ON RANDOM ERRORS OF MEASURING THE WIND VELOCITY WITH A CW COHERENT LIDAR

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Accuracy of assessment of the radial component of the wind velocity vector measured with a cw coherent lidar is analyzed in the paper as depending on the turbulence of atmospheric air flow as well as on the size of effective scattering volume. It is shown that in the case of large signal–to–noise ratio the increase of the scattering volume leads to decrease of the measurement error to a level determined by the turbulence intensity and by time of averaging the spectrum of photocurrent power.

The advent of coherent Doppler lidars provides a considerable scope for study of atmospheric dynamics. At present there are a few types of such systems. In particular, for measuring wind velocity fields in the boundary layer of atmosphere a ground–based cw coherent CO_2 lidar was created and repeatedly tested in field conditions.[1–6] The registered lidar returns make it possible to estimate the radial component of the wind velocity vector. The use of conical scan of a laser beam makes it possible to determine the three components of wind velocity vector by fitting these estimations to sine function of azimuth angle by means of least–square procedure.[2] Representativity of such measurements is mainly determined by random deviations of the estimations of radial wind velocity component. The random deviations of wind velocity estimations are determined mainly by two factors, namely, dynamic turbulence of the atmosphere and own noise of lidar receiving system.[3] The last factor can be neglected for a high signal–to–noise ratio.

In this paper the effect of turbulent fluctuations of wind velocity as well as a length of scattering volume on the error of measurement of radial component of wind velocity by a cw coherent lidar is analyzed.

Let the source of a cw laser radiation with the wavelength $\lambda = 10.6$ μm be in the plane $z' = 0$. Laser beam propagating along the z' axis is focused in the plane $z' = R$. Radiation is backscattered by aerosol particles drifting with the atmospheric air flow. The backscattered radiation is collected with the telescope and detected by a coherent technique. The current originated at the circuit of photodetector has a component j_s which contains an information on velocity of scattering particles. The component j_s is proportional to the complex amplitude of the backscattered wave field U_s at the plane of photodetector

$$j_s = BU_s , \quad (1)$$

where B is the proportionality coefficient.

By analogy with Ref. 7 we represent the scattered wave field at the moment $t + t'$ as

$$U_s(t + t') = \sum_{j=1}^{n} q_j \exp [2\, ik(z_j + V_r(z_j , t)\, t')] , \quad (2)$$

where the summation is made over the wave fields scattered by separate particles, n is the number of particles in the effective scattering volume, the length Δz of which (along the laser beam axis) is determined by parameters of the transceiving system of the lidar; $q_j = q(z_j)$ is the amplitude of the jth scattered wave; z_j is the projection of the coordinate of jth particle on the z' axis at the moment t; V_r is the projection of the velocity vector on the z' axis (the radial component of wind velocity); and, $k = 2\pi/\lambda$ is the wave number. Let us assume that in calculating the field U_s we can neglect the fluctuations of the number of scattering particles, an inhomogeneity of optical properties of particles, and variations of refractive index along the propagation path. Then two factors are left only which will determine the random variations of the field U_s, namely, the random location of scattering particles and turbulence of the air flow. Due to turbulence the velocity of flow V_r is a random variable of coordinates and time. Nevertheless, the velocity of each of the particles moving with the flow can be considered as an invariant during the duration of its stay in sensing volume. But the velocities of particles differ from one another in magnitude randomly. As a result, the distance between particles will change with time by a random way. This leads to fluctuations of amplitude and phase of the field U_s.

To analyze the statistical properties of scattered light, it is necessary to know the statistical moments of the field of different orders:

$$M_{mn} = E\, [U_s^{\,m}\, U_s^{*n}] = \langle \overline{U_s^{\,m}\, U_s^{*n}} \rangle , \quad (3)$$

where symbol $E[\ldots]$ denotes the ensemble averaging, bar denotes the averaging over the position of the particles z_j, and angular brackets denote the averaging through the turbulent variations of air flow velocity, $m, n = 0, 1, 2, \ldots$.

Let the initial positions of scattering particles z_j be of uniform probability distribution, and V_r be a stationary and statistically homogeneous random field with Gaussian distribution of probability density.

The technique of obtaining different–order moments of field U_s is given in detail in Ref. 9.

