CALCULATION OF A SIGNAL-TO-NOISE RATIO FOR A LAYERED INHOMOGENEOUS MEDIUM

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The behavior of a signal–to–noise (S/N) ratio is investigated depending on the optical dimensions of a medium and the quantum survival probabilities as well as the influence of the underlying surface and external sources is analyzed during radiation propagation through a multilayered medium. It has been shown that absorption by a medium affects the signal–to–noise ratio impoving it by a few orders of magnitude. The influence of the layered structure on the dependence of the S/N ratio upon the optical size of a scattering volume is also analyzed.

The investigation of light field formation in layered inhomogeneous media is the important problem of atmospheric and oceanic optics. The techniques for solving this problem are widely discussed in the literature[1]. The use of energy balance and correspondingly energy characteristics as the most advisable technique meets with a number of difficulties. One of them is the complexity of the direct solution of the transfer equation, because this problem calls for additional assumptions whose correctness is difficult to estimate[2]. However, the combination of the exact solution and the method of multiple reflections[3] allows one to estimate quantitavely the radiation balance and consequently to design properly the experimental researches. The necessity arises not only to determine and measure the total transmission and reflection, but also to reveal the regularities in the fraction of radiation exiting through the sides of a dispersed medium. In the process of light radiation transfer through a layered inhomogeneous medium consisting of n layers, the total transmission and reflection do not depend on the rearrangement of the layers only in the special case of unbounded medium. In bounded media the total transmission, reflection, and the fraction of radiation exiting through the sides become the functions of the order of arrangement of layers and essentially depend on the transverse size of a medium. Consequently, the number of experiments required for solving the practical problems on the search for the layered media with optimal parameters becomes great.

For this reason the works[4,5] devoted to the development of the theory of radiative transfer through inhomogeneous media are many in number. The application of general principles of invariance and symmetry allowed one to obtain a number of analytic solutions of the radiative transfer problem for inhomogeneous media. The complexity of the obtained analytic solutions has motivated the wide use of the numerical techniques for the calculation of the radiation field in such media.[4] The calculation of the energy characteristics of radiation propagaiting in a layered inhomogeneous medium, based on the direct solution of the radiative transfer equation, is rather complicated and calls for a number of additional assumptions whose correctness cannot always be easy estimated.

In this connection the attempt was made to describe the process of radiation propagation in layered inhomogeneous media by the method of multiple reflections parametrization of the scattering phase function and model based on the inhomogeneous medium consisting of homogeneous layers.

In spite of the artificial model of medium, a possibility to obtain the solution of the problem of radiative transfer in closed form for any scattering phase function taking into account the spatial boundness of a medium is undoubted advantage of this method, which provides the complete analysis of the regularities in the radiative transfer through layered inhomogeneous media for a wide range of variations of optical dimensions and parameters of the medium.

The fundamental parameters determining the solution of the radiative transfer problem in a layered inhomogeneous medium are: optical size of the medium τ_{x_i}, τ_{y_i}; τ_{z_i}; extinction coefficient in the ith layer $\alpha_i = \sigma_i + \kappa_i$; the quantum survival probability Λ_i; and, parametrized scattering phase function $\eta_i + \beta_i + 4\mu_i = 1$.

For such parametrization of the problem its the solution is found by the method of multiple reflections through the interaction of layers. Transmittance and reflectance can be written in the simple analytical form only for the medium, being unbounded in the plane, perpendicular to the propagation direction and consisting of the conservative scattering layers

$$t_n = \frac{1}{1 + \sum_{i=1}^{n} (\beta_i + 2\,\mu_i)\,\tau_{x_i}}, \qquad (1)$$

$$r_n = \frac{\sum_{i=1}^{n} (\beta_i + 2\,\mu_i)\,\tau_{x_i}}{1 + \sum_{i=1}^{n}(\beta_i + 2\,\mu_i)\,\tau_{x_i}}, \qquad (2)$$

where t_n is the transmittance and r_n is the reflectance of the medium consisting of n layers and τ_{x_i} is the optical depth of the ith layer.

The radiation distribution in the medium versus the running coordinate τ_{x_i} specified at the interface between the ith and $(i + 1)$th layers is determined as follows:

$$I_1(\tau_x) = I_0 \frac{1 + \sum\limits_{i=j+1}^{n} (\beta_i + 2\,\mu_i)\,\tau_{x_i}}{1 + \sum\limits_{i=1}^{n} (\beta_i + 2\,\mu_i)\,\tau_{x_i}} , \qquad (3)$$

$$I_2(\tau_x) = I_0 \frac{\sum\limits_{i=j+1}^{n} (\beta_i + 2\,\mu_i)\,\tau_{x_i}}{1 + \sum\limits_{i=1}^{n} (\beta_i + 2\,\mu_i)\,\tau_{x_i}} . \qquad (4)$$

It follows from the analysis of the obtained equations that the transmittance and reflectance of the unbounded conservative scattering medium do not depend on the order of arrangement of layers, whereas the radiation distribution in the medium transforms when the order of mutual arrangement of layers changes, and may have $n!$ different values.

