ON MEASUREMENT OF THE DISSIPATION RATE OF THE TURBULENT ENERGY WITH A CW DOPPLER LIDAR

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Various methods of measurement of the dissipation rate of the turbulent energy by means of a cw Doppler lidar are analyzed in the paper. It is shown that information on the dissipation rate can be derived from the structure function or the spectrum of wind velocity measured with Doppler lidar having arbitrary length of a sounded volume.

INTRODUCTION

Along with measurements of mean wind fields, Doppler lidars are used for estimation of the turbulence parameters.[1–4] In particular, attempts to use the Doppler lidars for measurement of the dissipation rate of the turbulent kinetic energy and the wind field structure constant from measurements of the width of the Doppler signal power spectrum were made in Refs. 2–4. However, the acceptable accuracy of the data can be achieved only for small (of the order of inertia range of turbulence) lengths of lidar sounded volume. When the length of the sounded volume becomes comparable with the outer scale of turbulence, this method fails and hence imposes restrictions on the height of measuring with ground–based cw Doppler lidars. In the paper, a feasibility of estimations of the dissipation rate from Doppler lidar data for arbitrary length of lidar sounded volume is investigated theoretically.

COHERENT DETECTION OF SCATTERED RADIATION

Let a cw Doppler lidar be placed in the plane $z = 0$. A laser beam having Gaussian profile and radius a_0 at the exit from a transmitting–receiving telescope propagates along z the axis of the Cartesian coordinates $\mathbf{r} = \{z, x, y\}$. It is focused at a distance R from the lidar. Radiation scattered by aerosol particles is collected by the telescope and together with a reference beam of the same wavelength λ as sounding one is incident on a photodetector. The valid component j_s of the current in a photodetector circuit is described by the expression in the complex form[5–7]

$$j_s(t) = B \sum_{m=1}^{n} a_m E^2(\mathbf{r}_m) \exp\left\{ 2ik\left[z_m + \int_0^t \mathrm{d}t'\, V_z(\mathbf{r}_m, t') \right] \right\}, \quad (1)$$

where $B = 2\, e\, \eta\, (h\nu)^{-1}\, \lambda \sqrt{P_L/P_T}$, e is the electron charge, η is the detector quantum yield, $h\nu$ is the photon energy, P_L is the power of reference beam and P_T is the power of sounding beam, n is the number of aerosol particles in the sounded volume of the atmosphere, a_m is the amplitude of scattering by the mth particle located at a point $\mathbf{r}_m$ at $t = 0$,

$$E(\mathbf{r}) = \frac{\sqrt{P_T/\pi}}{a_0\, g(z)} \exp\left\{ -\left(1 + i\,\frac{k\, a_0^2}{R}\right) \frac{x^2 + y^2}{2\, a_0^2\, g(z)} \right\}$$

is the complex field amplitude of sounding beam, $g(z) = (1 - z/R) + i\, z/(k\, a_0^2)$, and $k = 2\,\pi/\lambda$. At arbitrary moment the mth particle is at the point

$$\tilde{\mathbf{r}}(\mathbf{r}_m, t) = \mathbf{r}_m + \int_0^t \mathrm{d}t'\, \mathbf{V}(\mathbf{r}_m, t'),$$

where $\mathbf{V} = \{V_z, V_x, V_y\}$ is the vector of particle velocity (Lagrange wind velocity). The Lagrange velocity and the velocity $\mathbf{U}(\mathbf{r}, t) = \{U_z, U_x, U_y\}$ (Euler wind velocity) at a fixed point are related by the following equation:

$$\mathbf{V}(\mathbf{r}_m, t) = \mathbf{U}(\tilde{\mathbf{r}}(\mathbf{r}_m, t), t).$$

DOPPLER SPECTRUM ESTIMATE

A received signal $j_s(t)$ is fed to a spectrum analyzer where it passes through a linear bandpass filter with narrow bandwidth Δf. Then a squared modulus or modulus of the signal passed through such filter $J_s(t, f)$ at its central frequency f is averaged over integration period t_0 to obtain the estimates of spectral power density $W(t, f)$ or mean modulus of filtered–out signal $A(t, f)$. Let us represent $J_s(t, f)$ in the form

$$J_s(t, f) = \Delta f \int_{t-1/\Delta f}^{t} \mathrm{d}t'\, j_s(t') \exp\{-2\pi\, i\, f\, t'\}\,. \quad (2)$$