COHERENCE AND INTENSITY CORRELATION OF SCATTERED RADIATION

Among the major temporal parameters describing the statistics of the field of the scattered wave is the coherence function

$$\Gamma(\tau) = \langle \overline{U_s\,(t + \tau)\, U_s^*(t)} \rangle \quad (4)$$

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and intensity correlation function

$$K_p(\tau) = <\overline{|U_s(t+\tau)|^2 \, |U_s(t)|^2}> - <\overline{|U_s|^2}>^2 . \quad (5)$$

Having substituted Eq. (1) into Eq. (4) and averaging we obtain[7]

$$\Gamma(\tau) = <\overline{|U_s|^2}> \exp\,[2\, i\kappa <V_r> \tau - 2\, \kappa^2\, \sigma_r^2\, \tau^2] , \quad (6)$$

where $<\overline{|U_s|^2}> = (n/\Delta z) \int\limits_0^\infty dz'\, q^2(z')$ is the mean intensity of the scattered wave field; $< V_r >$ and σ_r^2 are the mean value and the variance of the radial component of wind velocity, respectively.

When deriving the formula for $K_p(\tau)$ on the basis of Eqs. (2) and (5) it is necessary to average the functional of the velocity difference $V_r(z') - V_r(z'')$, whose probability distribution, as known from Ref. 8, generally is different from the Gaussian one. Nevertheless, for rough estimates the assumption on Gaussian distribution of this difference can be used and, as a result, for K_p one can obtain the formula

$$K_p(\tau) = <\overline{|U_s|^2}>^2 \int\!\!\int\limits_0^\infty dz'\, dz''\, Q_s(z')\, Q_s(z'') \times$$

$$\times \exp\,\{-\, 4\sigma_r^2\, \kappa^2 [1 - \kappa_r(z' - z'')]\, \tau^2\} , \quad (7)$$

where $Q_s(z') = q^2(z') / \int\limits_0^\infty dz'\, q^2(z')$ is the function describing the spatial resolution, $\kappa_r(z' - z'') = <[V_r(z') - <V_r>][V_r(z'') - <V_r>]>/\sigma_r^2$ is the correlation coefficient of the radial component of wind velocity.

In line with the Refs. 5 and 6 the function $Q_s(z')$ can be represented as

$$Q_s(z') = \{\pi\kappa a_0^2 [(1 - z'/R)^2 + z'^2/(ka_0^2)^2]\}^{-1} , \quad (8)$$

where a_0 is the initial radius of a laser beam in the plane $z' = 0$. The expression for effective length of the sensing volume Δz which is defined by the formula $\Delta z = \int\limits_0^\infty dz'\, Q_s(z')/Q_s(R)$ under condition $ka_0^2 \gg R$ is reduced to[5,9]

$$\Delta z = \frac{\lambda}{2} \frac{R^2}{a_0^2} . \quad (9)$$

From whence it follows that the length of sensing volume depends on the focusing length of a beam R.

Having determined the integral time of coherence τ_c and the integral temporal scale of intensity (power) correlation τ_p of scattered wave field as

$$\tau_c = \int\limits_0^\infty d\tau\, |\Gamma(\tau)|^2 \,/ <\overline{|U_s^2|}>^2 \quad (10)$$

and

$$\tau_p = \int\limits_0^\infty d\tau\, K_p(\tau) / <\overline{|U_s^2|}>^2 , \quad (11)$$

from Eqs. (6), (10) and (7), (11) we obtain respectively

$$\tau_c = \frac{1}{2\sqrt{\pi}} \frac{1}{2\sigma_r} \quad (12)$$

and

$$\tau_p = \frac{1}{2\sqrt{\pi}} \frac{1}{2\sigma_r} \int\!\!\int\limits_0^\infty \frac{dz'\, dz''\, Q_s(z')\, Q_s(z'')}{\sqrt{1 - k_r(z' - z'')}} . \quad (13)$$

When comparing Eqs. (12) and (13) it is clear that $\tau_p > \tau_c$. This is a consequence of the non–Gaussian statistics of the wave field scattered by aerosol particles suspended in a turbulent flow ($|\Gamma(\tau)|^2 \neq K_p(\tau)$ at $\tau \neq 0$).[7] When the size of the scattering volume Δz is much larger than the outer scale of turbulence L_V ($\Delta z \gg L_V$), in Eq. (13) we can take $k_r \approx 0$. As a result, we have: $\tau_p \approx \tau_c$, that is, in this case the process under study is of Gaussian statistics.

