CODIRECTIONAL FOUR-WAVE INTERACTION UNDER CONDITIONS OF STRONG ENERGY TRANSFER BETWEEN THE WAVES

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Received June 1, 1989

The process of codirectional four-wave interaction of hypergaussian and gaussian wave beams under conditions of strong energy transfer between the waves is studied numerically. It is shown that as the collinear interaction length increases owing to the self-action of the beams the mutual energy transfer has an oscillatory character C even for hypergaussian beams with m = 10). The amplitude of the oscillations and their period are determined by the initial profile of the beams. Analogous dependences are realized for the figure of merit of wavefront reversal. The quality of reversal, especially for gaussian beams, is higher if the interaction of the waves occurs noncollinearly. The results obtained based on the fixed-field approximation and taking into account energy transfer between the pump waves are compared.

The nonlinear interaction of light waves has been attracting for many years the attention of a large circle of investigators. This is explained by the large number of applications of multifrequency interaction in different areas of optics, for example, in nonlinear spectroscopy, for frequency conversion as well as for producing light beams with a reversed wavefront.[1,2] In this paper the codirectional interaction of four beams (both coaxial and noncollinear) is studied in application to the problem of wavefront reversal. Unlike other publications on this subject (for example, Refs. 3–5), we studied the case of strong energy transfer between the waves, when the traditional analytical methods cannot be used to describe the processes adequately. For this reason we employed numerical modeling. In addition, the results obtained both based on the fixed-field approximation and taking into account energy transfer between the pump waves were compared.

FORMULATION OF THE PROBLEM. INVARIANTS. COMPUTATIONAL METHODS

The noncollinear, simultaneous interaction of four undiffracted beams propagating in the same direction (z-axis) in a transparent Kerr medium in the case when all induced gratings of the dielectric constant are equivalent is described by the following system of dimensionless equations:

$$\frac{\partial A_j}{\partial z}+\beta_j\frac{\partial A_j}{\partial x}+\nu_j\frac{\partial A_j}{\partial y} = -i\gamma_j\Phi_j;$$

$$\Phi_j = A_j\Big(\sum_{p=1}^{4}|A_p|^2-0.5|A_j|^2\Big)+\frac{\partial}{\partial A_j^*}(A_2A_4A_1^*A_3^*+A_1A_3A_2^*A_4^*),$$

$$j = 1\div 4, \qquad (1)$$

with the initial conditions

$$A_j = A_{0j}\exp\Big[-\Big(\frac{x}{a_{0j}}\Big)^m-\Big(\frac{y}{a_{0j}}\Big)^m+iF_j(x^2+y^2)\Big], \qquad (2)$$

where A_j is the complex amplitude of the j-th beam: A_1 and A_3 are the pump beams; A_2 is the signal wave; A_4 is the reversed wave; z is the longitudinal coordinate normalized to the length of the medium; x and y are dimensionless transverse coordinates, measured, for example, in units of the thickness of the crystal; the coefficients β_j and ν_j describe the drift of j-th beam along the x and y axes, respectively; γ_j are the nonlinear coupling constants; A_{0j} are the amplitudes of the beams; a_{0j} is the dimensionless starting radius of the beam; F_j is the focusing of the beam; and, the parameter $m = 2\div 10$ characterizes the amplitude distribution of the optical radiation.

The system of equations (1) has certain invariants, which we employed to monitor the correctness of the numerical results. Thus multiplying the system (1) by A_j^* and the system of equations conjugate to the system (1) by A_j and then summing and integrating over the transverse coordinates we obtain the following relation:

$$\frac{\partial}{\partial z}\sum_{j=1}^{4}P_j-\sum_{j=1}^{4}\Big\{\beta_j\int_0^{L_y}[|A_j(z,0,y)|^2-|A_j(z,L_x,y)|^2]dy+\nu_j\int_0^{L_x}[|A_j(z,x,0)|^2-|A_j(z,x,L_y)|^2]dx\Big\} = -\sum_{j=1}^{4}\beta_jI_{xj}-\sum_{j=1}^{4}\nu_jI_{yj},\quad P=\int_0^{L_y}\int_0^{L_x}|A_j(z,x,y)|^2dxdy, \qquad (3)$$