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Then for estimates $W(t, f)$ and $A(t, f)$ we can write the following expressions:

$$W(t, f) = \frac{1}{\Delta f}\frac{1}{t_0} \int\limits_{t-t_0}^{t} \mathrm{d}t' \; |J_s(t', f)|^2 , \tag{3}$$

$$A(t, f) = \frac{1}{t_0} \int\limits_{t-t_0}^{t} \mathrm{d}t' \; |J_s(t', f)| . \tag{4}$$

It is known that when $\Delta f\, \tau_P \ll 1$ and $t_0\, \Delta f \gg 1$, where τ_p is the correlation time of the received signal power, the estimate of the mean modulus of the signal $A(t, f)$ is proportional to the square root of the estimate of the signal power spectrum. It is easy to verify when taking into account that distribution of the probability density of the quantity $J_s(t, f)$ is Gaussian with zero mean $\overline{J_s} = 0$ and identical variances of its independent real and imaginary parts $\sigma^2_{J_s} = t_0^{-1} \int\limits_{t-t_0}^{t} \mathrm{d}t' \; \overline{|J_s(t', f)|^2} \equiv 0.5\, \Delta f \; \overline{W(t, f)}$. Here the bar denotes averaging over an ensemble of realizations a_m and $\mathbf{r}_m$, and the nonstationarity of $\sigma^2_{J_s}$ is also considered (its dependence on time t) caused by slow, compared with the fluctuations of the received signal power, turbulent variations of wind velocity (correlation time of wind velocity $\tau_V \gg \tau_P$). In the limiting case of extremely large length of sounded volume Δz, when $\Delta z \gg L_V$, where L_V is the outer scale of turbulence, or of averaging period $t_0 \gg \tau_V$, the variance $\sigma^2_{J_s}$ under conditions of stationary and homogeneous turbulence can be represented as $\sigma^2_{J_s} = 0.5 \langle \overline{|J_s(f)|^2} \rangle$, where angular brackets denote averaging over an ensemble of wind velocity realizations. It can be shown that relative errors of estimates under consideration $\varepsilon_W = [\overline{(W - \overline{W})^2}]^{1/2}/\overline{W}$ and $\varepsilon_A = [\overline{(A - \overline{A})^2}]^{1/2}/\overline{A}$ are determined by the relations: $\varepsilon_W = 1/\sqrt{\Delta f\, t_0}$ and $\varepsilon_A \sim 1/\sqrt{\Delta f\, t_0}$, where $\overline{A} = (\sqrt{\pi}/2)\, [\Delta f \overline{W}]^{1/2}$. Thus, for sufficiently large number of degrees of freedom ($\Delta f\, t_0 \gg 1$) we have $W \approx \overline{W}$, $A \approx (\sqrt{\pi}/2)\, [\Delta f\, \overline{W}]^{1/2}$. If a device measures directly the quantity $A(f)$, its power spectrum $W(f)$ can be obtained by taking the square of the measured values of A.

The inequality $1/\Delta f \ll \tau_\eta = (\nu_k/\varepsilon_2)^{1/2} \sim 0.1$s, where τ_η is the characteristic temporal microscale of Lagrangian wind velocity,[8] ν_k is the kinematic viscosity of air, and ε_2 is the dissipation rate of the turbulent energy, allows us to use the approximation

$$\int\limits_0^{t+\tau} \mathrm{d}t' \; V_z(\mathbf{r}_m, t') \approx \int\limits_0^{t} \mathrm{d}t' \; V_z(\mathbf{r}_m, t') + \tau \; V_z(\mathbf{r}_m, t) \tag{5}$$

in Eq. (1) for $|\tau| \le 1/\Delta f$.

