ITERATION METHOD FOR ADJUSTMENT OF A SEGMENTED MIRROR USING FUNCTIONALS OF THE EXTENDED SOURCE IMAGE

G.L. Degtyarev, A.V. Makhan'ko, S.M. Chernyavskii, and A.S. Chernyavskii

A.N. Typolev State Technical University, Kazan'
Received August 4, 1998

An instrumental realization of the Newton iteration method is proposed for a solution of the problem of modal reconstruction and compensation for angular aberrations of a segmented mirror using functionals of the extended source image. Some results of numerical modeling are presented for the mirror comprising six segments with consideration for the measurement noise.

We consider an adaptive optical system (AOS) with a mirror comprising n segments in which the wave-front (WF) aberration function $\Phi(\xi, \eta)$ at the aperture is represented by a finite series in a linearly independent system of functions $\{\Phi_k(\xi, \eta)\}$ with sufficient accuracy, namely,

$$\Phi(\xi, \eta) = \sum_{k=1}^{N} \zeta_k \Phi_k(\xi, \eta) , \qquad (1)$$

where $\zeta = (\zeta_1, \zeta_2, ..., \zeta_N)$ is the unknown mode vector. For the linearly independent system of functions we used piecewise-linear functions for which the aberration function within the kth segment can be represented as

$$\Phi_k(\xi, \eta) = (\alpha_k + \beta_k(\xi - \xi_k) + $$
$$+ \gamma_k(\eta - \eta_k))\, \delta(\xi - \xi_k, \eta - \eta_k) ,$$

where $(\xi_k, \eta_k) = \mathbf{r}_k$ are the coordinates of the center of the kth segment; β_k, γ_k specify the local WF tilts (angular aberrations normalized by the ratio (λ/a)); λ is the wavelength; a is the characteristic aperture size radius.

It is assumed that the AOS compensates for the mode vector ζ by means of the control vector $\zeta_u = (\zeta_{u1}, \zeta_{u2}, ..., \zeta_{uN})$. In this case, the control problem is reduced to finding of the control for which $\zeta - \zeta_u \to 0$.

The equation for the control vector of a point source has the form

$$H(\mathbf{f}, z, \zeta_u) = J(\mathbf{f}, z, \zeta) , \qquad (2)$$

where $H(\mathbf{f}, z, \zeta_u)$ is the optical transfer function (OTF) at the normalized spatial frequency $\mathbf{f} = (\xi, \eta)$ for the given defocusing z and the unknown mode vector ζ, and $J(\mathbf{f}, z, \zeta)$ is the experimentally measured OTF. Within a constant factor[1] it can be written as

$$H(\mathbf{f}, z, \zeta) = \int_{-\infty}^{\infty} G(\xi + \xi', \eta + \eta')\, G_0(\xi + \xi', \eta + \eta') \times$$
$$\times\, G^*(\xi', \eta')\, G_0^*(\xi', \eta')\, \partial r' , \qquad (3)$$

where $G_0(\xi, \eta) = P(\xi, \eta)e^{-iz(\xi + \eta)/2}$ is the pupil function comprising aberrations for the given defocusing z; $P(\xi, \eta)$ is the characteristic pupil function; $G = e^{i2\pi\Phi(\xi, \eta)}$ is the pupil function for the unknown aberration function; the symbol $*$ denotes complex conjugation. We solve Eq. (2) by the Newton method using the iteration scheme

$$\zeta_u^{n+1} = \left[\frac{\partial H(\mathbf{f}, z; \zeta_u^n)}{\partial \zeta}\right]^{-1} \times$$
$$\times \left[J(\mathbf{f}, z; \zeta) - H(\mathbf{f}, z; \zeta_u^n) + \frac{\partial H(\mathbf{f}, z; \zeta)}{\partial \zeta}\, \zeta_u^n\right] . \qquad (4)$$

Taking $\zeta_u^0 = 0$ as an initial approximation, we obtain in the first control step

$$\zeta_u^1 = \left[\frac{\partial H(\mathbf{f}, z; 0)}{\partial \zeta}\right]^{-1} [J(\mathbf{f}, z; \zeta) - H(\mathbf{f}, z; 0)] , \qquad (5)$$

which is immediately responded by the AOS. In what follows the scheme given by Eq. (4) with the new function $J(\mathbf{f}, z; \zeta - \zeta_u^1)$. Then we proceed to the next iteration of the algorithm according to Eq. (5). Therefore, for the exact instrumental realization the scheme described by Eq. (4) is simplified and can be written as

$$\frac{\partial H(\mathbf{f}, z; 0)}{\partial \zeta}\, \zeta_u^{n+1} = \Delta H(\mathbf{f}, z; \zeta) , \qquad (6)$$

