for the IR-range PC with the help of FWI is the most efficient. At the same time it was impossible to attain under real conditions PC of high quality in FWI with simultaneous large amplification of the probe wave. It is primarily connected with such factors as self-action and transfer of energy of the interacting waves and inequality of amplitudes of the pump waves.[1]

It should be noted that influence of noncollinearity of the interacting counterpropagating beams on the PC efficiency and quality was considered, as a rule, in the approximation of fixed pump-wave amplitudes. Phase conjugation of noncollinear beams was studied in Refs. 2 and 3 only for beams propagating in the same direction. The feasibility of mutual compensation for self-action, noncollinearity, and inequality of pump-wave amplitudes was demonstrated there. Meanwhile, an analysis of interaction in a nonlinear medium of two noncollinear beams, one of which, being oblique upon entering the medium, is then reflected from a screen, has shown that development of free oscillations of the beams' parameters (for example, positions of their energy centers and angle of exit of the reflected beam from the nonlinear medium) is possible. Their reason consists in violation of Snell's law of reflection because of complex nonlinear refraction of the incident beam on distributed lens, induced by the reflected beam, and refraction of the reflected beam on distributed lens, induced by the incident optical beam.

In addition, it is also well known about the instability, arising in an interaction of counterpropagating light beams.[1,6–10] For PC problems of a four-wave interaction of counterpropagating beams, influence of beam noncollinearity on an interaction pattern is of interest. Thus, the problem arises about a relationship among thresholds of instability of various physical natures.

The present paper studies the influence of noncollinearity of two or four counterpropagating beams on their interaction pattern.

2. BASIC EQUATIONS, STATEMENT OF THE PROBLEM, AND INVARIANTS

Phase conjugation in a four-wave interaction of noncollinear beams is described in dimensionless variables by the following system of the equations:

$$L_j A_j + i\,\gamma_j\,(F_{sj} + F_{cj}) = 0,\ \ j = 1–4, \qquad (1)$$

where A_j are the complex wave amplitudes normalized on the maximum amplitude; $A_{1,2}$ are the pump wave amplitudes, $A_4 \equiv A_{\rm back}$; $A_3 \equiv A_{\rm sign}$; L_j is the linear operator specified by geometry of interaction and in our case has the form

$$L_j = \frac{\partial}{\partial t} + \nu_j \frac{\partial}{\partial z} + \beta_j \frac{\partial}{\partial x} + iD_j \frac{\partial^2}{\partial x^2}\,;$$

$$t > 0,\ 0 < x < L_X,\ 0 < z < L_Z, \qquad (2)$$

where t is the normalized time; z is the longitudinal coordinate measured in diffraction length, $L_{\rm d} = 2ka^2$; a is the initial beam radius; k is the wave number; x is the transverse coordinate normalized on a; β is the angle between the incident beam propagation direction upon entering the nonlinear medium ($z = 0$) and the axis z and is measured in units of diffraction beam divergence in a linear medium $\beta_{\rm d} = 0.5ka$; D_j is the diffraction coefficient; ν_j takes into account the

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direction of propagation of the jth beam ($\nu_{1,3} = 1$, $\nu_{2,4} = -1$); $L_{X,Z}$ are the transverse and longitudinal dimensions of the interaction zone, respectively. Because we are interested in non-stationary effects, in the present paper we restrict ourselves by a case of plane geometry (coordinates (X, Z)).

Terms F_{sj} in Eq. (1) of the form

$$F_{sj} = A_j \left(\sum_{m=1}^{4} |A_m|^2 - 0.5\,|A_j|^2 \right),\ j = 1\text{–}4, \qquad (3)$$

describe self-action of a light beam under conditions of equality (absence of dispersion) of permittivity gratings in a transparent medium, and

$$F_{cj} = \frac{\partial}{\partial A_j^*} (A_3\, A_4\, A_1^*\, A_2^* + A_1\, A_2\, A_3^*\, A_4^*), \quad j = 1\text{–}4, \quad (4)$$

describes generation of the jth wave in a nondispersive medium.

In our case, the account of time in Eq. (1) is caused by non-stationary character of interaction. It is of fundamental importance here, because it allows one to describe adequately the processes occurring in the system.