Having substituted Eq. (1) into Eq. (2) and Eq. (2) into Eq. (3), performing averaging in Eq. (3) over a_m and $\mathbf{r}_m$, and taking into account Eq.(5) for the estimate of the signal power spectrum, we have

$$W(t, f) = \\ = B^2 \frac{1}{t_0} \int\limits_0^{t_0} \mathrm{d}t' \int\limits_0^{\infty} \mathrm{d}z \int\!\!\int\limits_{-\infty}^{\infty} \mathrm{d}x\, \mathrm{d}y\, \beta_\pi(\mathbf{r}, t - t')\, |E^2(\mathbf{r})|^2 \times \\ \times \frac{1}{\Delta f} \left[\frac{\sin\left(\frac{\pi}{\Delta f}\left[f - \frac{2}{\lambda} V_z(\mathbf{r}, t - t')\right]\right)}{\frac{\pi}{\Delta f}\left[f - \frac{2}{\lambda} V_z(\mathbf{r}, t - t')\right]} \right]^2 , \tag{6}$$

where $\beta_\pi = \sigma_\pi\, \rho$ is the backscattering coefficient, $\sigma_\pi = \overline{|a_m|^2}$ is the backscattering cross section, ρ is the particle number density (the number of particles in a unit volume).

ESTIMATES OF VELOCITY MOMENTS FROM THE POWER SPECTRUM

If we make the substitution $f = (2/\lambda) V$ in Eq. (6), where V is the velocity, the normalized power spectrum $W(t, (2/\lambda)\, V) / \int\limits_{-\infty}^{\infty} \mathrm{d}V \; W(t, (2/\lambda)\, V)$ can be considered as a probability density of velocity distribution of aerosol particles entering the sounded volume during period $[t - t_0, t]$. A frequency f_m at which the spectrum takes its maximum can be approximately considered as corresponding the most probable velocity of particles entering the center of sounded volume. The spectrum width Δf_s determined, for example, at $1/2\; W(f_m)$, characterizes the degree of statistical uncertainty in the estimate of the velocity of individual particle (instantaneous velocity at a fixed point of sounded volume).

Along with the power spectrum of valid signal $W(f)$, the measured spectrum comprises the spectral component $W_0(f)$ of near–zero frequency and broadband noise spectrum $W_N(f)$ throughout frequency range $f \in [0, f_N]$, where f_N is the highest frequency of a filter set. Eliminating $W_0(f)$ and $W_N(f)$ from the measured spectrum, we can determine arbitrary moments of the velocity from the power spectrum of valid signal only in the selected frequency range $[f_1, f_2]$ such that $f_1 < f_m < f_2$ and $(f_2 - f_1) \gg \Delta f_s$.

By virtue of the condition $\Delta f \ll \Delta f_s$, turning from discrete spectral distribution $W(t, l\, \Delta f)$

($l = 0,1,2, \ldots, N_s$, where N_s is the number of frequency channels, and $f_N = \Delta f\, N_s$) to continuous distribution $W(t, f)$, we can write down the expressions for the first moment of the velocity $V_D(t)$ and the second central velocity moment $V_s^2(t)$ in the form

$$V_D(t) = S^{-1}(t)\,(\lambda/2)\int\limits_{f_1}^{f_2} \mathrm{d}f\, f\, W(t, f), \tag{7}$$

$$V_s^2(t) = S^{-1}(t)\,(\lambda/2)^2 \int\limits_{f_1}^{f_2} \mathrm{d}f\,[f - (2/\lambda)\,V_D(t)]^2\, W(t, f), \tag{8}$$

where

$$S(t) = \int\limits_{f_1}^{f_2} \mathrm{d}f\, W(t, f) \tag{9}$$

is the signal power. If $(f_2 - f_1)\,\Delta f \ll (\Delta f_s)^2$ and $\Delta f_s \ll f_2 - f_1$, we substitute Eq. (6) into Eqs.(7)–(9) and take the limits $\Delta f \to 0$, $f_2 \to +\infty$, and $f_1 \to -\infty$. As a result, after integrating over variable f we have

$$V_D(t) = \int\limits_0^{t_0} \mathrm{d}t' \int\limits_0^{\infty} \mathrm{d}z \int\limits_{-\infty}^{\infty}\!\!\int \mathrm{d}x\, \mathrm{d}y\; V_z(\mathbf{r}, t - t')\; G(\mathbf{r}, t - t'), \tag{10}$$

