ADAPTIVE COMPENSATION FOR TIME-DEPENDENT THERMAL BLOOMING WITH LOCAL EXTREMA IN THE SPACE OF CONTROL COORDINATES

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Received July 31, 1998

Laser beam adaptive control with the use of a multidither algorithm is considered under the conditions of thermal blooming. If the algorithm is applied before the termination of transient processes in a medium, an adaptive system finds the region of the extremum but leaves it in a short while. Time required to find the extremum is defined by the value of a gradient step, lesser is the step, greater is the time. On the contrary, the time interval during which the system remains in the vicinity of the extremum is inversely proportional to the value of the step. The problem becomes even more complicated if local maxima appear in the space of control coordinates. In this case if the value of a gradient step is chosen too small the algorithm stops in a local maximum. It was shown that initially the control should be applied only to the tilt of the beam wave front, the step of the algorithm can be taken large that ensures the termination of the control in the vicinity of the global extremum. After that, to detect precisely the location of the maximum, the control algorithm could be applied to other coordinates.

1. INTRODUCTION

Influence of local extrema on efficiency of multidither algorithm realized with the use of steady-state parameters of a high-power laser beam was considered in our paper published in *Atmospheric and Oceanic Optics* (Ref. 1). It was shown that efficiency of adaptive correction for thermal blooming is 30% to 50% less if control stops in a local maximum. This difficulty can be resolved by an increase in the receiver aperture (the region over which the laser beam parameters are integrated). If the radius of the aperture is large enough, the goal function is smooth and has only one extremum.

In this paper the correction for time-dependent thermal blooming is considered when the local maxima are present. It was shown that in a short-time interval after a laser beam is switched on, the goal function has only one extremum, coordinates of which are changed with heating of the medium. So it is possible to assume that if the algorithm tracks precisely the location of the maximum during the development of the transient process, the system is always positioned at the global maximum. To prove this assumption in the present paper, we evaluate precision of detecting the extremum coordinates for a «smooth» hill (a goal function with one extremum). After that correction for thermal blooming is considered under the condition of local extremum development.

2. MATHEMATICAL DESCRIPTION OF A BEAM PROPAGATION. AN ALGORITHM OF CONTROL

Analysis of the control in a homogeneous medium was performed in approximation of nonstationary wind refraction. So the following system of equations can be used to describe the complex amplitude E of a field[2]:

$$2ik\frac{\partial E}{\partial z} = \Delta_{\perp}E + 2\frac{k^2}{n_0}\frac{\partial n}{\partial T}TE\ ; \qquad (1)$$

$$\frac{\partial T}{\partial t} + (\mathbf{V}\nabla)T = \frac{\alpha}{\rho_0 C_p}I\ , \qquad (2)$$

here k is the wave number, n_0 is unperturbed value of the refraction index n, z is the coordinate axis along which the beam propagates, T is the temperature of the medium, $\mathbf{V}$ is the wind velocity vector, α is the absorption coefficient, other designations being of common use.

The beam and medium interaction is characterized by the following dimensionless parameter:

$$R_v = \frac{2ka_0^2\alpha I}{n_0\rho_0 C_p V}\frac{\partial n}{\partial T}, \qquad (3)$$

which is proportional to the intensity I, squared initial radius of the beam a_0, and also depends on other parameters of the beam and medium.

The field of the laser beam in the observation plane is characterized by a criterion of focusing

$$J(t) = \frac{1}{P} \iint \exp\,(-\,(x^2 + y^2)/r_{\mathrm{a}}^2) \times$$

$$\times\, I(x, y, z_0, t)\,\mathrm{d}x\,\mathrm{d}y, \qquad (4)$$

actually $J(t)$ is the power of light within the aperture. In Eq. (4) r_{a} is the radius of the aperture, P is the total power of the beam.

The diffraction length $z_{\mathrm{d}} = 2ka_0$ of radiation was chosen as a spatial scale of the problem along the axis of the beam propagation (here k is the wave number). In the plane perpendicular to the beam propagation the scale was the initial radius a_0. The temporal scale of the problem was the wind clearing time $\tau_V = a_0/|\mathbf{V}|$ characterizing the development of transient processes associated with heating of the medium. Corresponding variables are normalized to these scales.

