PROBABILITY DENSITY OF THE INTENSITY AND FLUX FLUCTUATIONS OF OPTICAL RADIATION PROPAGATING AND REFLECTING IN THE TURBULENT ATMOSPHERE

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Received June 3, 1993

In this paper we analyze some results of recent investigations of theoretical models and experimental data on the probability density of the intensity and flux fluctuations of optical radiation in the turbulent atmosphere. In doing so we consider the K–, universal, Beckman, and 1–K distributions. We also analyze the errors in determining the highest normalized moments of the distributions and the influence of the instrumental dynamic range on the estimate of the probability density of fluctuations. As our comparative analysis of experimental data and model distributions has shown, the K–distribution is asymptotically more adequate for saturated fluctuations ($\beta_0 \geq 10$) than the lognormal distribution and in addition approximates well the experimental data in the case of reflection of radiation from an array of corner–cube reflectors during propagation of radiation in the rain at large optical thicknesses. The fluctuations of the light flux reflected from specular objects and received by a distributed aperture practically always obey the lognormal law.

Experimental studies allow us to conclude that for arbitrary values of the parameter β_0, characterizing the conditions of light propagation, superposition of fields whose amplitudes obey the K– and lognormal distributions, is preferable for the description of experimental data.

1. INTRODUCTION

Probability density of the intensity and flux fluctuations of optical radiation is the most complete single–point statistical characteristic, which determines the reliability and noise proofing of the optical communication systems[1–3] as well as the noise of the goniometric and range–finding optical systems operating in the atmosphere.[4] Probability density for weak intensity fluctuations was studied in detail by V.I. Tatarskii,[5] while the case of strong fluctuations was considered in Refs. 6–8. Currently some new experimental data have been obtained, and new theoretical models of the probability density of the optical wave fluctuations have been developed. This problem is of great interest because of its practical importance[1–4] as well as due to the fact that now there is not any reliable theoretical model describing the probability density over the whole variety of turbulence conditions at least for a forward propagation path, while the experimental data give no way to prefer one or other model for strong intensity fluctuations. Only a small number of papers considered the problem of the intensity and light flux fluctuations in the case of reflection in the turbulent atmosphere as well as in the rain.

In the monographs and reviews devoted to the intensity fluctuations[6,7,8] some problems concerning the measurement accuracy and being of great importance for the experiments have still received only insufficient study.

In this paper some available theoretical models of propagation and real accuracy of measuring the statistics of fluctuations are analyzed. Experimental data (obtained mainly by one of the co–authors) are compared with the theoretical models of fluctuations developed in recent years.

2. THEORETICAL MODELS OF PROBABILITY DENSITY OF THE LIGHT INTENSITY FLUCTUATIONS IN THE CASE OF RADIATION PROPAGATION THROUGH THE TURBULENT ATMOSPHERE

When radiation propagates through the medium with a random field of the refractive index, some part of the energy flux is scattered and the type of function describing the probability density of the intensity fluctuations is determined by scattering mechanism. The model of a single–ray propagation, allowing for the radiation being forward–scattered by the inhomogeneities located on the receiver–transmitter axis, can be considered as a simple example. If the resultant field is the sum of the great number of independently forward–scattered components, then applying the central limit theorem to the expression for logarithm of the wave field amplitude, one can conclude that both the amplitude and the intensity distributions are lognormal

$$P(I) = \left(\sqrt{2\pi\sigma}\, I\right)^{-1} \exp\left[-\,(1/2\sigma^2)\left(\ln I - \xi\right)^2\right], \qquad (1)$$

$$\sigma = \ln(1 + \beta^2), \quad \xi = \ln\left[\langle I\rangle/(1 + \beta^2)^{1/2}\right],$$

where $\beta^2 = (\langle I^2\rangle - \langle I\rangle^2)/\langle I\rangle^2$ is the normalized variance of the intensity I, in other words, the flicker parameter, and angular brackets denote an ensemble averaging. This model

0235-6880/93/11 760-09 $02.00

was first proposed and substantiated by V.I. Tatarskii[5] for plane waves and was further developed for the other ray geometry, in particular, for spherical waves.[8,9] There is sufficiently large experimental data base[6,10,11] confirming the lognormal statistics for the amplitude and radiation intensity. However, both the theoretical analysis and the experimental results show that this model can be applied only under specific propagation conditions. Really, a path length must be sufficiently long to ensure the applicability of the central limit theorem and at the same time sufficiently short to guarantee that the multiray effects due to scattering by off–axis vortexes contribute only insufficiently to the resultant field. The contribution of these effects is also determined by the optical strength of the atmospheric turbulence. The universal parameter characterizing the propagation conditions on the path is (see Ref. 6)

$$\beta_0(L) = 1.23\, C_n^2\, k^{7/6} L^{11/6},$$

where C_n^2 is the structural characteristic of the refractive index field, L is the path length, and $k = 2\pi/\lambda$ is the wave number. One can say that the intensity fluctuations are described well by the lognormal distribution for weak fluctuations when $\beta_0 < 1$ (in this case the path length L must be much longer than the outer scale of turbulence L_0 ($L_0 \ll L$)).

Some authors tried to extend the lognormal model over the strong fluctuations[12,13,14] to approximate the experimental distribution by a modified lognormal distribution[15] but they failed since they remained within the scope of the model of a single–ray propagation. Moreover, the lognormal model predicts infinite growth of the intensity fluctuations as β_0 increases,[16] but this is not the case. In fact, the fluctuations increase only up to certain maximum values and then decrease gradually with further increase of β_0.