The description of the process of radiative transfer is considerably complicated and the solution can not be obtained in the analytical form for a spatially bounded layered inhomogeneous medium. The key to the solution to the problem of radiative transfer through the layered inhomogeneous medium is the solution to the problem of radiative transfer through a spatially bounded homogeneous layer, on the basis of which the solution for inhomogeneous medium can be found. The desired solution is written in the form of recurrent relations obtained in the calculation of the interaction of homogeneous layers. The transmittance, reflectance, absorptance, and the fraction of radiation exiting through the side have the form:

$$t_n = t_1\, t_{n-1} / (1 - r_1\, r_{n-1}) , \qquad (5)$$

$$r_n = r_1 + [t_1^2\, r_{n-1} / (1 - r_1\, r_{n-1})] , \qquad (6)$$

$$S^*_n = S^*_1 + \{[t_1\,(r_n\, S^*_1 + S^*_{n-1})] / (1 - r_1\, r_{n-1})\} . \qquad (7)$$

where t_n is the transmittance of n layers; r_n is the reflectance; S^*_n is the sum of absorptances and the fraction of radiation exiting through the side; and, t_1, r_1, S^*_1, t_{n-1}, r_{n-1}, and S^*_{n-1} are the transmittance, reflectance, absorptance, and fraction of radiation exiting through the side of the first and $(n - 1)$th layers, respectively.

Let us study the behavior of the S/N ratio depending on the optical size of the medium and quantum survival probability and analyze the effects of the underlying surface and external sources on the radiation propagating through the medium consisting of two homogeneous layers. The layers are chosen with identical optical depths $\tau_x / 2$.

The values of the S/N ratio depending on the transverse optical size of the medium ($\tau_y = \tau_z$) for different quantum survival probabilities Λ are shown in Fig. 1, in which curves *1* and *3* correspond to the homogeneous media with scattering phase functions characterized by the parameters of elongation a being equal to 1.0 and 12.09, respectively; curve *2* is for the layered medium consisting of two layers with the same scattering phase functions. It can be seen from the figure that the S/N ratio decreases as transverse dimentions of the medium increase, and the layered structure with the number of layers n=2 has no effect on this dependence; two layers with different scattering phase functions are similar to the case of a homogeneous medium with some average scattering phase function. This is explained by the fact that the transmission of the medium does not depend on the order of arrangement of layers.

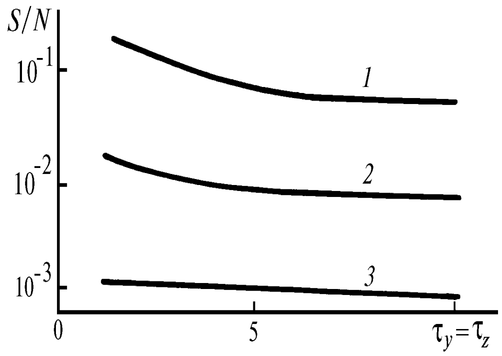

FIG. 1. Signal–to–noise ratio depending on the transverse optical dimentions of the medium at $\Lambda = 0.8$ *and* $\tau_x = 10$:$a = 1$ *(1), layered medium (2), and* $a = 12.09$ *(3).*

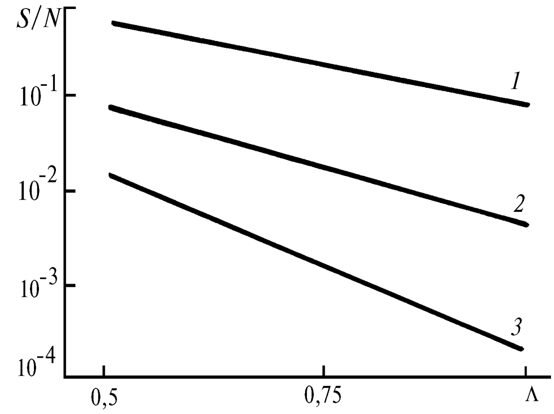

FIG. 2. The S/N ratio depending on the quantum survival probability at $\tau_x = 10$ *and* $\tau_y = \tau_z = 1$: *1)* $a = 1$, *2) layered medium, and 3)* a=12.09.

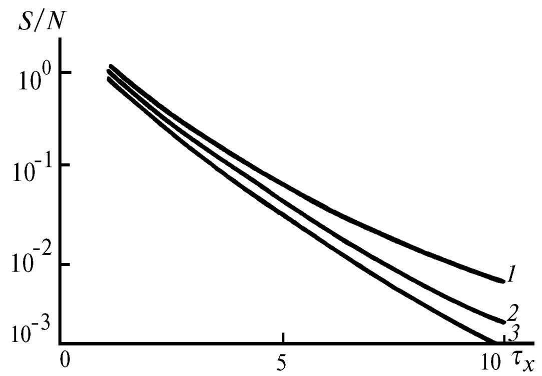

FIG. 3. The S/N ratio depending on the optical depth of layered inhomogeneous medium at Λ=1: *1)* τ_y=τ_z=1, *2)* τ_y=τ_z=5, *and 3)* τ_y=τ_z=10.

The S/N ratio depending on the quantum survival probability Λ is shown in Fig. 2. Curves *1* and *3* correspond to the homogeneous media characterized by the elongation parameters $a = 1.0$ and 12.09; curve *2* – to the two–layer medium. It can be seen from the figure that the absorption by the medium affects the S/N ratio improving it by a few orders of magnitude. This is explained by the decrease of the intensity of multiply scattered light as absorption by the medium increases.

The S/N ratio depending on the optical depth of the medium τ_x is shown in Fig. 3. The sharp decrease of the S/N ratio is found to occur as the optical depth of the medium increases for any optical transverse size, and layered structure has no effect on this dependence.

Thus, from the results obtained it may be concluded that:

– the S/N ratio does not depend on the order of arrangament of layers in a two–layer medium, because the transmission in this case remains unchanged;

– a medium consisting of two layers is similar to a homogeneous medium with identical optical size and some average scattering phase function.

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