EFFICIENCY OF THE MARKOVIAN FILTERING IN OZONE SOUNDING BY UV LIDARS

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Accuracy of retrieving the fluctuating profiles of concentration and total content of ozone by UV lidars when performing the optimal Markovian filtering of signals are analyzed analytically and with the help of numerical modeling. The maximum altitudes of efficient filtering are determined for sounding at various pairs of wavelengths in the Hartley and Huggins bands.

The optimal estimate of the fluctuating profiles of the gas concentration by the differential absorption lidar method using the apparatus of the Markovian filtering was considered in Ref. 1 in which the simplest analysis of the efficiency of single–channel filtering was performed for four hypothetical lidars with reasonable power as applied to ozone sounding in the Hartley and Huggins bands.

In this paper an analysis of the accuracy of optimal retrieving the fluctuating profiles of the ozone concentration and its total content in the sounded altitude range is perused. By way of numerical modeling the approximate altitude and spectral dependences of the errors in their determination under various sounding conditions are obtained with the use of atmospheric models. The maximum altitudes of efficient filtering are determined for sounding at various pairs of wavelengths in the Hartley and Huggins bands with regard to absorption at both wavelengths when operating in the mode of two–channel reception.

Dispersion equations. The accuracy in estimating the fluctuating profiles of the ozone concentration $N(h) = \overline{N}(h) + \Delta N(h)$ smoothed by the lidar pulse of the duration τ_p is given by the *a posteriori* correlation matrix[1]

$$K = \{K_{ij}\} = \overline{(\eta - \eta^*)(\eta - \eta^*)^T} .$$

Here η^* is the estimate of the state vector $\eta = \{\eta_1, \eta_2\}^T$, where $\eta_1(\tau)=\Delta N(h)/\sigma_N(h)$ is the normalized realization of the fluctuation of the O_3 concentration $\Delta N(h)$, $\eta_2(\tau)$ is the spatial realization proportional to the fluctuations of the current optical thickness due to absorption by ozone, $\overline{N}$ and $\sigma_N^2(h)$ are the *a priori* known profiles of mean value and variance of $N(h)$, $\tau = 2h/c$, and c is the speed of light.

The variance of the estimate $N^*(h)$ of the profile $N(h)$ is equal to

$$D[N^*(h)] = \mu^2(h)\, \overline{N}^{\,2}(h)\, K_{11}(h) , \tag{1}$$

where $\mu(h)$ is the coefficient of variation of $N(h)$. In its turn, $K_{11}(h)$ can be determined by integrating the Rikkati matrix equation of the form

$$\dot{K} = AK + KA^T + b - \left[\hat{\nu}^2_{s1}(\tau)/\hat{\nu}^2_{\Sigma 1}\right] K\boldsymbol{C}\boldsymbol{C}^T K , \tag{2}$$

which was derived in Ref. 1 when sounding was performed at the wavelengths λ_1 and λ_2 and radiation absorption at the wavelength $\lambda_2 > \lambda_1$ was neglected,

$$b = \begin{pmatrix} 2\alpha & 0 \\ 0 & 0 \end{pmatrix} , \ \boldsymbol{C} = \{0, -2\mu(h)\} , \ \alpha = 1/\tau_p .$$

It can be shown that when absorption at the wavelength λ_2 is taken into account the matrix A has the form

$$A = \begin{pmatrix} -\alpha & 0 \\ c\,\overline{\Delta\gamma}\,(h)/2 & 0 \end{pmatrix} ,$$

where $\overline{\Delta\gamma}\,(h) = \overline{\gamma}_1(h) - \overline{\gamma}_2(h)$ and $\overline{\gamma}_1(h)$ and $\overline{\gamma}_2(h)$ are the mean profiles of ozone absorption at the wavelengths λ_1 and λ_2. In practice, the estimates $\hat{\nu}_{s1}$ and $\hat{\nu}_{\Sigma 1} = \hat{\nu}_{s1} + \hat{\nu}_{N1}$ are determined by the variant of maximum likelihood from observations at λ_2. In so doing Eq. (2) depends on the sampling data and can be integrated as they become available.