where L_x and L_y are the boundaries of the region under study along x and y, respectively, on which the complex amplitudes vanish:

$$A_j(z,0,y) = A_j(z,L_x,y) = A_j(z,x,0) = A_j(z,x,L_y) = 0. \qquad (4)$$

Thus the total power of the light beams will be conserved In the Interaction process:

$$\sum_{j=1}^{4} P_j = \text{const}. \qquad (5)$$

Other invariants can also be written for the system (1):

$$\frac{\partial}{\partial z}(P_{1,3}+P_{2,4})+\beta_{1,3}I_{x1,3}+\beta_{2,4}I_{x2,4}+ +\nu_{1,3}I_{y1,3}+\nu_{2,4}I_{y2,4} = 0; \qquad (6)$$

$$\frac{\partial}{\partial z}(P_{1,2}-P_{3,4})+\beta_{1,2}I_{x1,2}-\beta_{3,4}I_{x3,4}+ +\nu_{1,2}I_{y1,2}-\nu_{3,4}I_{y3,4} = 0. \qquad (7)$$

Therefore if the conditions (4) are satisfied, the sums and differences of the powers are conserved along the z-axis.

We shall make the following remarks regarding Eqs. (3), (6), and (7). First, in the case of collinear interaction ($\beta_j = \nu_j = 0$) the corresponding invariants are given in Ref. 3. Second, Eqs. (6) and (7) indicate mutual transfer of energy. For this reason if for $z = 0$ the intensities of the pump waves are different, then their difference will be conserved along the axis of propagation and complete transfer of energy into the signal and reversed wave is impossible. An analogous result holds for the signal and reversed beams: the power of the signal wave will always be higher initially (at $z = 0$) than the power of the reversed wave.

A symmetric second-order nonlinear difference scheme was written for the system of equations (1) on a characteristic grid;[6,7] the steps h_x and h_y along the x and y axes were matched to the step h_z along the z-coordinate The conditions of matching were as follows:

$$h_{xj} = m_j K h_z = m_j h_x; \quad h_{yj} = l_j S h_z = l_j h_y, \qquad (8)$$

where the numbers K and S were chosen from the conditions

$$\beta_j/m_j = K; \quad \nu_j/l_j = S, \qquad (9)$$

and m_j and l_j are integers. The method of simple iteration was used to solve the difference problem. The iteration was terminated when the relative accuracy $\varepsilon = 10^{-4}$ was achieved. Tests showed that lower accuracy does not guarantee conservation of the invariants.

In performing the calculations we were interested in the following characteristics of the interacting beams: the positions of their centers of gravity x_0, y_0 and the radii of the beams a_x, a_y, determined in terms of the first and second moments of the intensity distribution, respectively; the powers of the beams P_j and the quality of reversal, defined by the familiar overlap integral,

$$\chi = \frac{\left|\int_0^{L_y}\int_0^{L_x} A_2 A_4 dxdy\right|^2}{P_2 P_4}. \qquad (10)$$

RESULTS OF NUMERICAL EXPERIMENTS

Figures 1–3 show the characteristic dependeces of the power of the reversed wave and the quality of the reversal of the initially defocused signal beam ($F_2 = -(1\div150)$, $F_{j\neq2} = 0$, $j = 1\div4$), obtained in the numerical experiments, on the propagation path of optical radiation with the parameters $a_{0j} = 0.4$ and $A_{01} = A_{03} = 1$ for hypergaussian beams with $m = 10$ or $A_{01} = A_{03} = 1.41$ for a gaussian beam with $m = 2$; $A_{02} = 0.1\,A_{01}$, $A_{04} = 0$, and $P_4 = P_1 = P_3 = 0.5$ or 0.25. For convenience, the propagation path is indicated on the horizontal axis in nonlinear lengths also (Fig. 1, bottom row of numbers). We note that since the invariants presented above are valid the powers of the other waves are not shown in these figures and can be easily determined through $P_{rev} = P_4$.