For strong fluctuations (or, more specifically, in the regime of saturation) when $\beta_0 \gg 1$, the resultant field is the superposition of multiply scattered waves. Under assumption that the components of the resultant field are statistically independent and numerous, the application of the central limit theorem results in the Rayleigh distribution of the total amplitude and, therefore, the exponential distribution of the intensity

$$P(I) = \langle I \rangle^{-1} \exp(-I/\langle I \rangle)\ . \tag{2}$$

From an asymptotic analysis of the behavior of normalized intensity moments[17] $\langle \tilde{I} \rangle$ the conclusion was drawn that the exponential distribution

$$\langle \tilde{I}^n \rangle = n!\, [1 + 0.21\, \beta_0^{-4/5}\, n(n-1)]$$

is applicable.

In the limiting case ($\beta_0 \to$ $) this expression leads to the relation for the intensity moments corresponding to the exponential distribution. However, in the real atmosphere β_0 takes the finite values; moreover, for the atmospheric turbulence with a wide range of inhomogeneity scales the components of the scattered fields prove to be partially correlated because of large inhomogeneities resulting in the deviation of the total intensity distribution from the exponential law. This model can be considered only as a limiting case for the very strong fluctuations, prerequisites to the formation of which in the atmosphere have not yet been clear.

The most widespread distribution for the saturation region is the so–called K–distribution[18–21]

$$\langle I \rangle P(I) = \left(2/\Gamma(y)\right) y^{(y+1)/2} I^{(y-1)/2} K_{y-1}\left[2(I\,y)^{1/2}\right], \tag{3}$$

$$y = 2/(\beta^2 - 1), \quad y > 0,$$

where $K_y(z)$ is the modified Hankel function.[22] It was derived under assumption that the radiation is scattered by the object population obeying the binomial distribution in the limiting case of the great average number of objects, and since the scattered field can be represented by a two–dimensional vector, the results obtained by the method of wandering in the plane with binomial distribution of the number of steps can be applied to the scattering process. The limiting case in solving this problem is the K–distribution. For very large values $\beta_0 \to \infty$ when the parameter $\beta \to 1$, it can be reduced to the exponential distribution. Since this distribution has been introduced, as shown in Refs. 18, 20, and 23, it is very useful in modeling of non–Gaussian statistical characteristics of radiation scattered by such different objects as the Earth's and sea surfaces as well as by the extended and localized turbulence, but its range of applicability is limited by the condition for the flicker parameter $\beta^2 \ge 1$. This makes it impossible to use this distribution for weak turbulence. Naturally, some attempts were undertaken to extend the K–distribution over this region. In particular, the model of generalized K–distribution was proposed by R. Baracat[24] and considered also in Refs. 25 and 26. According to this model

$$\langle I \rangle P(I) = \frac{2N}{\Gamma(N)} \left(\frac{N}{\xi}\frac{I}{\langle I \rangle}\right)^{(N-1)/2} \left(\xi + \frac{\upsilon^2}{4}\right)^{(N+1)/2} \times$$

$$\times I_0\left\{\upsilon\left[\left(\xi + \frac{\upsilon^2}{4}\right)\frac{I}{\langle I \rangle}\right]^{1/2}\right\} K_{N-1} \times$$

$$\times \left\{2\left[\xi N\left(\xi^2 + \frac{\upsilon^2}{4}\right)\frac{I}{\langle I \rangle}\right]^{1/2}\right\}, \tag{4}$$

$$\xi = 1 + \upsilon^2/4N,$$

where N is the number of scattering centres, υ is the parameter describing the phase fluctuations, and $I_0(x)$ is the zero order Bessel function. In this model the vector of a scattered electric field is the result of random wandering in a plane with the sense of displacement corresponding to a nonuniform phase distribution. Baracat assumed that the random phase obeys the Mises distribution[24] and the average number of steps tends to infinity. Resultant probability density of the intensity fluctuations obeys the K–distribution for the strong turbulence, while for the weak turbulence it approaches a functional form identical to the generalized Rice distribution and although, as shown in Ref. 26, a simple generalization of the Mises distribution over n measurements is not evident, the distribution proposed by Baracat is of great interest.

Generally the statistical models developed for the intensity fluctuations are based on the assumption that the atmospheric turbulence is stationary, uniform, and isotropic. Even if uniformity and isotropy are reasonable assumptions for many turbulence conditions, the assumption of stationarity can be valid only for very short periods of time during which the turbulence parameters remain significantly unchanged on the propagation path. For long periods of time required for experimental measurements these parameters of turbulence will probably fluctuate in a random way. This fact is responsible for the turbulence intermittance.[27] Taking into account these arguments, Andrews and Phillips[28,29] tried to generalize the K–distribution for weak fluctuations by means of introducing the I–K––distribution

$$P(I)=\begin{cases}(2\alpha/b_0)\left(\sqrt{I}/A\right)^{\alpha-1}K_{\alpha-1}\left(2A\sqrt{\alpha/b_0}\right)I_{\alpha-1}\left(2\sqrt{\alpha I/b_0}\right), I<A^2,\\(2\alpha/b_0)\left(\sqrt{I}/A\right)^{\alpha-1}I_{\alpha-1}\left(2A\sqrt{\alpha/b_0}\right)K_{\alpha-1}\left(2\sqrt{\alpha I/b_0}\right), I>A^2,\end{cases}\quad(5)$$