Corrections for nonlinear distortions are considered with the use of multidither[3] sensing according to which a change in the control coordinates of the adaptive corrector $\mathbf{F} = \{F_1, F_2, \ldots, F_N\}$ is performed by the following formula:

$$\mathbf{F}(t) = \mathbf{F}(t - \tau_{\mathrm{d}}) + \alpha(t - \tau_{\mathrm{d}})\,\mathbf{grad}\,J\,(t - \tau_{\mathrm{d}}). \qquad (5)$$

Coefficients of Zernike polynomials are used as components of the vector $\mathbf{F}$; $\alpha(t - \tau_{\mathrm{d}})$ is the coefficient that defines the value of the gradient step at each iteration. The components of the vector $\mathbf{grad}\,J\,(t - \tau_{\mathrm{d}})$ are derivatives $\partial J(t)/\partial F_i$ calculated during the test soundings. To calculate the derivatives, a small variations ΔF_i are prescribed to each of the control coordinates. Corresponding increase ΔJ of the J criterion is found as a solution to the problem of propagation.

The control with the use of non steady-state parameters is performed assuming that the period of test variations is much less than the wind clearing time τ_V, i.e., during the test variation the goal function remains constant.

All the numerical experiments, the results of which are presented in this article, have been carried out using the following set of parameters: $R_v = -\,100$, the path length $z = 0.5$, the length of the nonlinear portion of the path is taken to be less than the path length, $z_{\mathrm{nl}} = 0.1$. The radius of the aperture was taken to be equal to the initial radius of the beam that insures the distribution of the criterion J that has only one extremum in the observation plane. Local extrema have been observed when the aperture radius was four times decreased.

Control with the use of multidither sounding was also considered with other sets of the problem parameters (the conditions under which local extrema develop were assessed in Ref. 1). The main features of the algorithm remained the same.

3. ESTIMATES OF THE ACCURACY IN SEEKING THE EXTREMUM WHEN PERFORMING CONTROL USING NON STEADY-STATE PARAMETERS

The possibility of performing control with the use of non steady-state parameters (before termination of the transient processes in the beam − medium system) has been considered by the authors of Refs. 4 and 5. In particular, it was shown that convergence of the control algorithm depends on the value of a gradient step α. In this article we present some results of a more detailed investigation into this problem. The possibility is being assessed of not only determining the extremum during a finite time interval, but also of tracing the goal function's extremum location during the process of the medium heating by the beam.

The isolines in Fig. 1 show the distribution of the criterion $J(t)$ over the space of two control coordinates (tilt and defocusing) at different moments in time. Figure 2 illustrates the motion of the extremum (curve *1* in Figs. 2*a* and *b*). Corresponding changes in the criterion J at the point of maximum is shown in Fig. 2*c* (curve *1*). Obviously, positioning the adaptive system in the extremum at every moment in time would provide for the highest efficiency of control.

The speed with which a corrector profile is changed is determined by an interval between two successive iteration steps and by the coefficient α (the step in tilt α_{Tilt} can differ from that in focusing α_{Foc}). Varying these two parameters, interval between the steps and the coefficient α, enabled us to reveal that three variants of the control are feasible.

1. The coefficient α is small ($\alpha_{\mathrm{Tilt}} = 1.0$, $\alpha_{\mathrm{Foc}} = 0.5$). In this case the algorithm reaches maximum in $15\tau_V$ and remains in the extremum practically without any limitations on time. In this version the control is very slow, and no tracing of the extremum is feasible at the initial moments in time.

2. Increasing the gradient step, that is α approaches optimal value. In Figs. 2*a* and *b* this variant is characterized by curves *2*. In the interval from 0 to $4\tau_V$ the system moves slower than the maximum, after $4\tau_V$ the tilt, defocusing, and the criterion J are approach optimal values, but the algorithm does not stop in the extremum, the focusing continues to increase that finally leads to defocusing ($t > 6.5\,\tau_V$) and decreases the light field concentration on an object.

3. Further increase of α (curves *3* in Figs. 2*a*, *b*, and *c*). Although in this variant the tilt and defocusing differ, at the initial moments in time, from optimal values, those are nearer to optimum as compared to the above two examples. At $t > 2\tau_V$ the growth of tilt and defocusing continues and exceeds the optimal values. Criterion J in the interval from τ_V to $2\tau_V$ is almost equal to its value in the extremum (curves *1* and *3* in Fig. 2*c*), while then it essentially decreases.