$$V_s^2(t) = \int\limits_0^{t_0} \mathrm{d}t' \int\limits_0^{\infty} \mathrm{d}z \int\limits_{-\infty}^{\infty}\!\!\int \mathrm{d}x\, \mathrm{d}y\; V_z^2(\mathbf{r}, t - t')\; G(\mathbf{r}, t - t') - V_D^2(t), \tag{11}$$

$$S(t) = \int\limits_0^{t_0} \mathrm{d}t' \int\limits_0^{\infty} \mathrm{d}z \int\limits_{-\infty}^{\infty}\!\!\int \mathrm{d}x\, \mathrm{d}y\; F(\mathbf{r}, t - t'), \tag{12}$$

where $F(\mathbf{r}, t) = B^2\, t_0^{-1}\, \beta_\pi(\mathbf{r}, t)\, |E^2(\mathbf{r})|^2$, $G(\mathbf{r}, t - t') = F(\mathbf{r}, t - t')\, S^{-1}(t)$.

The noise component of measured photocurrent power spectrum $W_N(f)$ in the frequency range $f \in [f_1, f_2]$ can be represented as a sum of the mean

$$\overline{W}_N = (f_2 - f_1)^{-1} \int\limits_{f_1}^{f_2} \mathrm{d}f\, W_N(f)$$

and fluctuation component

$$W'_N(f) = W(f) - \overline{W_N}\,.$$

The latter is responsible for spikes in the measured spectrum distribution, whose amplitude may be comparable to the amplitude of valid signal for small signal–to–noise ratio. Thus, generally using Eqs.(7)–(9) to determine the moments of the velocity, we should obtain $V_D(t) + \Delta V_D(t)$ and $V_s^2(t) + \Delta V_s^2(t)$, where $\Delta V_D(t)$ and $\Delta V_s^2(t)$ are the errors in estimates of $V_D(t)$ and $V_s^2(t)$ caused by noise, rather than $V_D(t)$ and $V_s^2(t)$. In what follows we assume that the signal–to–noise ratio is sufficiently large and we can neglect errors ΔV_D and ΔV_s^2.

In practice, we can put β_π = const in Eqs.(10)–(12). Then for stationary and homogeneous turbulence, with integration time t_0 being so large that the condition $t_0 \gg \tau_V$ is fulfilled, the estimates V_D and V_s^2 comprise the mean $\langle V_z \rangle$ and the variance $\sigma_r^2 = \langle V_z^2 \rangle - \langle V_z \rangle^2$ of the radial component of wind velocity. However, for lidar systems, as a rule, $t_0 \lesssim 50$ ms (Refs. 9–11), and correlation time of wind velocity $\tau_V \sim 10$ s far exceeds t_0. That is why we can put $t_0 \to 0$ in Eqs.(10)–(12).

Using Taylor's hypothesis of frozen turbulence

$$V_z(\mathbf{r}, t) = U_z(\mathbf{r} + t <\mathbf{V}>, 0)$$

and taking into account that radii of sounding laser beam are small, which allows us to put

$$V_z = U_z(z + t <V_z>,\; t <V_x>,\; 0,\; 0) \quad (<V_y> = 0)$$

in Eqs.(10)–(12), for $R \ll k\, a_0^2$ we can derive from Eqs. (10)–(12) simpler formulas

$$V_D(t) = \int\limits_0^{\infty} \mathrm{d}z\; Q_s(z)\; V_r(z, t), \tag{13}$$

$$V_s^2(t) = \int\limits_0^{\infty} \mathrm{d}z\; Q_s(z)\; V_r^2(z, t) - V_D^2(t), \tag{14}$$

where $V_r(z, t) = U_z(z + t<U_z>, t<U_x>, 0, 0)$, and $Q_s(z) = \{\pi k\, a_0^2\, [(1 - z/R)^2 + z^2/(k a_0^2)^2]\}^{-1}$ is the function characterizing the spatial resolution of a lidar.[5,6,12] If we define the effective length of sounded volume as

$$\Delta z = \int\limits_0^{\infty} \mathrm{d}z\; Q_s(z)/Q_s(R),$$

for $R \ll k\, a_0^2$ it takes the form

$$\Delta z = (\lambda/2)\,(R^2/a_0^2). \tag{15}$$

DETERMINATION OF THE TURBULENT ENERGY DISSIPATION RATE FROM THE SECOND CENTRAL MOMENT OF THE VELOCITY

Assuming homogeneity and isotropy of the field of wind velocity fluctuations, after ensemble averaging of the second central moment of the velocity (squared width of the signal power spectrum), from Eq. (14) for $\sigma_s^2 = \langle V_s^2 \rangle$ we have