where α is the parameter determining the number of scatterers, b_0 is the absolute average intensity of a random component of an optical field, $I_v(z)$ and $K_v(z)$ are the modified Bessel functions of the first and second kind, respectively.[22] This distribution was obtained from representation of an optical wave by a bistochastic random process. Taking into account nonstationarity of the atmospheric turbulence, the optical field intensity was considered as an arbitrary random process obeying the Rice–Nakagami distribution as predicted by the Born approximation. The effect of random fluctuations of the parameters of turbulence is modeled by random variations in the mean intensity or variance of the field. Thus by averaging the Rice–Nakagami distribution for gamma–statistics of the fluctuating field variance, one can obtain the I–K–distribution as an absolute probability density for the intensity. Comparing the normalized moments of this distribution with their experimental values, the authors of the model obtained a good agreement for a wide range of variation of the parameter β_0. The main problem in application of the I–K–distribution is the determination of the functional form of its parameters as well as their relations with the physical parameters of the turbulence and optical wave. A technique for adjusting the parameters proposed in Ref. 30 is only approximate and induces some uncertainty in this point.

However, this model of the intensity fluctuations was first considered in Ref. 30 *a* in a physically more vivid representation. In this model the probability density $P(I)$ is based on the concept of two types of scattering inhomogeneities, namely, small–scale inhomogeneities producing a normalized wave field and large–scale ones, which modulate the small–scale turbulence thereby denormalizing the total wave field. For $\beta_0^2(L) \gg 1$ the probability density obtained by averaging over both types of scattering can be written down in the following form:

$$P(I)=\int\limits_0^\infty P_1(I, \langle I_l\rangle)\, P_2(\langle I_l\rangle)\, d\langle I_l\rangle,$$

where $P_1(I, \langle I_l\rangle) = \langle I_l\rangle^{-1}\exp[-I/\langle I_l\rangle]$ is the local exponential function of the intensity distribution due to scattering by small–scale inhomogeneities, $P_2(\langle I_l\rangle)$ is the probability density describing the effect of large–scale inhomogeneities. From the physical viewpoint the intensity fluctuations depend on both the mean number of rays which arrived at the point (effect of small–scale inhomogeneities) and the variance of this number (effect of large–scale inhomogeneities).

For the spectrum of the turbulent fluctuations of the refractive index of the atmosphere obeying the Kolmogorov–Obukhov law in accordance with Ref. 30 *b* the probability density of the intensity can be represented in the following form:

$$P(I)=\begin{cases}\exp(-I)[1+0.724\,\beta_0^{-0.8}(1-2I+1/2\,I^2)], & I\le\beta_0^{0.4},\\ \exp[-I(1-1/1.45\beta_0^{-0.8}I)], & \beta_0^{0.4}\le I\le\beta_0^{0.8},\\ 1.06\beta_0^{0.24}I^{-0.3}\exp[-1.5\beta_0^{0.24}I^{-0.7}], & I\ge\beta_0^{0.8}.\end{cases}\quad(5a)$$

This formula is compared with experimental data below; here we note only that $P(I)$ in the form of formula (5*a*) does not satisfy the normalization condition and has discontinuities near the joints of the intervals of the argument variations.

The models presented above describe the intensity fluctuations of the optical radiation mainly either for weak or saturated turbulence but at the same time they cannot adequately describe an intermediate rather wide region which is of greatest interest for practice. For this case in a number of papers it was suggested to use the so–called mixtures of distributions[31] in which the field of optical ray at the receiving point was assumed to be formed by the two components

$$A\,e^{i\varphi} = A_1\,e^{i\varphi_1} + A_2\,e^{i\varphi_2},$$

the first of which $A_1 e^{iz_1}$ is due to forward scattering of radiation by the inhomogeneities located on the transmitter–receiver axis and obeys the lognormal distribution of the amplitude A_1 and normal distribution of the phase φ_1. The second component $A_2 e^{iz_2}$ is due to multiple scattering by the off–axial inhomogeneities and obeys the Rayleigh distribution of the amplitude A_2 and uniform distribution of the phase φ_2. In particular, based on this model the authors of Ref. 32 proposed the Beckman distribution

$$P(I)=\frac{1}{\sqrt{2\pi}\sigma_{I_1}m_{I_2}}\int\limits_0^\infty\frac{dI_1}{I_1}\exp\left[-\frac{(\ln I_1-m_{I_1})^2}{2\sigma_{I_1}^2}-\frac{I+I_1}{m_{I_2}}\right]I_0\left(\frac{2\sqrt{II_1}}{m_{I_2}}\right),\quad(6)$$

where σ_{I_1} and m_{I_1} are the variance and mean value of the intensity logarithm of the lognormal component, correspondingly, m_{I_2} is the mean value of the intensity of the field Rayleigh component, $I_0(x)$ is the modified Bessel function of the first kind, while the authors of Ref. 33 suggested to use the universal distribution

$$P(I)=\frac{1}{2}\int\limits_0^\infty zJ_0(z\sqrt{I})\,{}_1F_1\left(M,1;-\frac{cz^2}{4M}\right){}_1F_1\left(m,1;-\frac{bz^2}{2m}\right)dz,\quad(7)$$