where $\Delta H(\mathbf{f}, z; \zeta) = J(\mathbf{f}, z; \zeta) - H(\mathbf{f}, z; 0)$. An instrumental realization of the algorithm described by Eq. (6) is considered in Ref. 2 in more detail. It was demonstrated in Ref. 3 that the low-frequency OTF, whose magnitudes have meanings of normalized displacements in the pupil plane by the vector $\mathbf{f} = -(\xi, \eta)$ in Eq. (3), can be simplified and within a constant factor is represented by the formula

$$H(\mathbf{f}, z; \zeta) = \sum_{k=1}^{n} e^{-i\, z\, \mathbf{f}\, r_k}\, e^{i\, 2\pi\, \mathbf{f} \zeta_k} . \qquad (7)$$

0235-6880/98/11 1063-03 $02.00

which can be used to analyze the matrix of derivatives.

For low spatial frequencies the effect of phasing is insignificant and hence the modes of the segment in Eq. (7) are $\zeta_k = (\beta_k, \gamma_k)$. This means that for these frequencies the problems of adjusting and phasing can be considered separately. In addition, for an extended source,[3] the problem of adjusting for low spatial frequencies can be solved using the iterative scheme described by Eq. (6) with the help of the approximate inequality

$$H(\mathbf{f}, z; \zeta) \approx [H(\mathbf{f}, 0; 0) + i2\pi S\mathbf{f}\zeta_{\mathrm{m}}] \times$$

$$\times J(\mathbf{f}, z; \zeta)/J(\mathbf{f}, 0; \zeta) , \quad (8)$$

where S is the area of the pupil aperture; ζ_{m} is the mean WF tilt determined by the first-order moments of the image intensity distribution $I(x, y)$

$$\zeta_{\mathrm{m}} = \left(\frac{M_{10}}{M_{00}}, \frac{M_{01}}{M_{00}}\right) ;$$

$$M_{st} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x, y)\, x^s\, y^t\, \mathrm{d}x\, \mathrm{d}y,$$

and can be measured. It should be noted that the OTF given by Eq. (8) is measured in two planes $z \neq 0$ and $z = 0$.

In Refs. 2 and 3 the problem of finding of the mode vector was reduced to a solution of the system of equations

$$\frac{\partial H(f_j, z; 0)}{\partial \zeta}\, \zeta_u^{n+1} = \Delta H\, (f_j, z; \zeta) \;, \; j = \overline{1, N} \;, \quad (9)$$

derived from Eq. (6) by means of selection of N frequencies that ensure the best conditioned matrix of derivatives whose elements can be easily derived from Eq. (7)

$$\frac{1}{i2\pi}\left\{\frac{\partial H(\mathbf{f}, z; 0)}{\partial \zeta}\right\}_{N\times n} = \{\mathbf{f}\mathrm{e}^{-iz\mathbf{f}r_k}\}_{N\times n} .$$

A problem of selection of the frequencies f_j is $2N$-dimensional. It was solved by the trial-and-error method which considerably complicated as the number of segments increased. Its analytic solution was found only for a mirror with 3 segments.

In the present paper, we suggest to fulfil the control ζ_u using the functionals of the point source image with the help of iterative scheme (6) in the following form:

$$\frac{\partial \Re_j\, [H(\mathbf{f}, z; 0)}{\partial \zeta}\, \zeta_u^{n+1} = \Re_j\, [\Delta H(\mathbf{f}, z; \zeta)] \;, \; j = \overline{1, N} \;, \quad (10)$$

where $\Re_j$ are the linear functionals of H which in its turn depends on the spatial frequency. This method has allowed us to select the functionals $\Re_j$ so that the matrix of derivatives in Eq. (10) has predetermined properties. In addition, the use of the entire range of low frequencies excludes the possibility that the selected frequencies f_j necessary to fulfil the iteration scheme described by Eq. (9) are lacking. The condition for the functional can be written in the form

$$\int_{\theta(\mathbf{f})} \frac{\partial H(\mathbf{f}, z; 0)}{\partial \zeta_k}\, F_j(\mathbf{f})\, \mathrm{d}\mathbf{f} = \delta_{jk} \;, \; k = \overline{1, N} \;, \quad (11)$$

where δ_{ik} is Kronecker's delta symbol, and $\theta(\mathbf{f})$ specifies the low-frequency domain of integration. Here, the unknown scalar function $F_j(\mathbf{f})$ has the form

$$F_j(\mathbf{f}) = \sum_{s=1}^{n}\left[\frac{\partial H(\mathbf{f}, z; 0)}{\partial \zeta_s}\right]^{*} \lambda_{js} \;. \quad (12)$$