As a rule, in practice the opposite condition $\Delta z \ll L_V$ is realized. Then, setting in Eq. (13) $1 - k_r(z' - z'') = 0.92\varepsilon_T^{2/3}\, |z' - z''|^{2/3}/\sigma_r^2$ (Ref. 10), where ε_T is the dissipation rate of turbulent energy and using Eq. (8) under condition $R \ll ka_0^2$, we obtain

$$\tau_p = \tau_c\; 1.4\; \sigma_r\, (\varepsilon_T\, \Delta z)^{-1/3} . \quad (14)$$

At $\sigma_r = 1$ m/s and $\lambda = 10.6$ μm the coherence time τ_c equals 1.5 μs. If $\varepsilon_T = 10^{-2}$ m^2/s^3 and $\Delta z = 10$ m, then, according to Eq. (14), τ_p is three times greater than τ_c.

The estimates of radial component of flow velocity V_D and Doppler frequency shift f_D are related by the simple equation

$$V_D = \frac{\lambda}{2} f_D . \quad (15)$$

The Doppler frequency shift can be determined from the power spectrum of photocurrent $W(t, f)$.

POWER SPECTRUM OF PHOTOCURRENT

Let us use the finite Fourier transform and represent, taking into account Eq. (1), the measured (averaged) power spectrum of photocurrent as

$$W(t, f) = \frac{|B|^2}{m\, t_s} \sum_{j=1}^{m} \Big| \int\limits_{-t_s/2}^{t_s/2} dt'\, U_s(t_j + t')\, e^{-2\pi i f t'} \Big|^2 , \quad (16)$$

where m is the number of isolated spectra (unsmoothed estimates) measured during the integral time t_0, t_s is the time for measuring a single spectrum (respectively, $t_0 = mt_s$), and $t_j = t + [j - (m+1)/2]t_s$. The time t_s for measuring a single spectrum will be considered as far exceeding the time of power correlation of the receiving signal τ_p ($t_s \gg \tau_p$).

Due to setting the Gaussian distribution of probability density for radial velocity, the expression for ensemble averaged spectrum from Eqs. (2) and (16) takes a form

$$E[W(f)] = \overline{P}\,\frac{\lambda}{2}\,\frac{1}{\sqrt{2\pi}\,\sigma_r}\exp\left\{-\frac{\left(\frac{\lambda}{2}f - \langle V_r\rangle\right)^2}{2\,\sigma_r^2}\right\}, \qquad (17)$$

where $\overline{P} = |B|^2 \langle \overline{|U_s|^2} \rangle$ is the mean photocurrent power.

When sounding a wind velocity in the atmospheric boundary layer by a ground–based Doppler lidar, as usual, the measurement time t_0 is within the range of ~1–50 ms (Ref. 2), that is, far shorter than the correlation time of wind velocity fluctuations $\tau_V \sim 5$–20 s.[10] That is why when deriving the formulas for spectrum moments of higher orders from Eq. (16), the velocity V_r can be considered as time–independent (the approximation of "frozen turbulence").

Taking into account the inequalities $t_s \gg \tau_p$ and $\tau_V \gg t_0$ we can obtain from the Eqs. (2), (16), and (17) the next formula for the square of relative error of the spectrum estimate $\varepsilon_w^2 = E[W^2]/(E[W])^2 - 1$ at the frequency $f = (2/\lambda)\langle V_r \rangle$,

$$\varepsilon_w^2 = (A - 1) + \frac{1}{m}A\,, \qquad (18)$$

where

$$A = \int\!\!\int_0^\infty dz'\,dz''\,Q_s(z')\,Q_s(z'')\,/\,\sqrt{1 - k_r^2(z' - z'')}\,.$$

When the length of scattering volume is large ($\Delta z \gg L_V$) we can assume, that the ratio $\Delta z/L_V \to \infty$ holds and take $k_r \approx 0$ in Eq. (18). As a result, we obtain the formula $\varepsilon_w \approx 1/\sqrt{m}$ describing the error of measurement of spectrum of Gaussian random process.[11] In this case, the increase of the number of degrees of freedom $n_d = 2m$ enables one to perform the averaging of measurement error of $W(f)$ to the wanted magnitude. In a general way the increase of n_d can reduce the error ε_w only down to the level determined by the ratio $\Delta z/L_V$. Thus, with increasing the ratio $\Delta z/L_V$ and the number of degrees of freedom n_d the error in spectrum estimate decreases and, hence, the accuracy of determining the mean velocity $\langle V_r \rangle$ increases.