$c = \langle A_1^2\rangle$, $b = \langle A_2^2\rangle$,

where ${}_1F_1(\alpha, \beta, z)$ is the confluent hypergeometric function.[22] The Beckman distribution was investigated in Ref. 32 in which the relation of its parameters with the characteristics of the atmospheric turbulence and propagation path was found. However, these relations obtained with a phenomenological model are rather approximate and make it impossible to compare them correctly with the experimental data. The universal distribution was studied in Ref. 34. It was shown that this distribution cannot be reduced to the lognormal one for weak turbulence and significantly deviates from the experimental data in this region. A barrier to the application of the mixtures is the mathematical complication of these models. The point is that in contrast to all the above–mentioned two–parametric distributions, the probability density for mixtures depends on the three parameters, two of which characterize the distributions forming the mixture, while the third so–called parameter of mixture r determines their ratio $r = \langle A_1^2\rangle / \langle A_2^2\rangle$. Thus to compare these distributions with the experimental data it is necessary to match the first three moments of distribution with experimental moments, while independence of the third moment of the second one leads, in its turn, to uncertainty of finding the distribution parameters.

So, we have the lognormal distribution for weak fluctuations, the K–distribution for saturated fluctuations, but for the intermediate range we have not yet found the distribution providing a good approximation for the experimental data.

The case of reflection from a spatial array of corner–cube reflectors which are used, for example, in laser detection and ranging of space objects[35] is of great practical interest. The fluctuation characteristics of intensity and radiation flux under the joint action of the atmospheric turbulence and interference of waves reflected from individual reflectors forming the array were studied in Ref. 36, but the expression for the probability density of the intensity fluctuations obtained in it gave the variance of the intensity fluctuations that does not agree with experimental data.[34] Apparently, in accordance with the model description, the K–distribution will approximate the intensity fluctuations quite well provided that the number of corner–cubes (scattering centers) is sufficiently large.

3. SOME ASPECTS OF EXPERIMENTAL MEASUREMENT OF THE PROBABILITY DENSITY OF THE INTENSITY FLUCTUATIONS

To estimate proximity of the distribution function of a random process to a certain theoretical law, one can use the description of this process by the sequence of its moments.[37] This is most often used for the processes of propagating the optical waves,[38–41] apparently, due to the fact that until recently the moments could be measured in a rather simple way. In addition, there are some developed methods for constructing the probability density from the finite number of the moments.[42]

Earlier it was noted that for the saturated intensity fluctuations in the theoretical investigations of the probability density of the fluctuations the most extensively used method is the method of moments. In this case it is necessary to take into account the instrumental and statistical measurement errors because in the real atmosphere the estimates of the highest moments may introduce considerable errors. There are a number of papers[43,44,45] in which the statistical errors in estimating the highest moments were studied as well as the problems of consistency of experimental estimates of the measurement accuracy with the theoretical results were considered. However, in Refs. 43 and 44 only finite period of the measurement T was taken into account and it was implicitly assumed that during the course of measurement any improbable value of random process from the infinite interval could be realized. Such a model of a random signal is not adequate to the technical essence of the process of measurements and in estimating the highest moments it gives the results which deviate widely from the real values because instruments for measurements have always a restricted dynamic range and by virtue of a character of the measurement procedure, they are tuned to the most probable values of a signal to use it more completely. This leads to the fact that improbable values of a random process corresponding to large but finite spikes or deep fadings will be distorted, for example, the spikes will be cut off, while fadings will be distorted by the instrumental noise. The joint effect of both factors, namely, finite instrumental dynamic range and measurement period, on the accuracy of experimental determination of the highest moments of time series was first considered in Ref. 46. In particular, the truncated moment $\langle I_y^n\rangle$ corresponding to the real measurements was calculated

$$\langle I_y^n\rangle = \int\limits_{I_{min}}^{I_{max}} I^n P(I)\, dI,$$

where I_{min} and I_{max} are the minimum and maximum values of a signal in realization. For the lognormal distribution the authors derived the relations for the bias of the estimate of the nth real moment $\langle I_y^n\rangle$ against its model value $\langle I^n\rangle$

$$\delta_n \simeq \frac{\langle I_y^n\rangle}{\langle I^n\rangle} = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{\ln I_{max} - \xi}{\delta\sqrt{2}} - \frac{n\delta}{\sqrt{2}}\right)\right], \tag{8}$$

where erf(z) is the error integral,[22] and for the relative root–mean–square error in estimating the truncated moment

$$\delta_{ny} = \frac{d_{ny}}{\langle I_y^n\rangle} 2\left(\frac{\tau_c}{T}\right)^{1/2} \frac{\exp\left(\dfrac{n^2\delta^2}{2}\right)}{n\delta} \times$$

$$\times \frac{\left[1 + \mathrm{erf}\left(\dfrac{\ln I_{max} - \xi}{\delta\sqrt{2}} - \dfrac{2n\delta}{\sqrt{2}}\right)\right]^{1/2}}{1 + \mathrm{erf}\left(\dfrac{\ln I_{max} - \xi}{\delta\sqrt{2}} - \dfrac{n\delta}{\sqrt{2}}\right)} \left[1 - e^{-1}\right]^{1/2}, \tag{9}$$

where τ_c is the correlation length of a lognormal process.