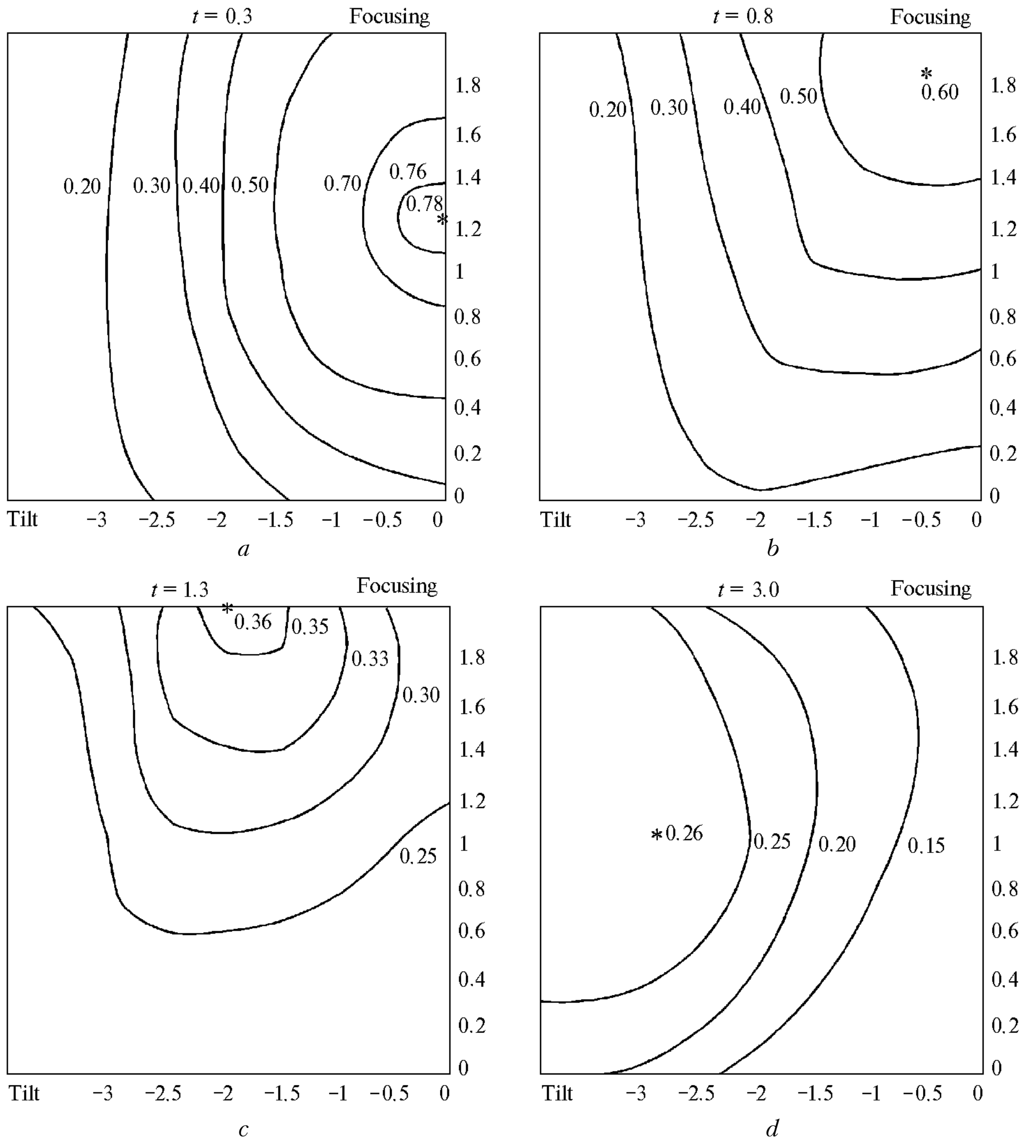

FIG. 1. Distribution of the criterion J(t) (Eq. (4)) in the space of coordinates of tilt and focusing recorded at different moments in time. The initial conditions have been taken one and the same in all calculations, namely, $R_v = -100$; $z = 0.5$; $z_{\rm nl} = 0.1$, $r_{\rm a} = a_0$.

If summarizing, one may arrive at a conclusion that it is impossible to find out the optimal value of the gradient step that would allow the adaptive system to detect coordinates of the extremum at the initial moments in time and to hold the optimal focusing after termination of the medium heating, that is, if the control is too slow, the system is behind the maximum motion, otherwise the system leaves the vicinity of the extremum in a while.