ESTIMATE OF RADIAL COMPONENT OF WIND VELOCITY

Of all the known methods of determining the Doppler frequency shift from measured spectrum $W(t, f)$ we use the formula for the first–order random moment of frequency

$$f_D(t) = \frac{1}{P(t)}\int_{-\infty}^{+\infty} df\,f\,W(t, f)\,, \qquad (19)$$

where

$$P(t) = \int_{-\infty}^{+\infty} df\,W(t, f) \qquad (20)$$

is a measured power of photocurrent. Having substituted Eq. (16) into Eq. (20) we can obtain

$$P(t) = |B|^2\,t_0^{-1}\int_{-t_0/2}^{t_0/2} dt'\,|U_s(t + t')|^2\,. \qquad (21)$$

Taking into account the condition $t_0 \gg \tau_p$ for the relative variance of measured photocurrent power $\sigma_P^2 = E[P^2]/(E[P])^2 - 1$ from Eqs. (21), (7), and (11) we have

$$\sigma_P^2 = 2\tau_p\,/\,t_0\,. \qquad (22)$$

For $\tau_p = 2.5$ μs and $t_0 = 50$ ms the value σ_P equals 0.01. Thus, in Eq. (19) within negligible error we can put

$$P \approx E[P] \equiv \overline{P}\,. \qquad (23)$$

From Eqs. (2), (15), (16), and (19), in view of Eq. (23), we have unbiased estimate of velocity

$$E[V_D] = \langle V_r \rangle\,. \qquad (24)$$

For the variance of estimate of the mean radial component of wind velocity $\sigma_D^2 = E[V_D^2] - \langle V_r \rangle^2$ from Eqs. (2), (15), (16), and (19) after rearrangement, taking into account the conditions $\tau_V \gg t_0$ and $t_s \gg \tau_p$, we obtain

$$\sigma_D^2 = \sigma_r^2\int\!\!\int_0^\infty dz'\,dz''\,Q_s(z')\,Q_s(z'')\,k_r(z' - z'') + $$
$$+\frac{\lambda\,\sigma_r}{8\sqrt{\pi}\,t_0}\int\!\!\int_0^\infty dz'\,dz''\,Q_s(z')\,Q_s(z'')\left[\frac{1 + k_r(z' - z'')}{\sqrt{1 - k_r(z' - z'')}}\right]. \qquad (25)$$

The first term in the right–hand part of Eq. (25) represents the variance of radial component of wind velocity averaged along laser beam axis

$$\overline{V}_D = \int_0^\infty dz'\,Q_s(z')\,V_r(z')\,. \qquad (26)$$

The second term in Eq. (25) σ_a^2 owes its origin to imperfect averaging the photocurrent power fluctuations due to finiteness of the spectrum measurement time t_0. This term can be considered as a measure of statistical uncertainty (rms error) in measuring the parameter $\overline{V}_D$

$$\sigma_a^2 = E[(V_D - \overline{V}_D)^2]\,. \qquad (27)$$

In the limiting case $\Delta z/L_V \to \infty$, setting in Eq. (25) $k_r \approx 0$, we obtain[12]

$$\sigma_a^2 = \lambda\,\sigma_r\,/\,(8\sqrt{\pi}\,t_0)\,. \qquad (28)$$

To estimate the rms error σ_a in the case $\Delta z \ll L_V$, we use the formula

$$\sigma_a = \sigma_r\sqrt{0.2\,\lambda(\varepsilon_T\,\Delta z)^{-1/3}\,t_0^{-1}}\,, \qquad (29)$$

which is obtained as a result of the use in second term of Eq. (25) the approximations $1 + \kappa_r(z' - z'') \approx 2$ and $1 - k_r(z' - z'') \approx 0.92\,\varepsilon_T^{2/3}\,|z' - z''|^{2/3}/\sigma_r^2$ and integrating it with respect to variables z' and z''. For $\lambda = 10.6$ μm, $\sigma_r = 1$ m/s, $\varepsilon_T = 10^{-2}$ m^2/s^3, $t_0 = 50$ ms, and $\Delta z = 10$ m, according to Eq. (29), $\sigma_a \approx 0.01$ m/s. This value is less than the resolution of the measured velocity $\sim \lambda/(2t_s)$ ($t_s = 50$ μs) by one order of magnitude and it can be neglected. Thus, we can put $V_D \approx \overline{V}_D$.