Calculations of the bias carried out for most experimental conditions show that for the first moment it can be neglected. The underestimation of the highest moments for $\beta > 1$ becomes pronounced when $I_{max} < (10–15)\langle I\rangle$. Such an underestimation of the highest moments was found in the experiments on propagation of laser radiation through the atmosphere.[6] Calculations of the relative root–mean–square error of the experimental moment from Eq. (9) agreed satisfactory with the theoretical data.[43] In addition to the analytical results, the data on model lognormal process that confirm the advantage of the proposed approach were also obtained in Ref. 46. The authors of Refs. 40 and 41 have arrived at the same conclusions as in Ref. 46, but in their papers no consideration has been given to the variance of the estimates of the moments for real sample. In Ref. 47 a procedure for calculating was different from the procedure employed in Ref. 46. It was used to derive the relation for the bias of the normalized moments for this model with allowance for the limited instrumental dynamic range for the K–distribution

$$\delta_n = -\frac{2}{y^n\Gamma(y)}\sum_{i=1}^{n}\frac{\Gamma(n+1)\Gamma(n+y)}{\Gamma(n+1-i)\Gamma(n+y-i)}\left(\frac{z_{DS}}{2}\right)^{2n+y-2i}\times$$

$$\times\left[K_y(z_{DS}) + \frac{z_{DS}}{2(n+y-i)}K_{y-1}\left(z_{DS}\right)\right], \qquad (10)$$

where $z_{DS} = 2(I_{\max}\, y)^{1/2}$, $\Gamma(z)$ is the gamma function, and $K_v(z)$ is the modified Hankel function.[22] Relation for the bias of the moments of exponential distribution modulated by the lognormal distribution, which was proposed in Ref. 48 to describe the saturated fluctuations, was also derived in Ref. 47

$$\delta_n = -\sum_{i=1}^{n}\frac{n!}{(n-i)!}\, I_{\max}^{n-i}(1 + I_{\max}\,\sigma_z^2\,\xi_i)^{-1/2}\times$$

$$\times\exp\left[-i\ln\xi_i - I_{\max}\,\xi_i - (s_z^2/2)\,(i + I_{\max}\,\xi_i)^2\right], \qquad (11)$$

where $\sigma_z^2 = \ln(\langle I^2\rangle/2)$ and ξ_i is determined from the equation $-\, i + 1/2 - I_{\max}\,\xi_i = \ln\xi_i/\sigma_z^2$.

By comparing the results obtained in Ref. 47 with those obtained in Refs. 46 and 49, one can see that an account of the limited instrumental dynamic range most strongly affects the lognormal distribution, has less of an effect on the exponential distribution modulated by the lognormal one, and the least bias has the K–distribution. In this connection, the truncated moments of the above–mentioned distributions are rather close together and in comparison with the experimental data with allowance for the statistical spread it proved to be rather complicated to recognize proximity of the probability distribution to one or another law; therefore, it is necessary to analyze histograms of instantaneous values of the intensity.

The behavior of the highest moments of the distribution mixtures should be specially mentioned. So–called "loop effect", consisting in the fact that normalized moments grow with increase in the turbulence intensity (β_0) up to some maximum values and then with further increase in the turbulence intensity they decrease down to their minimum values $\beta = 1$ corresponding to the exponential distribution, and the reverse dependence of moments differs from the direct one, thereby resulting in the loop formation. Such a behavior of moments can be explained on the basis of the empirical dependence of the root–mean–square deviation of the intensity fluctuations on the propagation conditions along the path.[6] Presence of a hump in this dependence testifies that for two different turbulence conditions one value of the second normalized moment is possible; therefore, an additional parameter is required to distinguish between these conditions. As for the mixture of distributions, this is the "mixture parameter". However, if the statistical data spread for the highest moments becomes comparable with the loop width, it becomes impossible to identify the branch they belong and hence an analysis of histograms of instantaneous values of the intensity is also required here.

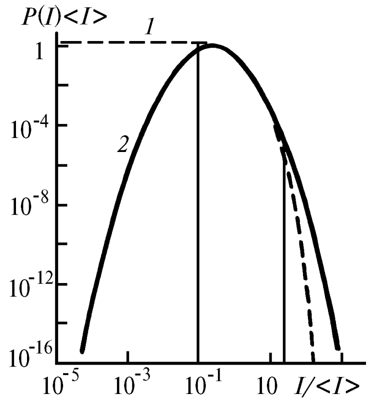

FIG. 1. A comparison of the K–distribution (curve 1) with the lognormal distribution (curve 2) for $\beta = 1.3$.

When analyzing the probability density by the histogram method, it is of great importance that the data would be measured by the instrument whose dynamic range is sufficiently wide for such measurements. This statement is illustrated by Fig. 1 which shows the probability densities of the K– and lognormal distributions for $\langle I\rangle = 1$. As a rule, in all the experiments the data were obtained for $0.1 \le I/\langle I\rangle \le 25$. It is obvious that both experimental normalized moments and histograms of the intensity will agree to within the statistical errors which have been neglected until recently in the experiment.[20] Note that the first five moments of the intensity fluctuations for distributions (1) and (3) practically coincide in the above–indicated range of variation of the normalized intensity (difference is much less than the statistical measurement error). Hence not only from the theoretical but also from practical viewpoint some distributions are ambiguously determined from the infinite number of their moments.[50] Really, as an experience shows, in the experiment in the real atmosphere within the time over which it is considered to be stationary (as a rule, no more than 20–30 min), the above–mentioned range of variation of the normalized intensity is realized for the sample length of about 1–2 millions provided that the sampling frequency does not exceed 1 kHz. For this reason it seems impossible to estimate the probability density in some interval when this probability density is less than 10^{-5}.