Thus, the determination of the functional is reduced to the selection of the vector λ_j. The substitution of Eq. (10) into Eq. (11) gives the system of equations for λ_j

$$i\,2\pi \sum_{s=1}^{n}\left[\int_{\theta(\mathbf{f})} \frac{\partial H(\mathbf{f}, z; 0)}{\partial \zeta_k}\left(\frac{\partial H(\mathbf{f}, z; 0)}{\partial \zeta_s}\right)^{*} \mathrm{d}\mathbf{f}\right]\lambda_{js} = \delta_{jk} \;,$$

$$k = \overline{1, n} \;,$$

or

$$j\,2\pi\, \Gamma\lambda_j = \mathbf{c}_j \;, \quad (13)$$

where Γ is the $n\times n$ Hankel matrix with block elements

$$\Gamma_{ks} = \int_{\theta(\mathbf{f})} \frac{\partial H(\mathbf{f}, z; 0)}{\partial \zeta_k}\left[\frac{\partial H(\mathbf{f}, z; 0)}{\partial \zeta_s}\right]^{*} \mathrm{d}\mathbf{f} =$$

$$= \mathbf{f}^{\mathrm{T}}\, \mathbf{f} \int_{\theta(\mathbf{f})} \mathrm{e}^{-i\,z\,\mathbf{f}\,(r_k - r_s)}\, \mathrm{d}\mathbf{f} \;;$$

$\mathbf{c}_j$ is the vector with elements $c_{jk} = 1$ and zero values of the remaining elements. The components of the vector of derivatives $\mathrm{d}H(\mathbf{f}, z; 0)/\mathrm{d}\zeta$ are linearly independent functions and hence a solution of system (13) for λ_j, $j = 1, N$ does exist. This choice of the vectors $\mathbf{c}_j$ specifies the unit matrix in the left side of Eq. (10), and the iterative scheme assumes the form

$$\zeta_u^{n+1} = \Re_j\, [\Delta H(\mathbf{f}, z; \zeta)] \;, \quad j = \overline{1, N} \;. \quad (14)$$

The functionals selected in this way make the iteration scheme realizable. It only remains to ensure its convergence. This can be done by the trial-and-error method of selection of the optical system parameters. They are: the coordinate z of the image plane, the range of variation of measurable parameters, and the Tikhonov regularization parameter.

We solved the system of equations (14) for functionals (11) with the domain of integration representing a circle with the radius $|\mathbf{f}| = 0.15$ for aberration modes of the order $w = 0.64$ and defocusing $z = 5$. In the normalized coordinates, accepted by us, $w = 0.61$ corresponds to the displacement of the central ray of the segment by the Airy circle radius. The initial mode distribution was within $\pm w$.

The algorithm for the segmented mirror adjustment was modeled under assumption that, for point source, $J(\mathbf{f}, z; \zeta)$ had been measured in one plane $z = 0$ of the domain $\theta(\mathbf{f})$. It had been measured in the planes $z \neq 0$ and $z = 0$ for an extended unknown source. The ratio $J(\mathbf{f}, z; \zeta)/J(\mathbf{f}, 0; \zeta)$ for system (9) was modeled as a distorted variant of the ratio $H(\mathbf{f}, z; \zeta)/H(\mathbf{f}, 0; \zeta)$. The noise was modeled as a normal random variable with a given standard deviation for which the maximum error in calculating ζ by the scheme given by Eq. (15) did not exceed 5% of the maximum coordinate $|\zeta| = w$.

Figure 1 shows the norms $\|\zeta_s\|/\|\zeta_{max}\|$ of variations of the vector of corrected WF tilts for a mirror with 6 segments. Curves *1* and *2* illustrate the convergence of the algorithm for point and extended sources, respectively. It can be seen that the iterative process converges to a certain constant rather than to zero. This algorithm compensates only the WF aberrations that comprise only low-frequency constituents. This is a consequence of the fact that the OTF was measured only at low frequencies. Curves *3* and *4* illustrate the iterative process when the measurements noise is taken into consideration. It oscillates near the fixed value.

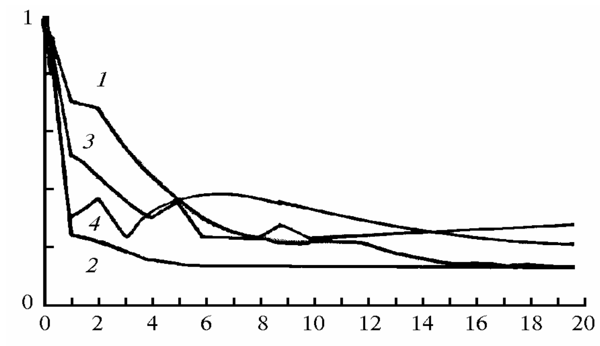

FIG. 1.

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