$$\sigma_s^2 = \sigma_r^2 - \sigma_D^2, \tag{16}$$

where

$$\sigma_D^2 = <[V_D - <V_D>]^2> =$$

$$= \int\int_0^\infty \mathrm{d}z_1\,\mathrm{d}z_2\,Q_s(z_1)\,Q_s(z_2)\,B_r\,(z_1 - z_2) \qquad (17)$$

is the variance of the velocity measured with the Doppler lidar, $B_r\,(z_1 - z_2) = <V'_r(z_1)\,V'_r(z_2)>$ is the correlation function of the radial wind velocity component, and $V'_r = V_r - <V_r>$ $(<V_r> \equiv <V_z> = <V_D>)$.

Let us define the spatial scale of velocity correlation (outer scale of turbulence) L_V by the integral

$$L_V = \int_0^\infty \mathrm{d}z'\,B_r(z')/\sigma_r^2\,. \qquad (18)$$

Then representing the correlation function B_r in the form

$$B_r(z') = \int_{-\infty}^{+\infty} \mathrm{d}\kappa\,S_r(\kappa)\,\mathrm{e}^{i\kappa z'}, \qquad (19)$$

where $S_r(\kappa)$ is the one–dimensional longitudinal spectrum of radial wind velocity fluctuations, from Eqs.(18) and (19) we can obtain

$$L_V = \pi\,S_r(0)/\sigma_r^2\,. \qquad (20)$$

In accordance with Eqs.(17) and (19), the variance σ_D^2 can be represented as

$$\sigma_D^2 = \int_{-\infty}^{+\infty} \mathrm{d}\kappa\,S_r(\kappa)\,H(\kappa), \qquad (21)$$

where $H(\kappa) = \left|\int_0^\infty \mathrm{d}z\,Q_s(z)\,\mathrm{e}^{i\kappa z}\right|^2$ is the transfer function of low–frequency spatial filter determined by the distribution of laser beam intensity along the propagation axis. With $k\,a_0^2 \gg R$, for $H(\kappa)$ we have

$$H(\kappa) = \exp\{-\,(2/\pi)\,\Delta z\,\,|\kappa|\}. \qquad (22)$$

From Eqs.(16), (19), (21), and (22) we obtain the following expression :

$$\sigma_s^2 = \int_{-\infty}^{+\infty} \mathrm{d}\kappa\,S_r(\kappa)\,[1 - \exp\{-\,(2/\pi)\,\Delta z\,\,|\kappa|\}]. \qquad (23)$$

For $\Delta z \ll L_V$ for $S_r(\kappa)$ we can use the Kolmogorov–Obukhov spectrum model[8]

$$S_r(\kappa) = \{C\,\varepsilon_2^{2/3}/[3\,\Gamma\,(1/3)]\}\,\,|\kappa|^{-5/3}, \qquad (24)$$

where $C \approx 1.83$ is the Kolmogorov constant, and $\Gamma(x)$ is the gamma function. Having substituted Eq. (24) into Eq. (23) and integrated over variable κ we obtain the expression

$$\sigma_s^2 = C\,(2/\pi)^{2/3}\,(\varepsilon_2\,\Delta z)^{2/3}. \qquad (25)$$

Thus, by measuring (for $\Delta z \ll L_V$) the parameter σ_s^2, we can determine the dissipation rate of the turbulent energy ε_2 using formula (25).

In the case of large sounded volume, when $\Delta z \gg L_V$, we can put in Eq. (23)

$$\int_{-\infty}^{+\infty} \mathrm{d}\kappa\,S_r(\kappa) = \sigma_r^2$$

and

$$\int_{-\infty}^{+\infty} \mathrm{d}\kappa\,S_r(\kappa)\exp\{-(2/\pi)\,\Delta z\kappa\,|\} \approx$$

$$\approx S(0)\int_{-\infty}^{+\infty} \mathrm{d}\kappa\,\,\exp\{-(2/\pi)\Delta z\,|\kappa|\}.$$

As a result, in view of Eq. (20), we obtain the formula

$$\sigma_s^2 = \sigma_r^2\,[1 - L_V/\Delta z], \qquad (26)$$

from which it follows that as $\Delta z/L_V \to \infty$, σ_s^2 saturates at a level σ_r^2.