We shall make two remarks. In the calculations for the case of coaxial propagation of the waves the width of the region along x and y was chosen to be 1.6. For this reason, in the case of a beam with an initial gaussian profile the values of the radii $a_{0j} = 0.4$ correspond to the situation when the transverse cross section of the nonlinear medium is entirely filled with radiation: the intensity of the beam at the boundary of the medium is equal to approximately 0.015. Further, it follows from the calculations that both the quality of reversal and the power of the reversed wave are practically (less than 1%) independent of the starting focusing beam, chosen from the interval of values presented above, so that in what follows we shall set $F_2 = -1$. In addition, if the radius of the signal beam is also not indicated, then its value is assumed to be equal to 0.4. We shall now analyze the figures.

Analysis of the curves in Fig. 1, which correspond to the case $\beta_j = \nu_j = 0$, $A_{01,3} = 1.5$, and $\gamma = 40$, leads to the following conclusions. The process of generation of the reversed wave is nonmonotonic even in the case when the amplutude distribution of the beam is close to uniform: after the maximum energy transfer into the signal and reversed wave is achieved energy is transfered back into the pump beam. In the case of gaussian beams with the same starting power (we recall that for them $A_{01,3} = 1.41$) reverse trasfer starting aerlier, the period of the oscillations is shorter, and the average power around which the oscillations occur is higher than for hypergaussian beams This circumstance is connected with fact that for initially gaussian beams, owing to their self-action, the breakdown of the optimal ratio of the phases of different rays (x, y) occurs in different sections z. As a result of this, generation of a reversed wave occurs in some sections of the beam and the reverse process occurs in other sections. The more rapid initional growth in the power of the reversed wave with $m = 2$ is caused by the higher values of the peak intensity of the pump values compared with the peak intensity of the hypergaussian waves. When the starting peak intensities of the gaussian and hypergaussian pump beans are equal, i.e., for $A_{01,3} = 1$ (in this case the beam powers are not equal), energy transfer occurs more rapidly in the case of radiation with a hypergaussian initial profile.