The peculiarities of multidither sensing with the use of non steady-state parameters are well known so the first variant may be explained, but realization of the control in the variants two and three calls for further investigations (i.e., the question should be answered: "Why adaptive system leaves extremum after finding it ?").

When performing control, we assumed that multidither sounding provides for detecting the extremum of the function presented in Fig. 1. Every point determined by this function is a solution to the problem of nonstationary wind refraction given certain initial conditions. If the frequency is high (time between the test variations is much shorter than the characteristic time of thermal lens change), the function does not change during the time of the gradient step change. Thus, one may expect, within the framework of the assumption made, that, as a result of heating of the medium, at a gradient step larger than the optimal one (the gradient step is kept constant during the control) the control coordinates and criterion of focusing would oscillate in the vicinity of extremum and the larger is coefficient α, the larger is the amplitude of oscillations. However, this situation has never been observed in the numerical experiments.

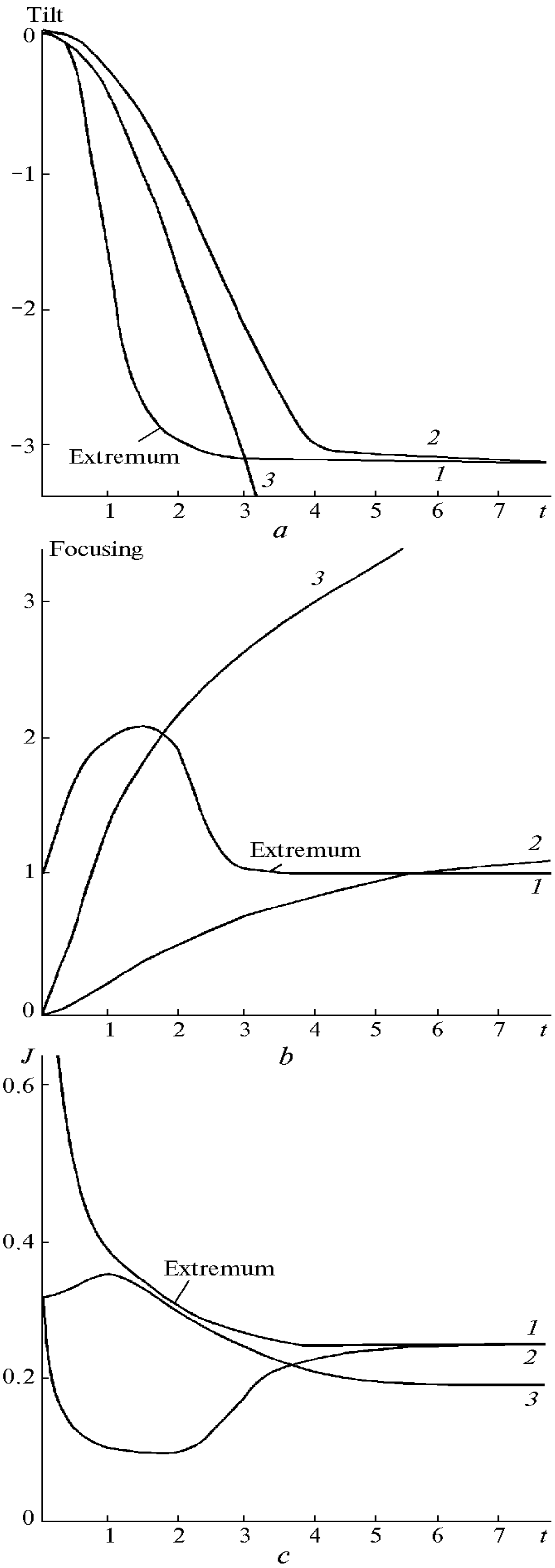

FIG. 2. Changes of the extremum coordinates (tilt (a) and defocusing (b)) and the criterion J(t) (c) at heating of a medium by a beam (curve 1 in all the figures) and corresponding changes in those due to control (curve 2 corresponds to $\alpha_{\rm Tilt} = 2.5$, $\alpha_{\rm Foc} = 0.2$, *curve 3 to* $\alpha_{\rm Tilt} = 2.5$, $\alpha_{\rm Foc} = 0.5$).