Averaging Eq. (2) over the Poisson fluctuations of the photoelectrons in the channel at λ_2, we obtain analogous equation for the absolute *a posteriori* matrix $\langle K\rangle$ in the form

$$\langle\dot{K}\rangle = A\langle K\rangle + \langle K\rangle A^T + b -$$
$$- \left[\langle\overline{\nu}_{s1}\rangle^2/\langle\overline{\nu}_{\Sigma 1}\rangle\right]\langle K\rangle \boldsymbol{C}\boldsymbol{C}^T\langle K\rangle , \tag{3}$$

where instead of the estimates of the conditional mean intensities of the signal and total photoelectron fluxes are their absolute mean values $\langle\overline{\nu}_{s1}\rangle$ and $\langle\nu_{\Sigma 1}\rangle$; angular brackets denote averaging over an ensemble of fluctuations in the number of photoelectrons. In contrast to Eq. (2), Eq. (3) can be solved *a priori*. The initial conditions are preset at the point $\tau_0 = 2h_0/c$: $\langle K_{11}(\tau_0)\rangle = 1$ and $\langle K_{12}(\tau_0)\rangle = \langle K_{22}(\tau_0)\rangle = 0$, where h_0 is the minimum altitude of sounding.

If we use the approximation

$$\langle K_{12}(h)\rangle \simeq \overline{\Delta\gamma}\,(h)\, h_L \langle K_{11}(h)\rangle , \tag{4}$$

where

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$$h_L = \begin{cases} h - h_0 \text{ , for } h - h_0 \ll L \text{ ,} \\ L \text{ ,} \quad \text{ for } h - h_0 \gg L, \end{cases}$$

then in system (3) the equation for $\langle K_{11}(h)\rangle$ is no longer dependent on $\langle K_{12}\rangle$ and $\langle K_{22}\rangle$, has the form

$$\frac{\mathrm{d}\langle K_{11}(h)\rangle}{\mathrm{d}h} = -\frac{2}{L}\left\{\langle K_{11}(h)\rangle - 1 + Q(h;\, \lambda_1,\, \lambda_2)\langle K_{11}(h)\rangle^2\right\}, \quad (5)$$

and can be solved independently of the other equations ($L = c\tau_{\mathrm{p}}/2$). Here the quantity

$$Q(h;\, \lambda_1,\, \lambda_2) = \frac{\langle \bar{\nu}_{s1}\rangle^2 \tau_{\mathrm{p}}\, \mu^2(h)}{\langle \bar{\nu}_{\Sigma 1}\rangle}\, \bar{\tau}^{\,2}_{12}(0,\, h_L) \text{ ,} \quad (6)$$

which can be called the generalized signal–to–noise ratio, is the generalization of $Q(h;\, \lambda_1)$ introduced in Ref. 1, in which it was proportional to the optical thickness $\bar{\gamma}_1(h)\, L$ due to absorption at λ_1. In our case, if the absorption at λ_1 and λ_2 is taken into account, Q depends on the differential optical thickness $\bar{\tau}_{12}(0,\, h_L) = \overline{\Delta\gamma}\,(h)\, L$.

Using again Eq. (4), we have for $\langle K_{22}(h)\rangle$

$$\frac{\mathrm{d}\langle K_{22}(h)\rangle}{\mathrm{d}h} = 2\,\overline{\Delta\gamma}\,(h)\bar{\tau}_{12}(0,\, h_L)\langle K_{11}(h)\rangle -$$

$$-\frac{2}{L}\,\frac{Q(h;\, \lambda_1,\, \lambda_2)}{\bar{\tau}^{\,2}_{12}(0,\, h_L)}\,\langle K_{22}(h)^2\rangle \text{ .} \quad (7)$$