Having neglected in Eq. (25) by the second term and using the condition $ka_0^2 \gg R$ we obtain for σ_D^2

$$\sigma_D^2 = \sigma_r^2 \frac{1}{\Delta z} \int\limits_0^\infty \frac{dz' \; \kappa_r(z')}{1 + (\pi / 2)^2 (z'/\Delta z)^2} . \quad (30)$$

It follows from Eq. (30) that with increasing Δz the variance σ_D^2 decreases monotonously. If we specify the outer scale of turbulence by the formula $L_V = \int\limits_0^\infty dz' \kappa_r(z')$ then under condition $\Delta z \gg L_V$ the denominator of the integrand in Eq. (30) can be taken as unity. As a result, we have

$$\sigma_D^2 = \sigma_r^2 \, L_V / \Delta z . \quad (31)$$

The square of the spectrum width can be determined as the central second–order random moment of frequency

$$\Delta f^2(t) = \frac{1}{P(t)} \int\limits_{-\infty}^{+\infty} df \, [f - f_D(t)]^2 \; W(t, f) . \quad (32)$$

This parameter being ensemble–averaged and scaled in accordance with Eq. (15), and expressed in terms of velocity as $\sigma_s^2 = (\lambda/2)^2 E\,[\Delta f^2]$ is related to σ_r^2 and σ_D^2 by the simple equation

$$\sigma_s^2 = \sigma_r^2 - \sigma_D^2 , \quad (33)$$

where σ_D^2 is described by the formula (30).

In the case of small scattering volume ($\Delta z \ll L_V$), we can put in Eq. (30) $\kappa_r(z') = 1 - 0.92\, \varepsilon_T^{2/3} \mid z' \mid^{2/3} / \sigma_r^2$ and perform the integration with respect to variable z'. As a result, for effective spectrum width we have

$$\sigma_s = 1.16\, (\varepsilon_T \, \Delta z)^{1/3} . \quad (34)$$

For $\varepsilon_T = 10^{-2}$ m^2/s^3 and $\Delta z = 10$ m it follows from Eq. (34) that $\sigma_s = 0.54$ m/s. Since with increasing Δz the parameter σ_D^2 tends to zero, as it is evident from Eq. (31), and σ_s tends to σ_r.

Consider a rms deviation of the measured velocity V_D from the real instantaneous velocity V_r at the moment t at the point $z' = R$: $\Delta V_D = \sqrt{<(V_D - V_r(R))^2>}$. Having used the formula (26) for V_D and taking into account the condition $ka_0^2 \gg R$ it can be shown that ΔV_D coincides with σ_s. Thus, if Doppler frequency shift is estimated by Eq. (19), the mean square of the effective spectrum width is defined as the variance of errors of measuring the instantaneous velocity $V_r(R)$. It follows from formulas (30) and (33) that the accuracy of determining the mean velocity $< V_r >$ increases with increasing the length of scattering volume. In contrast to this, the accuracy of measurement of instantaneous velocity at a point $V_r(R)$ decreases as the sensing volume length increases.

CONCLUSION

In this paper the analysis of accuracy of measuring a wind velocity by a cw Doppler lidar is performed for the case of large signal–to–noise ratio (SNR). It is shown that the estimate of the velocity by the use of Eqs. (19) and (15) may be considered as the radial component of wind velocity averaged over sensing volume (located at the point $z' = R$ along the laser beam axis) to a high accuracy. Therefore, random deviations of the velocity estimates V_D from the mean radial component of wind velocity $< V_r >$ are determined by the effect of large–scale turbulent vortexes of sizes $l > \Delta z$. At the same time, the error of measuring the instantaneous velocity at a point $V_r(R)$ is determined by turbulent variations of air flow caused by small–scale vortexes $l < \Delta z$. Generally, under arbitrary signal–to–noise ratios, the error of estimate of wind velocity will depend, in addition, on the SNR, integral time of measurement t_0, and the way of processing the signal received.

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