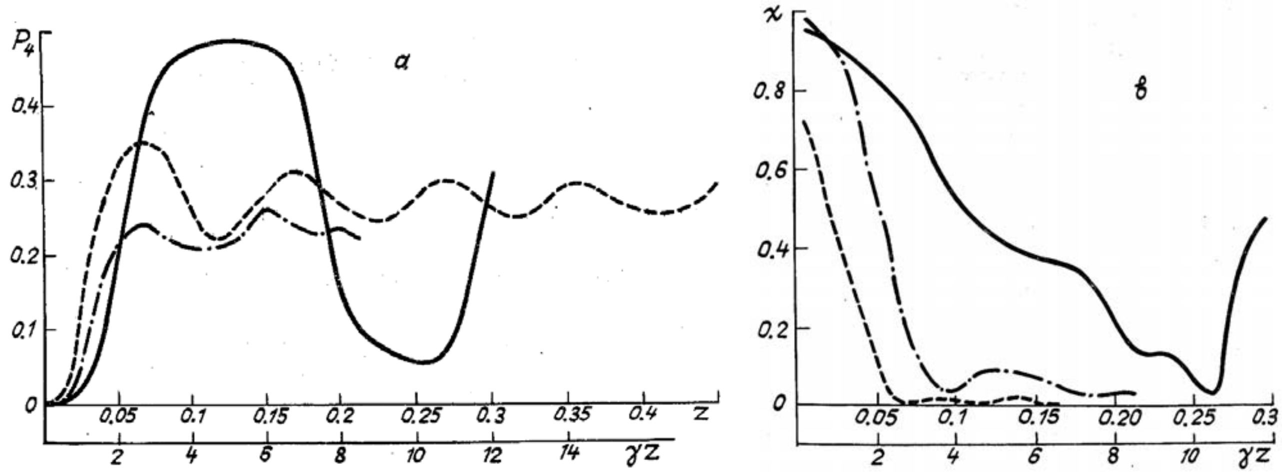

FIG. 1. The evolution of the power (a) and the quality (b) of the reversed wave in a nonlinear medium in the case of coaxial propagation of all waves. The solid curves correspond to the parameters $m_j = 10$ and the dashed and dot-dashed curves correspond to $m_j = 2$. For the dot-dashed curve $a_{02} = a_s = 0.15$.

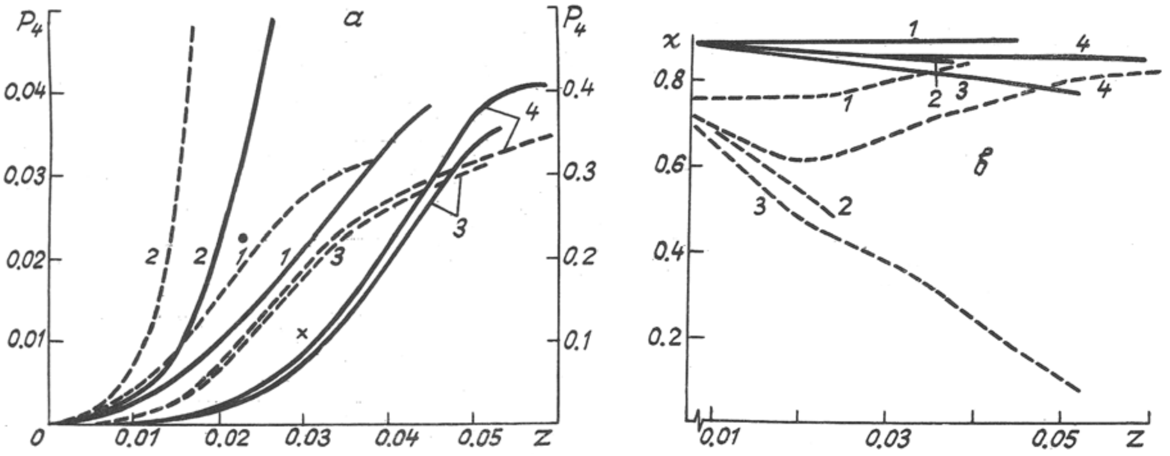

FIG. 2. Evolution of the power (a) and quality of reversal of the wavefront (b) for a gaussian signal beam in the case of coaxial propagation of the waves. The solid curves are hupergaussian pump beams and the dashed curves are for gaussian pump beams. Curves 1 and 2 correspond to the fixed-field approximation; for curves 3 and 4 the depletion of the pumps is taken into account. Curves 2 and 4 correspond to no self-action of the waves; curves 1 and 3 correspond to the presence of self-action. The cross mark refers to solid curve 2 and the dot refers to dashed curve 2. The ordinate axis on the left corresponds to curves 1 and 2 and the ordinate axis on the right corresponds to 3 and 4.

The dependence of the quality of wavefront reversal on the propagation path for both Gaussian and hypergaussian beams is strongly nonmonotonic. If, however, for beams with $m = 10$ the oscillations of χ on paths corresponding to the first minimum P_{rev} are superposed on the monotonic decrease of χ with increasing z, then for gaussian beams with $a_{02} = 0.4$ χ at first decreases monotonically as z increases, reaches its first minimum, and then oscillates as the propagation path increases. The period of the oscillations of χ approximately corresponds to the period of the power oscillations and their amplitude increases as the number of the oscillations increases (up to 5% in the case under study). In contradistinction to gaussian beams, for hypergaussian beams a second maximum of χ approximately equal to 50% of the first maximum, is observed.