Among the causes of inaccurate detection of the extremum coordinates may be the difference between the "frozen" hill and the function presented in Fig. 1. To confirm this hypothesis, the following numerical experiment has been conducted. We have simulated propagation of the beam under conditions of nonstationary wind refraction up to $3\tau_V$ time with control coordinates corresponding to the maximum in the goal function (tilt equals to 3.0, focusing to 1.0). Then, having fixed the thermal lens, we have varied the control coordinates and calculated the light field on the object (solution of the problem on beam propagation under conditions of a constant thermal lens).

In that way we have simulated the distribution of the goal function as it is sensed by fast multidither sounding. Thus obtained distribution is presented in Fig. 3. As seen from this figure, it differs from the function shown in Fig. 1*d* (this function was calculated for time of $3\tau_V$). In particular, the coordinates of its maximum are as follows: tilt equals 3.0, defocusing 2, that means that the optimal defocusing for the frozen hill is almost two times larger, while the optimal tilt being the same. Evidently, such a distribution of the criterion explains the tendency in the algorithm operation toward overfocusing.

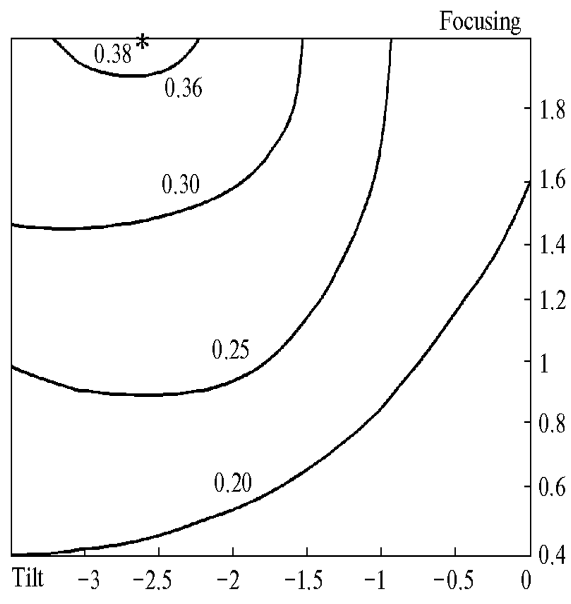

FIG. 3. Distribution of J(t) for a frozen thermal lens.

The thermal lens that has been calculated assuming different initial conditions differs from those shown in Fig. 1*d* and in Fig. 3. In this case the optimal defocusing is less while the tilt being the same as in the two examples considered above. The function shown in Fig. 3 transforms into the function shown in Fig. 1*d*, if the problem on the beam propagation under nonstationary wind refraction during $(2–3)\tau_V$ is being solved given each set of the control coordinates.

Since the goal function that has been introduced into the control algorithm is different than the function whose extremum is being sought, one can arrive at a conclusion that multidither sounding using non-steady-state parameters of the problem is inherently unstable. The algorithm does not always provide for accurately determining the extremum (one may expect only approaching the maximum neighborhood) while, in any case, the adaptive system leaves, in a while, the vicinity of the extremum and moves toward overfocusing. The time during which the adaptive system is in the vicinity of extremum depends on the gradient step value. If the choice of α is wrong, this time can be quite short (as, for example, that shown in Fig. 2 when the parameters correspond to curve *3*; the adaptive system was in the vicinity of the maximum only during $1\tau_V$ to $2\tau_V$).

4. PECULIARITIES OF MULTIDITHER SOUNDING UNDER THE PRESENCE OF LOCAL EXTREMA

The development and effect of local extrema on the efficiency of multidither sounding under conditions of steady state thermal blooming has already been considered in Ref. 1. An additional maximum appears in the space of control coordinates, for high-power beam propagation ($R_v = -100$, $z = 0.5$, $z_{\rm nl} = 0.1$), at the radius of the receiving aperture being 1/4 of the initial beam radius.

Solving the problem on propagation of laser beams under condition of time-dependent thermal blooming enables one to show that at the initial moments in time ($t < 3\tau_V$) only one extremum occurs, the second one taking place only at $t \geq 3\tau_V$ (see Fig. 4). So, one may assume that if the algorithm is capable of accurately detecting the motion of the extremum in the space of control coordinates the local maxima would not influence the control efficiency. The troubles may arise in the control solely because it is not so easy to keep the adaptive system in the vicinity of the global maximum.