The problem on the accuracy of the estimates of the probability density for continuous realization of a process over the period of observation T was considered in Ref. 51 in which the relations for bias were obtained

$$b \approx (W^2/24)\, P''(x), \qquad (12)$$

where W is the length of clustering interval and $P''(x)$ is the second derivative of the probability density $P(x)$ with respect to the argument x, and for the variance of the estimate of the probability density

$$D[x] \approx (P(x)/N\,W), \qquad (13)$$

where N is the length of the sample of independent counts of random variable. Further, as follows from a theorem on discrete representation of a process with time,[51] the realization of a random process with the frequency band B defined on the interval T can be completely described by $N = 2BT$ discrete counts. These N discrete counts will not be necessarily statistically independent. Nevertheless, for any given stationary random ergodic process each of its realization contains $n = N/c^2$ independent sampling counts, where c is the constant depending on the form of the covariance function of a process and on the length of the sampling interval. Hence it follows from Eq. (13) that

$$D[x] \approx c^2 P(x)/2BTW. \quad (14)$$

Total mean–square error in estimating the probability density is a sum of the variance given by Eq. (14) and square bias given by Eq. (12), hence the normalized mean–square error is equal to

$$\varepsilon^2 \approx c^2/2BTWP(x) + (W^4/576)\left[P''(x)/P(x)\right]^2. \quad (15)$$

Taking the square root of Eq. (15) we obtain the normalized root–mean–square error ε. As follows from relation (15), in measuring the probability density the corridor width W must meet contradictory requirements. On the one hand, to decrease the random error it is desirable to take large values of W. On the other hand, to decrease the bias it is necessary to narrow the corridor W. But the problem is the limited time over which the atmosphere can be considered stationary in the course of measurements. In the best case T is no more than a few tens of minutes; therefore, the probability density $P(x)$ for vary narrow corridor can be estimated only with significant error.

4. ANALYSIS OF EXPERIMENTAL DATA

Virtually all the experimental data obtained up to 1976 for the strong fluctuations were well approximated by the lognormal distribution. The most reliable results were obtained in Ref. 52 in which much care was taken to control the meteorological parameters (homogeneity of the turbulent characteristics along a path, $\beta_0 \approx 5$) and the radiation parameters; moreover, the receiving aperture had adequate spatial and temporal resolution. An analysis of data by the method of histograms and moments with allowance for the limited range of counts of the random process obtained during the measurement period unambiguously indicated an advantage of the lognormal model over the exponential model for histograms in the range of signal spikes above an average level. From the viewpoint of the results obtained in the above section, the recording instrument[52] had insufficiently wide dynamic range that gives no way to obtain a reliable estimate in the range of deep fadings of signals.[10,53]

Proximity of the experimental data to the lognormal distribution can be also seen in the case of reflecting a spherical wave from a specular plane[10] for weak and strong intensity fluctuations when $\beta_0 \le 2.5$.

For strong intensity fluctuations of optical waves near their focus ($\beta_0 \simeq 2$) the K–distribution was first proposed in Ref. 20 in which the experimental relative intensity moments were shown to deviate from model (1). They approach the moments of the K–distribution given by Eq. (3). For large values $\beta_0 \simeq 4$ these conclusions were confirmed in the analogous experiments.[38] However, in Ref. 10 for the same values of the parameter $\beta_0 \simeq 2$ with allowance for the dynamic range of the magnitudes of the intensity realized in the experiment according to the considerations given in the preceding section it was pointed out from the analysis of the data that the conclusions made in Refs. 20 and 38 were inadequately warranted, and large difference between the experimental histogram and the model K–distribution for deep intensity fadings $I \ll \langle I \rangle$ was indicated. In addition, the probability of fadings and histogram mode position indicated closer proximity to the lognormal distribution in comparison with the K–distribution for $\beta_0 \simeq 2$.

The intensity fluctuations of a plane wave studied recently for much greater values $\beta_0 \simeq 5$ in comparison with Refs. 10 and 52 showed that under these conditions the K–distribution got closer to the experimental data than the lognormal distribution in the range of deep fadings. As to the rest of the magnitudes of the intensity, these data become so much close to each other that they can hardly be distinguished.[53]

This conclusion for strong intensity fluctuations ($\beta_0 \simeq 10$–12) is in qualitative agreement with conclusions of Ref. 39 which presented the estimates of the corrections for the K–distribution for saturated intensity fluctuations. A comparison of histogram of the saturated intensity fluctuations for a plane wave with model (5a) is shown in Fig. 2. The values of the probability density $P(I)$ are closest to the histogram in the range $0.1 < I/\langle I \rangle < 2.5$. For deep fadings $0.01 \le I/\langle I \rangle \le 0.025$ the values of $P(I)$ exceed the histogram by 2–2.5 orders of magnitude, for moderate fadings $0.02 \le I/\langle I \rangle \le 0.1$ the difference is about 0.5–1 orders of magnitude, and in the range of spikes $I/\langle I \rangle \ge 10$ the difference is about 3 orders of magnitude. Discontinuities in the distribution density $\Delta P(I)$ hear the ends of the intervals of argument variations in model (5a) are as follows: $\Delta P(I_1) = 9.11$ at $I_1 = 2.66$ and $\Delta P(I_2) = 0.112$ at $I_2 = 7.06$. In other words, here the K–distribution fits the experimental data[53] much more better than model (5a).