Generalized signal–to–noise ratio. Taking into account the complicated dependences of $\langle \bar{\nu}_{s1}\rangle$ and $\langle \bar{\nu}_{\Sigma 1}\rangle$ on h and many factors determining the profiles $\bar{\gamma}_1(h)$ and $\bar{\gamma}_2(h)$, the Markovian filtering efficiency is advisable to analyze by the methods of numerical modeling. The filtering is efficient at the altitudes at which $Q > 1$ (see Ref. 1). Thus modeling of the profiles Q is the most important step in the analysis of the efficiency of sounding. It enables one, first, to select the lidar energy parameters, necessary spatial and temporal resolutions, etc.; second, to determine the altitude range in which the filtering makes sense; and, third, to estimate qualitatively the possibilities of filtering for various pairs of sounding wavelengths.

Formula (6) describes the well–known effect requiring optimization of the selection of pairs of the wavelengths in different altitude ranges, because the absorption by ozone, on the one hand, decreases the signal – in our case $\langle \bar{\nu}_{s1}\rangle$ – and on the other, it increases the differential optical thickness – in our case $\bar{\tau}_{12}(0,\, L)$ – which determines the sensitivity of the differential absorption method.

For ordering the selection of a large body of the lidar and atmospheric parameters and sounding conditions, the model homogeneous complex was considered, whose energy parameters were the same at all wavelengths, while the rest of the parameters were close to the real ones.

TABLE I.

Wavelengths, nm	282, 291.6, 308, 313, and 353
Lasers	XeBr, KrF + SRS D_2^2, XeCl, KrF + SRS H_2^2, and XeCl + SRS H_2
Pulse radiation energy, J	0.1
Total efficiency of optical train	0.012
Photodetector quantum efficiency	0.15
Receiving aperture area, m^2	0.785
Pulse length, m	300, 1000, and 2000

Figure 1 shows the profiles $Q(h;\, \lambda_1,\, \lambda_2)$ for various pairs of the sounding wavelengths and $L = 300$ m (the upper scale of the abscissa axis) and 1000 m (the lower scale). It can be seen that practically all the selected pairs can be used for sounding in the troposphere. Supplemental information about the applicability of these pairs provides the maximum altitudes h_{m} of effective filtering determined below. From Fig. 1 it follows that when sounding the stratospheric ozone, the worst conditions for optimal filtering are realized at a wavelength of 282 nm.

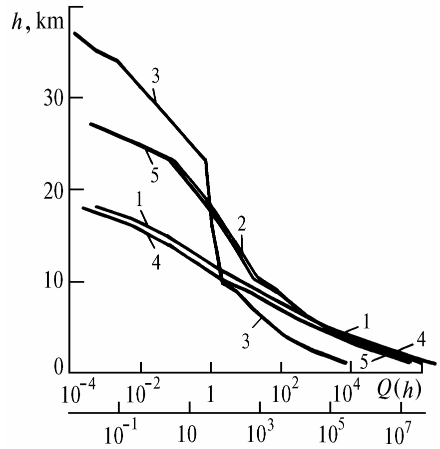

FIG. 1. Profiles of the generalized signal–to–noise ratio for various pairs of the sounding wavelengths λ_1 and λ_2 and for $L = 300$ m *(the upper scale of the abscissa axis) and* 1000 m *(the lower scale). 1)* 282 *and* 353 nm, *2)* 291.6 *and* 353 nm, *3)* 308 *and* 353 nm, *4)* 282 *and* 291.6 nm, *and* 5) 291.6 *and* 308 nm.