To illustrate this situation, Fig. 5 presents the change of tilt (*a*), defocusing (*b*), and of the criterion *J* (*c*) for the case of control when local extrema appear at different values of the gradient step α. As a result of thorough selection, we have managed to achieve the situation that algorithm identifies the global extremum ($\alpha_{\rm Tilt} = 1.0$, $\alpha_{\rm Foc} = 0.6$) and the adaptive system is being kept in its vicinity for quite a long time (curves *1* in Figs. 5*a*, *b*, and *c*). As the coefficient α decreasing, the adaptive system moves towards a local extremum (curves *3*), as α increasing the algorithm diverges (curves *2*). In contrast to the problem where the hill is smooth, the small value of the gradient step does not provide for reaching maximum.

One of the possibilities to perform the control with the use of non steady-state parameters is in the control only over tilt at the very initial moments in time, with the focusing being fixed. Such a procedure allows the algorithm to reach the vicinity of a global extremum within a wide range of $\alpha_{\rm Tilt}$ values and then to proceed to control over the tilt and focusing simultaneously. This possibility is illustrated by data presented in Fig. 6. The focusing remained constant being equal to the optimal one, in a linear medium (the calculations have been performed assuming a flat mirror and the defocusing being a half of the optimal one).

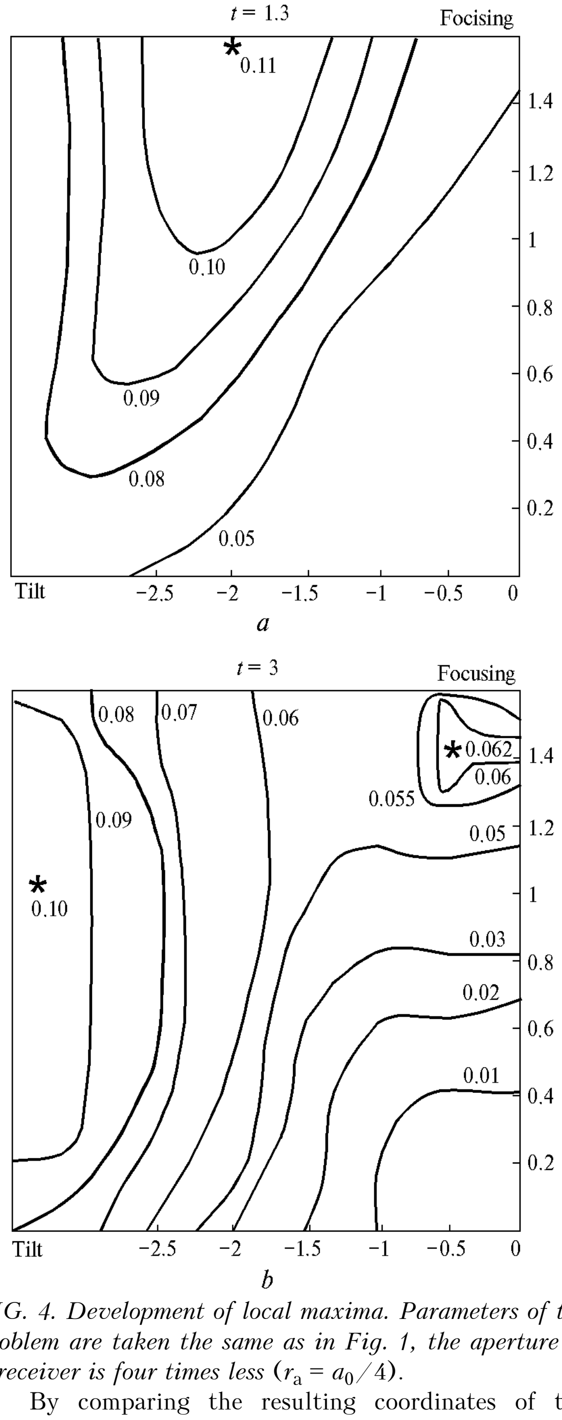

FIG. 4. Development of local maxima. Parameters of the problem are taken the same as in Fig. 1, the aperture of a receiver is four times less ($r_{\rm a} = a_0/4$).

By comparing the resulting coordinates of the global maximum with those from Fig. 4 one may see that the algorithm almost reaches the corresponding coordinate of the global maximum (curves *1* and *2* in Fig. 6*a*). Similar results have also been obtained for all the focusing values considered. The step over tilt may be chosen large enough (the results represented in Fig. 6 were obtained with $\alpha_{\rm Tilt} = 5$ and 10, but calculations were also performed with $\alpha_{\rm Tilt} = 20$).