It is important to emphasize that if the radii of the signal beam and the pump beams are equal, then generation of the reversed wave starts with a distorted wavefront: in the section $z = 0.0075$ $\chi \sim 72\%$, $P_{\text{rev}} < 10^{-3}$, which is connected both with the self-action of the beams and, apparently, the effect of the amplitude profile of the beams on the quality of the recorded grating (see below). For this reason, even in the case of pump beams with a hypergaussian intensity profile the quality of reversal does not reach 100%: it is of the order of 90%. If, however, the radius of the signal wave is decreased (by not less than a factor of 2.5) to less than the radii of the gaussian pump beams (for example, to 0.15), then the quality of the reveresal improves substantially (up to 97%, see Fig. 1b), but in this case energy transfer from the pump waves into the reversed wave is degraded (by a factor of 1.5 in the first maximum, see Fig. 1a). Therefore there exists an optimal ratio of the radii of the pump and signal beams. We note that a decrease in the radius of the signal beam by 0.05 in the interval from 0.4 to 0.15 is accompanied each time by an increase in χ near the front boundary of the nonlinear medium by approximately 5%. The lowest value of χ achieved at the end of the medium also increases at the same time.

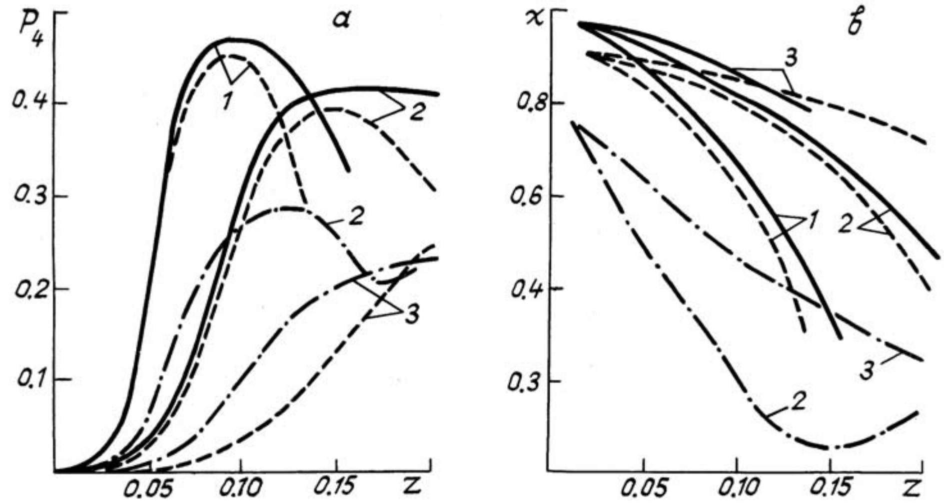

FIG. 3. The power of the reversed wave (a) and the quality of reversal (b) versus the path for noncollinear propagation of the beams with $\gamma = 40$ (1), 20 (2), and 10 (3). The solid curves correspond to hypergaussian beams ($m = 10$), the dashed curves correspond to hypergaussian pump beams and a gaussian signal beam, and the dot-dashed curves correspond to gaussian pump and signal beams.