Boundary and initial conditions for system (1) have the forms

$$A_j(z, x, t = 0) = 0, \quad j = 1\text{–}4,$$

$$A_j(z=0, x, t) = (1 - \exp(-\tau t)\exp\{-(x - x_{cj})^2\}, \quad j = 1, 3;$$

$$A_j(z = L_Z, x, t) = R_0\, A_{j-1}(z = L_Z, x, t) \times$$

$$\times \exp(i\,(x - x_{cj})^2/R_{\rm m}), \quad j = 2, 4. \qquad (5)$$

Here, x_{cj} describes the initial position of the jth beam center, $R_{\rm m}$ is the radius of curvature of the reflecting mirror, R_0 is its amplitude reflectance, and τ is the relaxation time.

In the stationary regime ($\partial/\partial t = 0$ in Eqs. (2)) the system (1) has some invariants, which were used by us to control the correctness of results of numerical modeling.

Thus, during interaction the total power of light beams is conserved

$$\sum_{j=1}^{4} P_j = \text{const};$$

$$P_j = \sum_{0}^{L_X} |A_j|^2\, {\rm d}x. \qquad (6)$$

A difference between a sum of powers of the first and third beams and a sum of powers of the second and fourth beams is also conserved along the Z axis:

$$\frac{\partial}{\partial z} (P_{1,3} - P_{2,4}) = 0. \qquad (7)$$

According to the aim of the present paper, we consider at first the influence of noncollinearity on an interaction of two counterpropagating beams.

3. CONDITIONS OF INSTABILITY DEVELOPMENT IN AN INTERACTION OF TWO NONCOLLINEAR COUNTERPROPAGATING BEAMS

3.1. Statement of the problem and its solution for a system of two counterpropagating noncollinear beams

The basic problem of phase conjugation is creation of conditions, at which the conjugated wave will be of the highest quality. It is well known[1] that to obtain such conditions in FWI, an optically homogeneous working medium and reference waves homogeneous throughout the interaction volume are required. At the same time, in order that PC-FWI had high energy efficiency, the intensity of reference waves should be rather large. However, smooth waves of large intensity are unstable in nonlinear reactive media. Therefore, instability study in such systems is of great interest at the moment not only for PC. Thus, for example, in Refs. 1 and 8 parametric wave generation and amplification were examined, in Ref. 7 oscillations of the beam characteristics in a system of counterpropagating beams were studied, in Ref. 9 modulation instability caused by phase cross-modulation of picosecond laser pulses was investigated, and in Ref. 10 instability in a system of two collinear beams propagating in opposite directions through a medium with the Kerr nonlinearity was investigated with the account of relaxation processes.

By analogy to the method suggested in Ref. 1, where instability of two collinear beams propagating in opposite directions through a nonlinear reactive medium was investigated, we derive conditions of instability development for noncollinear beams. The process is described by the following system of the dimensionless equations:

$$\frac{\partial A_+}{\partial z} + \beta\,\frac{\partial A_+}{\partial x} + iD\,\frac{\partial^2 A_+}{\partial x^2} = -i\gamma\,(0.5\,|A_+|^2 + |A_-|^2)\,A_+,$$

$$\frac{\partial A_-}{\partial z} - \beta\,\frac{\partial A_-}{\partial x} - iD\,\frac{\partial^2 A_-}{\partial x^2} = i\gamma\,(0.5\,|A_-|^2 + |A_+|^2)A_-, \quad (8)$$

where A_+ is the forward wave amplitude, and A_- is the backward wave amplitude. The other parameters are specified above. We emphasize that in the examined case each wave is incident independently on the nonlinear medium from the opposite sides, i.e. in the cross sections $z = 0$ and $z = L_z$, respectively.

For the further analysis it is convenient to introduce new functions $A_\pm = \tilde{A}_\pm \exp[(i\beta x)/(2D)]$. Then the equations (8) can be written down as

$$\frac{\partial \tilde{A}_+}{\partial z} + \frac{i\beta^2}{4D}\,\tilde{A}_+ + iD\,\frac{\partial^2 \tilde{A}_+}{\partial x^2} = -i\gamma\,(0.5\,|\tilde{A}_+|^2 + |\tilde{A}_-|^2)\,\tilde{A}_+,$$

$$\frac{\partial \tilde{A}_-}{\partial z} - \frac{i\beta^2}{4D}\,\tilde{A}_- - iD\,\frac{\partial^2 \tilde{A}_-}{\partial x^2} = i\gamma\,(0.5\,|\tilde{A}_-|^2 + |\tilde{A}_+|^2)\,\tilde{A}_-. (9)$$