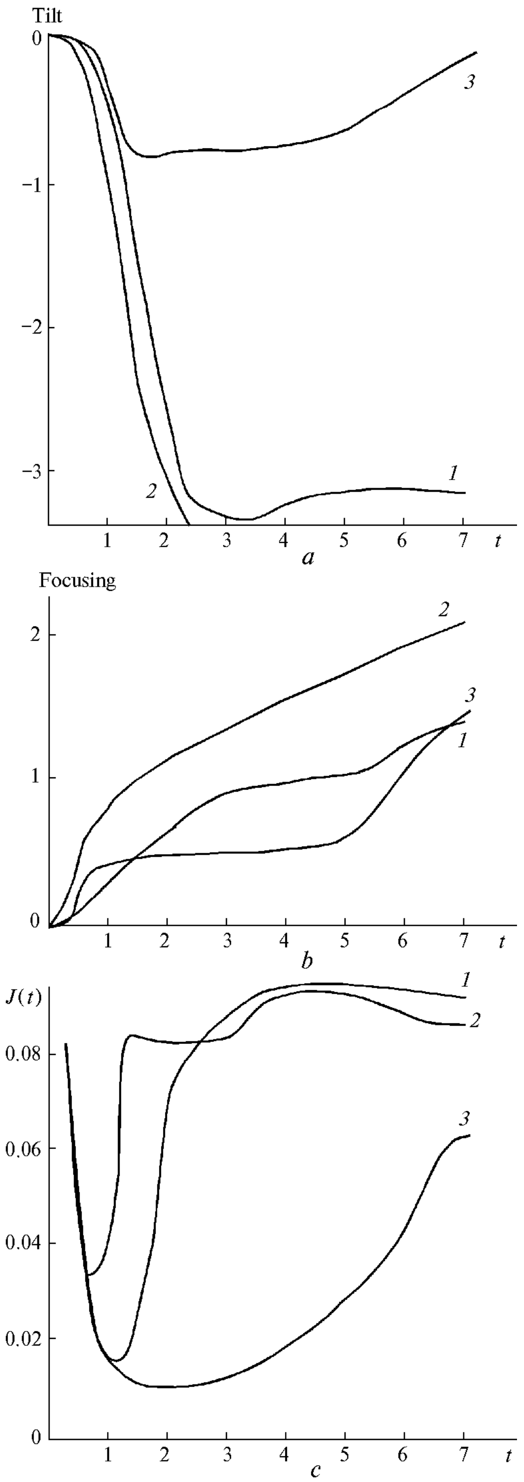

FIG. 5. Changes of tilt (a), defocusing (b), and criterion of focusing (c) due to the control performed. Curve 1 corresponds to $\alpha_{\mathrm{Tilt}} = 10$, $\alpha_{\mathrm{Foc}} = 0.7$ *(catching the global extremum), curve 2 to* $\alpha_{\mathrm{Tilt}} = 10$, $\alpha_{\mathrm{Foc}} = 0.8$ *(the gradient step has been increased and the control algorithm leaves the vicinity of maximum), and curve 3 to* $\alpha_{\mathrm{Tilt}} = 8$, $\alpha_{\mathrm{Foc}} = 0.7$ *(termination of the control in a local maximum). Coordinates of extrema and corresponding values of J(t) are presented in Fig. 4.*