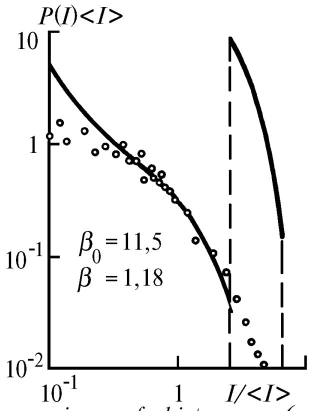

FIG. 2. A comparison of histogram (circles) of the normalized intensity for a plane wave with model (5a) (solid lines).

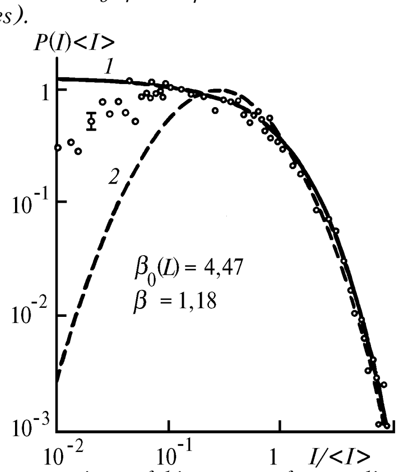

FIG. 3. A comparison of histogram of normalized intensity with the K–distribution (curve 1) and with the lognormal distribution (curve 2) for a spherical wave reflected from a corner–cube reflector.

Proximity of the experimental data to the K–distribution can be seen in the case of reflecting a quasispherical wave from a corner–cube reflector with the diameter d_{ref} = 2.5 cm on a path of length L = 1250 m for the parameter $\beta_0(L)$ = 4.47. This is illustrated by Fig. 3. Vertical bars denote the histogram variance estimated according to Eq. (13). Bias of histogram estimate in the range of deep fadings is insignificant and can be neglected. Still closer proximity is observed in the case of reflecting from an array of 12 corner–cube reflectors[34] arranged compactly (see Fig. 4). In this case the probability density in the range of the saturated intensity fluctuations remains practically unchanged and lies within the limits of statistical spread beginning with $\beta_0(L) > 3$. The last fact also takes place in the turbulent atmosphere at large optical thicknesses $\tau > 3.6$ under precipitation conditions[54] both in the case of direct propagation and reflection from an array of corner–cube reflectors (see Fig. 5).

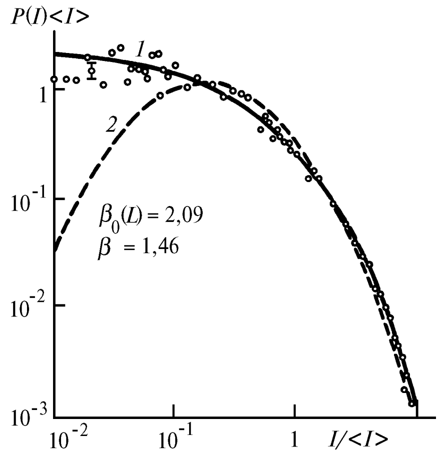

FIG. 4. A comparison of histogram of the normalized intensity with the K–distribution (curve 1) and with the lognormal distribution (curve 2) for a spherical wave reflected from an array of 12 corner–cubes.

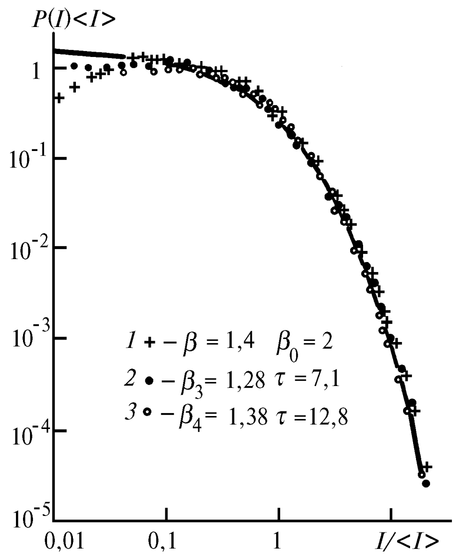

FIG. 5. Histogram of the intensity fluctuations of a narrow collimated beam in the rain (2 and 3) and turbulent atmosphere (1) when the beam is reflected from an array of 12 corner–cubes.

An additional parameter determining the statistical characteristics of a received optical signal is the correlation between the size of receiving aperture and spatial correlation length of the intensity fluctuations. There are a comparatively small number of works in which the effect of this factor was studied experimentally. Previous experimental results were reported in Ref. 6 in which the stability of lognormal distribution of the probability density of the light flux fluctuations in the sense of variations of the diameter of a receiving aperture was pointed out. In other words, with increase of the diameter of the receiving aperture only the variance of fluctuations decreases, while the probability density remains lognormal one. This conclusion holds up to the largest diameters $d \approx 1$ m, for which the distinction between the lognormal and Rayleigh–Rice distributions becomes insignificant from the practical viewpoint.