3.2. Equations for the instability increments

We shall search for the solution of Eq. (9) as perturbation of plane waves:

$$\tilde{A}_+ = A_+^0 \exp(-0.5\, i\gamma\, (I_+ + 2I_-)\, z) - (1 + u(x, z)),$$

$$\tilde{A}_- = A_-^0 \exp(0.5\, i\gamma\, (I_- + 2I_+)\, z) + (1 + v(x, z)), \quad (10)$$

where $A_\pm^0$ are the wave amplitudes upon entering the medium, $I_\pm = |A_\pm^0|^2$, and $u(x, z)$ and $v(x, z)$ are perturbations ($u, v \ll 1$).

Substituting Eq. (10) into Eq. (9) and linearizing the equation, we obtain in the first approximation on X and v the following equations for perturbations:

$$\frac{\partial u}{\partial z} + iD\frac{\partial^2 u}{\partial x^2} + \frac{i\beta^2}{4D}(u + 1) =$$

$$= -0.5\, i\gamma\, \{I_+\, (u + u^*) + 2I_-\, (v + v^*)\},$$

$$\frac{\partial v}{\partial z} - iD\frac{\partial^2 v}{\partial x^2} - \frac{i\beta^2}{4D}(v + 1) =$$

$$= 0.5\, i\gamma\, \{2I_+\, (u + u^*) + I_-\, (v + v^*)\},$$

and pair of the conjugated equations. Further, assuming that

$$u = \tilde{u} + C_1,\ u^* = \tilde{u}^* + C_2;$$

$$v = \tilde{v} + C_3,\ v^* = \tilde{v}^* + C_4,$$

where C_i are the coefficients determined under assumption of system homogeneity and having the form

$$C_{1,2} = -\frac{i\beta^2\,(i\beta^2 - 4i\gamma\, I_-\, D)}{-\beta^4 - 4\gamma D\beta^2 I_+ - 4\gamma D\beta^2 I_- + 48\gamma^2 D^2 I_+ I_-},$$

$$C_{3,4} = -\frac{i\beta^2\,(i\beta^2 - 4i\gamma\, I_+\, D)}{-\beta^4 - 4\gamma D\beta^2 I_+ - 4\gamma D\beta^2 I_- + 48\gamma^2 D^2 I_+ I_-},$$

we obtain

$$\frac{\partial \tilde{u}}{\partial z} + iD\frac{\partial^2 \tilde{u}}{\partial x^2} + \frac{i\beta^2}{4D}\tilde{u} = -0.5\, i\gamma\, \{\, I_+\, (\tilde{u} + \tilde{u}^*) +$$

$$+ 2\, I_-\, (\tilde{v} + \tilde{v}^*)\}, \quad (11)$$

$$\frac{\partial \tilde{v}}{\partial z} - iD\frac{\partial^2 \tilde{v}}{\partial x^2} - \frac{ib^2}{4D}\tilde{v} = 0.5\, i\gamma\, \{2I_+\, (\tilde{u} + \tilde{u}^*) + I_-\, (\tilde{v} + \tilde{v}^*)\},$$

and pair of the conjugated equations.

To study development of perturbations, we shall search for the solution of Eq. (11) in the form

$$\tilde{u}(x, z) = U_0 \exp(iqx + i\mu z + i\varphi);$$

$$\tilde{v}(x, z) = V_0 \exp(iqx + i\mu z + i\varphi);$$

$$\tilde{u}^*(x, z) = U_0^* \exp(iqx + i\mu z + i\varphi); \quad (12)$$

$$\tilde{v}^*(x, z) = V_0^* \exp(iqx + i\mu z + i\varphi).$$

Here, $U_0 = (p + ih)$ and $V_0 = (m + in)$ are the perturbation amplitudes, μ is the instability increment, q is the transverse wave number, and φ is the initial phase.