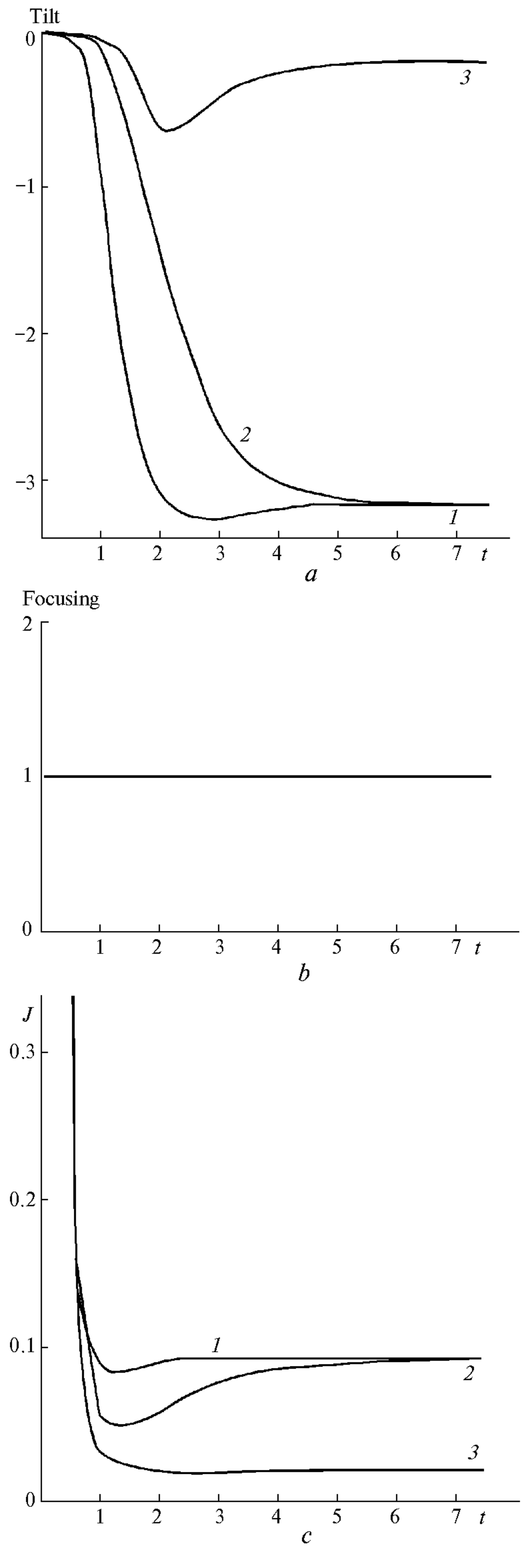

FIG. 6. Control over tilt, $\alpha_{\mathrm{Tilt}} = 10$ *(curve 1) and* $\alpha_{\mathrm{Tilt}} = 5$ *(curve 2), termination of the control in vicinity of the global maximum;* $\alpha_{\mathrm{Tilt}} = 2.5$ *(curve 3), termination of the control in a local maximum.*

On the whole one may conclude that the control algorithm is capable of tracing the tilt variations due to heating of the medium by a beam quite accurately what enables the adaptive system to reach the vicinity of the extremum. At the same time no exact determination of the global maximum coordinate over focusing is possible (it should be kept in mind that these conclusions are only true for the control over non steady-state parameters).

5. CONCLUSIONS

The multidither sounding algorithm, when performed over the non steady-state parameters, provides only for an approximate determination of the extremum coordinates even for the case when the goal function has a single maximum. In a certain time interval the algorithm leaves the vicinity of the extremum while moving toward the increasing focusing. The time during which the adaptive system remains near the maximum depends on the gradient step. The smaller is the coefficient α, the longer is time.

It is practically impossible to choose α that ensures reaching the extremum during the process of heating the medium. If α is too small the algorithm moves slower than the extremum and the adaptive system finds its coordinates only after termination of the heating. If α is too large the algorithm quickly leaves the vicinity of the extremum.

If local maxima are present in the space of control coordinates low speed of the control can result in termination of the control in a local extremum.

Control only over the tilt (the focusing is kept constant) allows one to accurately identify the corresponding coordinates of the extremum at the initial time moments and afterwards as well. This variant ensures termination of the correction in the vicinity of the global maximum. To identify the coordinates of the maximum more accurately, the switch on a control over two coordinates is possible with a small gradient step. In this case local extrema do not influence the efficiency of the control.

REFERENCES

1. F.Yu. Kanev, V.P. Lukin, and L.N. Lavrinova, Atmos. Oceanic Opt. **11**, No. 9, 883–888 (1998).
2. S.A. Akhmanov, M.A. Vorontsov, V.P. Kandidov, A.P. Sukhorukov, and S.S. Chesnokov, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **23**, No. 1, 1–37 (1980).
3. M.A. Vorontsov and V.I. Schmal'gausen, *Principles of Adaptive Optics* (Nauka, Moscow, 1986), 453 pp.
4. I.V. Malafeeva and S.S. Chesnokov, Atmos. Oceanic Opt. **6**, No. 12, 861–863 (1993).
5. F.Yu. Kanev and S.S. Chesnokov, Atmos. Opt. **2**, No. 11, 1015–1019 (1989).

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