Based on Eq. (14), we define the relative variance of squared width of the signal power spectrum

$$\varepsilon_s^2 = <[(V_s^2 - \sigma_s^2)]^2>/\sigma_s^4.$$

The obtained expression for ε_s^2 contains the fourth moment of the velocity difference $V_r(z_1) - V_r(z_2)$ whose probability distribution, as is known from Ref.13, is non–Gaussian. Nevertheless, for a rough estimate we use the hypothesis by Millionshchikov[8] to represent the fourth moment of the velocity difference as a product of its second moments. As a result, we have

$$\varepsilon_s^2 = \frac{2}{\sigma_s^4}\int\int_{-\infty}^{\infty} \mathrm{d}\kappa_1\,\mathrm{d}\kappa_2\,S_r(\kappa_1)\,S_r(\kappa_2)\,\times$$

$$\times\left[\exp\left\{-\frac{\Delta z}{\pi}\,|\kappa_1 + \kappa_2|\right\} - \exp\left\{-\frac{\Delta z}{\pi}\,(|\kappa_1| + |\kappa_2|)\right\}\right]^2, \qquad (27)$$

where σ_s^2 is described by formula (23).

As calculations show, for $\Delta z \ll L_V$, when we can use formula (24) for $S_r(\kappa)$ in Eq.(27), $\varepsilon_s \approx 0.5$. In the other limiting case $\Delta z \gg L_V$, we can neglect the second exponent in Eq.(27) and put $\sigma_s^4 \approx \sigma_r^4$. As a result, we obtain the approximate formula

$$\varepsilon_s = \mu\,\sqrt{L_V/\Delta z}\,, \qquad (28)$$

where the coefficient

$$\mu = \left[2 \int\limits_{-\infty}^{+\infty} \mathrm{d}\xi \; S_r^2(\pi\xi / L_V) / S_r^2(0) \right]^{1/2}$$

for the von Karman model of the spectrum $S_r(\kappa)$ (Ref. 14) equals 0.93. Thus relative fluctuations of estimate of the parameter $V_s{}^2$ from a single power spectrum of the signal are maximum when $\Delta z \ll L_V$ and decrease monotonically with increasing length of sounded volume Δz.

DETERMINATION OF THE DISSIPATION RATE OF THE TURBULENT ENERGY FROM THE STRUCTURE FUNCTION AND SPECTRUM OF THE FIRST MOMENT OF DOPPLER VELOCITY

The rate of turbulent energy dissipation ε_2 can be determined from measurements of the mean squared width of the signal power spectrum only when the length of sounded volume is small compared with the outer scale of turbulence: $\Delta z \ll L_V$. Despite increasing outer scale of turbulence L_V with height h (in particular, in the surface layer the linear growth of L_V is observed, and above this layer its rate of growth is slowed down[15]) the length of sounded volume Δz is proportional to R^2, as follows from Eq. (15), and consequently may increase with height faster than L_V even for directions of beam propagation close to vertical. Therefore, for ground–based lidar measurement the condition $\Delta z \ll L_V$ is violated beginning with a certain height, and consequently the Doppler spectrum width is no longer informative of the quantity ε_2.

Let us analyze the feasibility of determining the turbulent energy dissipation rate from the structure function

$$D(\tau) = \langle [V'_D(t + \tau) - V'_D(t)]^2 \rangle \tag{29}$$

and spectrum

$$S_D(\omega) = \frac{1}{2\pi} \int\limits_{-\infty}^{+\infty} \mathrm{d}\tau \; \langle V'_D(t + \tau) \; V'_D(t) \rangle \; \mathrm{e}^{-i\omega\tau} \tag{30}$$

of wind velocity measured by a Doppler lidar, where $V'_D = V_D - \langle V_D \rangle$.