After simple transformations we derive from Eq. (11) the system of the equations for the perturbation amplitudes:

$$i\mu U_0 - iq^2 D U_0 + \frac{i\beta^2}{4D} U_0 = -0.5 i\gamma\, \{I_+\, (U_0 +$$

$$+ U_0^*) + 2I_-\, (V_0 + V_0^*)\},$$

$$i\mu V_0 + iq^2 D U_0 - \frac{i\beta^2}{4D} V_0 = 0.5 i\gamma\, \{2I_+\, (U_0 +$$

$$+ U_0^*) + I_-\, (V_0 + V_0^*)\},$$

$$i\mu U_0^* + iq^2 D U_0^* - \frac{i\beta^2}{4D} U_0^* = 0.5 i\gamma\, \{I_+\, (U_0 +$$

$$+ U_0^*) + 2I_-\, (V_0 + V_0^*)\},$$

$$i\mu V_0^* - iq^2 D V_0^* + \frac{i\beta^2}{4D} V_0^* = -0.5 i\gamma\, \{2I_+\, (U_0 +$$

$$+ U_0^*) + I_-\, (V_0 + V_0^*)\}. \quad (13)$$

We note that at $\mu = 0$ from Eq. (11) it follows that for $q^2 D = \beta^2/(4D)$ h and n are arbitrary (otherwise amplitudes are equal to zero), i.e. we deal with development of free oscillations of finite amplitude. Thus, beam noncollinearity results in existence of a free oscillation mode on a fixed spatial frequency q determined by β and D.

Further, the transformation of Eq. (13) under condition $q^2 D \neq \beta^2/(4D)$ yields the system

$$\left\{\frac{\beta^2}{4D} + \gamma I_+ - q^2 D - \left(\frac{\beta^2}{4D} - q^2 D\right)^{-1} \mu^2\right\} p + 2\gamma I_-\, m = 0,$$

$$(14)$$

$$-2\gamma I_+ p - \left\{\frac{\beta^2}{4D} + \gamma I_- - q^2 D - \left(\frac{\beta^2}{4D} - q^2 D\right)^{-1} \mu^2\right\} m = 0,$$

which at $\beta = 0$ transforms into the equations investigated in Ref. 1.

The condition of existence of nontrivial solution for homogeneous system (12) is equality to zero of its determinant:

$$\begin{vmatrix} \left\{\frac{\beta^2}{4D} + \gamma I_+ - q^2 D - \left(\frac{\beta^2}{4D} - q^2 D\right)^{-1} \mu^2\right\} & 2\gamma I_-\, m - \\ -2\gamma I_+ - & \left\{\frac{\beta^2}{4D} + \gamma I_- - q^2 D - \left(\frac{\beta^2}{4D} - q^2 D\right)^{-1} \mu^2\right\} \end{vmatrix} = 0.$$

$$(15)$$

In the particular case of $I_+ = I_- = I$, from Eq. (15) the equation of the fourth degree for the increment μ follows:

$$-\left\{\frac{\beta^2}{4D} + \gamma I - q^2 D - \left(\frac{\beta^2}{4D} - q^2 D\right)^{-1} \mu^2\right\}^2 = 4\gamma^2 I^2. \quad (16)$$

For a self-focusing medium ($\gamma > 0$), Eq. (16) is equivalent to a pair of the equations

$$\mu^2 = \left(\frac{\beta^2}{4D} - q^2 D\right)^2 + 3\gamma I \left(\frac{\beta^2}{4D} - q^2 D\right),$$

$$\mu^2 = \left(\frac{\beta^2}{4D} - q^2 D\right)^2 - \gamma I \left(\frac{\beta^2}{4D} - q^2 D\right), \qquad (17)$$

whereas for a defocusing medium ($\gamma < 0$), it is equivalent to a pair of the equations

$$\mu^2 = \left(\frac{\beta^2}{4D} - q^2 D\right)^2 - 3|\gamma| I \left(\frac{\beta^2}{4D} - q^2 D\right),$$

$$\mu^2 = \left(\frac{\beta^2}{4D} - q^2 D\right)^2 + |\gamma| I \left(\frac{\beta^2}{4D} - q^2 D\right). \qquad (18)$$

3.3.1. Propagation of beams through a focusing medium ($\gamma > 0$)

For convenience of the analysis of noise development, we present the dependences

$$\mu^2 = x^2 - ax;$$

$$x = q^2 D - \frac{\beta^2}{4D}; \quad a = 3\gamma I, -\gamma I \qquad (19)$$

graphically (Fig. 1*a*).