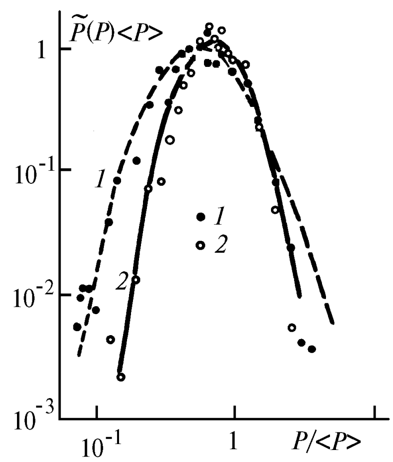

FIG. 6. A comparison of histograms of the instantaneous values of the light flux reflected from a specular discs of different diameters with the lognormal distribution for $\beta_0(L) \sim 3$*: 1)* d_{ref} = 12.5 cm, β = 0.93; *2)* d_{ref} = 2.5 cm, β = 0.67; *and, 3)* d_{ref} = 5.5 cm, β = 0.48.

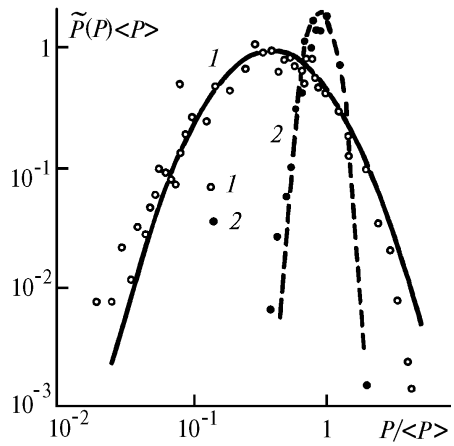

FIG. 7. A comparison of histogram of the instantaneous values of the light flux reflected from a specular disc of the diameter d_{ref} = 12.5 cm *with the lognormal distribution: 1)* $\beta_0(L)$ = 3.35, β = 0.93; *2)* $\beta_0(L)$ = 1.98, β = 0.21.

The same situation is also observed in backward scattering from specular objects (disc, corner–cube, array of corner–cube reflectors) over a wide variety of propagation conditions for different values of the parameter $\beta_0(L)$. Figure 6 shows the probability density of the light flux fluctuations $\tilde{P}(P)$ of a spherical wave reflected from specular discs of different diameters in the case of practically complete interception of a reflected beam by an aperture with a diameter of 500 mm for close values of the parameter β_0. It can be seen that only in the ranges of spikes and fadings the distribution deviates from lognormal one. In this case in the range of spikes it lies above the lognormal distribution. An increase in the turbulence intensity under the same conditions results in variation of the variance of the flux fluctuations, while the functional form of the probability density remains close to the lognormal distribution (see Fig. 7).

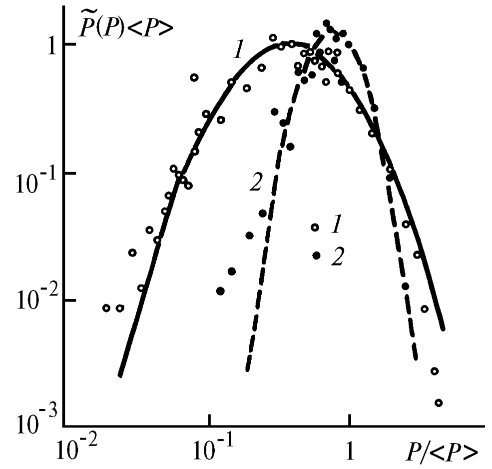

FIG. 8. A comparison of histograms of the normalized values of the light flux reflected from a specular disc (1) and array of 12 corner–cubes (2) with the lognormal distribution for $d_{\text{ref}} = 12.5$ cm *and* $\beta_0(L) = 4$: *1)* $\beta = 0.93$ *and 2)* $\beta = 0.41$.

Figure 8 illustrates the probability density of the light flux fluctuations across the aperture with a diameter of 500 mm in the case of reflection from a mirror with a diameter of 12.5 cm and an array of 12 corner–cube reflectors with approximately the same aperture. Due to the fact that under considered propagation conditions the array of corner–cubes is a self–focusing system,[34] the spatial correlation length of the intensity fluctuations will less than that in the case of reflection from a specular disc of equivalent size resulting in a greater degree of averaging of fluctuations over the aperture, in spite of the fact that the flux fluctuations across the reflector are approximately identical in both cases (difference in the parameter β_0 is small). As can be seen, in all the above–considered cases the light flux fluctuations are close to the lognormal distribution.

Thus the experimental data on the probability density of the intensity fluctuations obtained in recent ten years unambiguously point out the applicability of the so–called *K*–distribution for describing the probability density of the intensity fluctuations in the following cases: 1) for the values of the parameter $\beta_0 \geq 10$ in the case of direct propagation, 2) in the case of propagation through the atmosphere in the rain at large optical thicknesses, and 3) in the case of reflection from an array of corner–cube reflectors. In the case of reflection from artificial specular small objects the probability density obeys the lognormal distribution law. In the range of β_0 close to the focus of the intensity fluctuations the probability density, apparently, must be described fairly well by the combination of the lognormal and *K*–distributions.

In conclusion the authors would like to acknowledge A.P. Rostov, A.I. Petrov, and O.A. Pelymskii for their participation in the experiments.

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