Having substituted Eq. (13) into Eq. (29) and averaged we obtain, taking into account Eq. (22), the formula

$$D(\tau) = \int\int\limits_{-\infty}^{+\infty}\int \mathrm{d}\kappa_z \; \mathrm{d}\kappa_x \; \mathrm{d}\kappa_y \; F_z(\boldsymbol{\kappa}) \; \exp\left\{ -\frac{2}{\pi} \Delta z \; |\kappa_z| \right\} \times$$
$$\times \left[1 - \exp\{ i \; \kappa_z \langle V_z \rangle \tau + i \; \kappa_x \; \langle V_x \rangle \tau \}\right], \tag{31}$$

where $F_z(\boldsymbol{\kappa})$ is the three–dimensional spatial spectrum of fluctuations of radial component of wind velocity, and $\boldsymbol{\kappa} = \{\kappa_z, \kappa_x, \kappa_y\}$. In the inertia subrange of wave numbers when $|\langle \mathbf{V} \rangle \tau| \ll L_V (\langle \mathbf{V} \rangle = \{\langle V_z \rangle, \langle V_x \rangle, 0\})$, we can use in Eq. (31) for $F_z(\boldsymbol{\kappa})$ the expression of the form[8]

$$F_z(\boldsymbol{\kappa}) = \frac{1}{4\pi} \frac{55}{27 \; \Gamma\,(1/3)} \; \varepsilon_2^{2/3} \; |\boldsymbol{\kappa}|^{-11/3} \left(1 - \frac{\kappa_z^2}{|\boldsymbol{\kappa}|^2} \right). \tag{32}$$

After substitution of Eq. (32) into Eq. (31), change of the variables $\kappa_z = \kappa \sin\varphi$ and $\kappa_x = \kappa \cos\varphi$, and integration over κ_y and κ, we have

$$D(\tau) = C_1 \varepsilon_2^{2/3} (\Delta z)^{2/3} \int\limits_0^{\pi} \mathrm{d}\varphi \left[1 - \frac{8}{11} \sin^2 (\varphi - \gamma) \right] \times$$
$$\times \left[\mathrm{Re} \left(|\sin(\varphi - \gamma)| + i \frac{\pi}{2} \sin\varphi \; |\langle \mathbf{V} \rangle \tau| / \Delta z \right)^{2/3} - \right.$$
$$\left. - \; |\sin(\varphi - \gamma)|^{2/3} \right], \tag{33}$$

where $\gamma = \arcsin(\langle V_x \rangle / |\langle \mathbf{V} \rangle|)$ is the angle between the beam axis and the wind direction, $C_1 = (2/\pi)^{2/3} \, 55 \; \Gamma\,(1/3) \; C / [54 \sqrt{\pi} \Gamma\,(11/6)] \approx 1.2 \; C$. Note that formula (33) is inapplicable if the conditions

$$\Delta z / |\langle V_z \rangle| \gg \tau_\eta \qquad \text{and} \qquad |\langle V_x \rangle| < \sigma_x, \tag{34}$$

where $\sigma_x^2 = \langle V_x^2 \rangle - \langle V_x \rangle^2$, both are fulfilled simultaneously.

For $\Delta z \ll |\langle \mathbf{V} \rangle \; \tau| \ll L_V$ in Eq. (33) we can take $\Delta z \to 0$ (regime of point sounded volume), and $D(\tau)$ is described by the well–known expression[8]

$$D(\tau) = C \; [1 + (1/3) \sin^2\gamma] \; \varepsilon_2^{2/3} \; |\langle \mathbf{V} \rangle \tau|^{2/3}. \tag{35}$$

In the other limiting case in which $|\langle \mathbf{V} \rangle \; \tau| \ll \Delta \; z \ll |\tau \; \langle \mathbf{V} \rangle / \sin\gamma|$ (regime of large sounded volume with the beam axis aligned with the wind direction $(\gamma = 0)$), from Eq. (33) we have the asymptotic formula

$$D(\tau) = (2/9) \, (\pi/2)^{4/3} \, C \, \varepsilon_2^{2/3} \, (\Delta \; z)^{-4/3} \; |\langle V_z \rangle \tau|^2. \tag{36}$$

In this case conditions (34) can be fulfilled simultaneously.