From Fig. 1*a* existence of two regions of instability development can be seen with the values of the parameters lying below the X axis. Noncollinearity of interaction is manifested, first, through occurrence of an additional range of spatial frequencies for which the noise can develop. Second, it shifts the range of spatial frequencies of absolute instability toward larger frequencies. In general, the minimum frequency $q^2_{\min}$ is determined by the angle of light beam incidence on the nonlinear medium and the initial beam power. On the basis of Fig. 1*a* we can write down the total range of spatial frequencies

$$q^2 = \begin{cases} \left(0; \dfrac{\beta^2}{4D^2}\right), \ \beta^2 < 4\gamma ID, \\ \left(\dfrac{\beta^2}{4D^2} - \dfrac{\gamma I}{D}; \dfrac{\beta^2}{4D^2}\right), \dfrac{\beta^2}{4D^2} \neq q^2 D, \beta^2 > 4\gamma ID, \\ \left(\dfrac{\beta^2}{4D^2}; \dfrac{\beta^2}{4D^2} + \dfrac{3\gamma I}{D}\right), \dfrac{\beta^2}{4D^2} \neq q^2 D. \end{cases} \qquad (20)$$

We note that the maximum instability increment for different regions differs 3 times and is determined by the initial beam power.

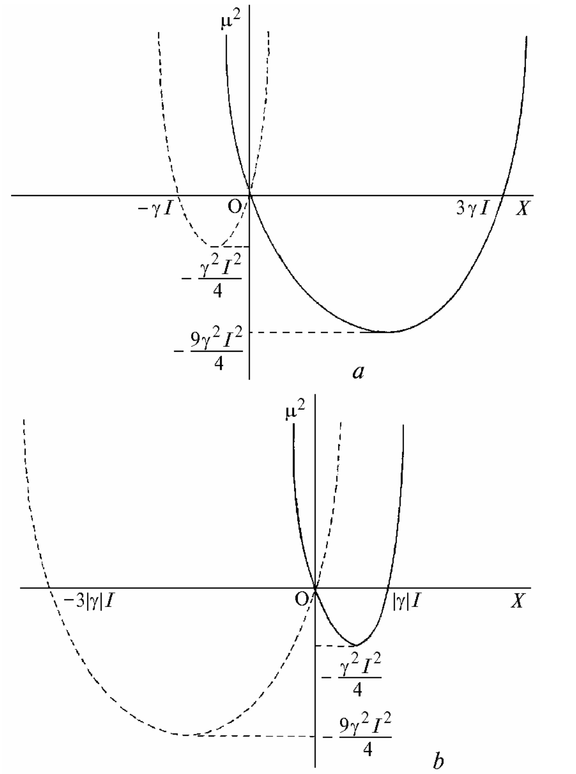

FIG. 1. Dependence of the square increment of noise amplification from the parameter $q^2D - \beta^2/(4D)$ in an interaction of light beams in self-focusing (a) and defocusing (b) media for $a = 3\gamma I$ (solid curve) and $-\gamma I$ (dashed curve).

3.3.2. Propagation of beams through a defocusing medium ($\gamma < 0$)

Similar to Fig. 1*a*, Fig. 1*b* has been drawn for a defocusing medium. It illustrates analogous dependences. The basic difference from self-focusing medium consists in change of the region of spatial frequencies for which the maximum amplification is attained; moreover, under certain conditions imposed on the angle of interaction the spectral range for the self-focusing medium can become not only wider, but also narrower than that for the defocusing medium. The total range of frequencies of instability development is defined as follows:

$$q^2 = \begin{cases} \left(0; \dfrac{\beta^2}{4D^2}\right), \ \beta^2 < 12|\gamma| ID, \\ \left(\dfrac{\beta^2}{4D^2} - \dfrac{3|\gamma| I}{D}; \dfrac{\beta^2}{4D^2}\right), \dfrac{\beta^2}{4D^2} \neq q^2 D, \beta^2 > 12|\gamma| ID, \\ \left(\dfrac{\beta^2}{4D^2}; \dfrac{\beta^2}{4D^2} + \dfrac{|\gamma| I}{D}\right), \dfrac{\beta^2}{4D^2} \neq q^2 D. \end{cases} \qquad (21)$$

In this case, the perturbations will grow most fast in the range of frequencies $0 < q < \beta/2D$, i.e. for smaller values of the wave vectors in comparison with collinear beams.

4. CONCLUSIONS

Summarizing our results, we can conclude that noncollinearity of interacting beams expands the spectral range of instability development and changes the range of maximum amplification. The enrichment of the spectral range depends on the sign of self-action. In a case of defocusing in an interaction of noncollinear beams, the largest amplification increment is reached at frequencies, at which the instability is not realized in an interaction of collinear beams.

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