We shall study the effect of the self-action of the beams on the efficiency and quality of reversal and we shall compare the parameters of the generation of the reversed wave determined in the approximation of fixed pumping beams (widely employed in the literature) and taking into account energy transfer between the waves. The results of the numerical experiments are presented in Figs. 2a and b. Comparating the dependences of the power of the reversed wave on the propagation path shows that when the self-action of the waves is neglected the fixed-field approximation substantially overestimates the value of the power P_4, especially in the case of gaussian pump beams. For generation under the conditions of self-action of the waves, however, the opposite situation is realized: the power of the fourth wave is significantly (by a factor of eight for pump waves a gaussian profile of the amplitude) lower in the scheme without self-action. We note that for gaussian beams the saturation of the dependence of P_4 on z occurs in approximately the same manner (see dashed curves 1 and 3 in Fig. 2a). It is also important to emphasize that in the scheme without self-action as z increases the power of the reversed wave increases monotonically and the process of reverse pumping is not observed right up to the last computed section $z = 12l_{\text{nl}}$. Thus taking into account the self-action of the waves fundamentally changes the energetics of the process of generation of the reversed wave and for this reason it is necessary to do so. The fixed-field approximation strongly distorts the values of the absolute powers of the interacting waves and without taking into account the self-action of the waves it gives an incorrect energetic z-dependence of P_4.

We shall analyse the dependence of the quality of wavefront reversal on the propagation path (see Fig. 2a) in the situation studied above. Comparing curves 1 and 3, which correspond to the interaction of waves with self-action, shows that the fixed-field approximation describes well the process of generation of the reversed wave for hypergaussian pump beams (solid curves) on the entire path presented in the figure although it gives too low a value for χ), while for gaussian pump beams good agreement is observed only for $z \leq 0.8$. When the length of the medium is further increased the quality of the reversal starts to increase, while according to the approximate theory it decreases. Therefore the fixed-field approximation for the situation studied here, as a rule, gives the wrong dependences of the quality of reversal on the length of the medium, especially for spatially modulated pump beams.

We shall now analyze the effect of nocol1inearity of beam propagation on the efficiency and quality of reversal. Some results of calculations performed for $\nu_j = 0$, $\beta_{1,2} = 1$, $\beta_{3,4} = -1$, $\gamma = 40, 20, 10$, $P_{1,2} = 0.54$ (the remaining parameters are the same as in the cases studied above) are presented in Figs. 3a and b. From a comparison of the dependences presented in Fig. 3a we can draw the following conclusions. In the case of the interaction of waves with $m = 10$ (the maximum computed path is equal to $2l_{nl}$), for both all four hypergeometric beams ($m = 10$) and hypergeometric pump beams and a gaussian beam for the signal wave the same power is achieved for $z \geq 0.1$, and it is approximately 1.5 times greater (in the section $z = 0.2$) than the corresponding value reached in the case of collinear interaction of hypergaussian beams. In the case of the interaction of waves with an initially gaussian intensity profile the noncollinearity of the propagation does not degrade the converted power compared with the case of coaxial propagation of the waves for $z \leq 0.1$. It is significant that for $\gamma = 10$ the curve of power dependence for gaussian beams lies above the curve realized in the case of the interaction of beams with hypergaussin amplitude profiles of the pumps in the same interval as in Fig. 1a. For other nonlinear coupling constants γ_j the power converted into the reversed wave is 1.15–1.35 times lower than the case of coaxial propagation.The power pumped into the fourth wave also decreases in the case of interaction of nongaussian beams. Therefore for large values of the nonlunear coupling constant the reduction of the optimal ratio of the phases owing to the noncollinearity of the interaction starts to predominate.

For some paths the quality of reversal is higher than for the case of coaxial propagation of the waves. Thus for $\gamma_j = 40$ and $m_j = 10$ χ in Fig. 3b in the section $z = 0.1$ is 10% greater than the value of χ in Fig. 1b, while for $\gamma_j = 10$ and $m_j = 2$ it is 2.5 times greater. For other paths (like for beams with other profiles) with $z \geq 0.15$, however, the opposite situation can also be realized. Thus there exists an optimal ratio of the parameters γ_j, β_j, and z, which depends on the beam profiles and which leads to a significant (for example, by a factor of 2.5) increase of the quality of reversal without a reduction of the converted power.

Thus, together with the introduction of nonlinear absorption, by choosing the optimal ratio of the signal and pump radii[5] it is possible to suppress effectively the self-action of gaussian beams by introducing some detuning from coaxial propagation of the pump waves.

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