For $\Delta \; z \gg |\tau \; \langle \mathbf{V} \rangle / \sin\gamma|$ (regime of large sounded volume with cross wind $(\gamma \neq 0)$), from Eq. (33) we obtain

$$D(\tau) = C_2 \; \varepsilon_2^{2/3} \; |\langle V_x \rangle \tau|^{5/3} \; / \; \Delta \; z \;, \tag{37}$$

where $C_2 = 11 \sqrt{3\pi} \, \Gamma\,(1/3) \; C / [36 \; \Gamma\,(11/6)] \approx 2.67 \; C$. When the condition $\langle V_x \rangle^2 \gg \sigma_x^2$ is additionally fulfilled, formula (37) can be applied for arbitrary Δz including $\Delta z \gg L_V$. Thus, in the case of strong cross wind the dissipation rate of the turbulent energy can be estimated from Doppler lidar data for arbitrary length of the sounded volume Δz. The necessary information on $\langle V_x \rangle$ can be obtained by the velocity–azimuth–display scan technique.[9–12]

Manipulations analogous to that used to derive Eq. (33) yield the formula for spectrum $S_D(\omega)$ which in its high–frequency region $\omega \gg |<\mathbf{V}>|/L_V$ takes the form

$$S_D(\omega) = S_r(\omega)\, H(\omega), \tag{38}$$

where

$$S_r(\omega) = (C/[3\,\Gamma\,(1/3)]) \times$$
$$\times\, [1 + (1/3)\,\sin^2\gamma]\,\varepsilon_2^{2/3}\, |<\mathbf{V}>|^{2/3}\,\omega^{-5/3}$$

is the temporal spectrum of radial wind velocity at a fixed point ($z = R$),

$$H(\omega) = \frac{55}{27}\frac{1}{4\sqrt{\pi}}\frac{\Gamma(1/3)}{\Gamma(11/6)}\left(1 + \frac{1}{3}\sin^2\gamma\right)^{-1} \times$$
$$\times \int\limits_{-\infty}^{+\infty} d\xi\,(1+\xi^2)^{-4/3}\left[1 - \frac{8}{11}\,\frac{(\cos\gamma + \xi\,\sin\gamma)^2}{1 + \xi^2}\right] \times$$
$$\times\, \exp\left\{-\frac{2}{\pi}\,\frac{\Delta\, z\, \omega}{|<\mathbf{V}>|}\,|\cos\gamma + \xi\,\sin\gamma|\right\} \tag{39}$$

is the transfer function of a low–frequency temporal filter. The applicability of Eq. (38) is restricted by conditions (34) as well.

From Eq. (39) it follows that when $\Delta z \to 0$, the function $H(\omega) \to 1$. When $\gamma \to 0$, $H(\omega) \to \exp\{-(2/\pi)\Delta z\omega/|<\mathbf{V}>|\}$. In the case of principal practical interest $\Delta z \gg |<\mathbf{V}>\,\omega^{-1}/\sin\gamma|$ (regime of large sounded volume with cross wind), the integrand in Eq. (39) is equal to $\pi\,|<\mathbf{V}>|\,|\sin\gamma|^{5/3}\,(\Delta z\,\omega)^{-1}$. In this case, the spectrum $S_D(\omega)$ is described by the formula

$$S_D(\omega) = C_3\,\varepsilon_2^{2/3}\,|<V_x>|^{5/3}\,\omega^{-8/3}/\Delta z, \tag{40}$$

where $C_3 = 55\sqrt{\pi}\,C/[324\,\Gamma\,(11/6)] \approx 0.32\,C$. Using Eq. (40), we can determine ε_2 from spectrum $S_D(\omega)$ measured for $\Delta z > L_V$.

CONCLUSION

A feasibility to determine the dissipation rate of the turbulent energy for arbitrary length of sounded volume from Doppler lidar measurements of the structure function or the spectrum of wind velocity has been demonstrated theoretically. It has been found that for large sounded volume and strong cross wind the structure function $D(\tau)$ is proportional to $\tau^{5/3}$, and the spectrum $S_D(\omega)$ is proportional to $\omega^{-8/3}$. Obtained results provide a basis for developing efficient methods of reconstruction of vertical profiles of the rate of dissipation at the turbulent energy throughout the entire boundary layer of the atmosphere with the use of ground–based Doppler lidar.

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