Let Q_{a} be the minimum acceptable value of $Q(h;\, \lambda_1,\, \lambda_2)$. Then the maximum altitude h_{m} is determined from the condition

$$Q(h_{\mathrm{m}},\, \lambda_1,\, \lambda_2) = Q_{\mathrm{a}} \text{ .} \quad (8)$$

Setting $Q_{\mathrm{a}} = 1$ in Eq. (8) and applying the exponential interpolation of the profiles of the generalized signal–to–noise ratio $Q(h;\, \lambda_1,\, \lambda_2)$ in the range $[h_{i-1},\, h_i]$, we derive the relation for h_{m} in the form

$$h_{\mathrm{m}} = h_{i-1} + \frac{(h_i - h_{i-1})\, \ln Q(h_{i-1})}{\ln[Q(h_{i-1})/Q(h_i)]} \text{ .} \quad (9)$$

Table II presents the values of h_{m} which allow us to estimate qualitatively the possibilities of ozone sounding in the troposphere and stratosphere. It can be seen that for $L = 300$ m most promising for sounding are the pairs of wavelengths 291.6 and 353, 291.6 and 313, and 291.6 and 308 nm, respectively, allowing us to reach an altitude of ~ 18 km. For $L = 1000$ m sounding is efficient at the pair of wavelengths 308 and 353 nm. By choosing a XeCl laser with a radiation wavelength of 308 nm as a reference one, let us analyze the efficiency figures in retrieving the profiles of the O_3 concentration $N(h)$ and total content

$$J(h) = \int_{h_0}^{h} dh' N(h').$$

TABLE II. Maximum altitudes h_m (m) of the efficient filtering for various pairs of wavelengths λ_1 and λ_2 in ozone sounding.

$(\lambda_1; \lambda_2)$, nm	L, m	
	300	1000
282; 291.6	10890	14843
282; 308	11828	15538
282; 313	11881	15577
282; 353	11923	15607
291.6; 308	17687	23956
291.6; 313	18027	24069
291.6; 353	18272	24151
308; 313	8949	26984
308; 353	16985	29384
313; 353	9190	27622

Analysis of sounding efficiency in the troposphere and stratosphere. Let us turn to retrieving the profiles of ozone concentration and total content in the troposphere and stratosphere. Variances of their estimates are unambiguously related to the components of the matrix $\langle K \rangle$, thus the analysis of the efficiency reduces to determination of altitude behaviour of these elements. Relations between the variances of the atmospheric parameters and $\langle K \rangle$ depend on the chosen method of introduction of the state vector. In particular, for the estimate of the concentration $N^*(h)$ this dependence is given by relation (1).

Since the scale of variation of the profiles $\overline{\Delta\gamma}\,(h)$ and $\sigma_N(h)$ is much larger than L, it can be shown that

$$\eta_2(2h/c) \simeq \left[\, \overline{\Delta\gamma}\,(h)/\sigma_N(h) \right] \Delta J(h) \,, \qquad (10)$$

where

$$\Delta J(h) = \int_{h_0}^{h} dh' \Delta N(h') \,.$$

According to Eq. (10) for the variance $D[\eta_2^*] = \langle K_{22}(h) \rangle$ we have

$$\langle K_{22}(h) \rangle \simeq \frac{\overline{\Delta\gamma}^{\,2}(h)}{\sigma_N^2(h)} D[\Delta J^*(h)] \,. \qquad (11)$$

In analogy with Eq. (1) we finally derive the relation for the variance $D[J^*] = D[\Delta J^*]$ of the optimal estimate J^* of the total O_3 content in the altitude range $[h_0, h]$ in the form

$$D[J^*] \simeq \left[\, (h) / \overline{\Delta\gamma}^{\,2}(h) \right] \langle K_{22}(h) \rangle \,. \qquad (12)$$

Using Eq. (1) for the relative error in optimal retrieving the profile of the ozone concentration $N(h)$, we have

$$(h) = \mu(h) \sqrt{\langle K_{11}(h) \rangle} \,, \qquad (13)$$

where $\langle K_{11}(h) \rangle$ is the solution of Eq. (5). By setting in Eq. (5) $d\langle K_{11}(h) \rangle / dh = 0$ we obtain the quadratic equation for $\langle K_{11}(h) \rangle$ whose solution has the form

$$\langle \tilde{K}_{11}(h) \rangle = \frac{1}{2Q(h; \lambda_1, \lambda_2)} \left\{ \sqrt{1 + 4Q(h; \lambda_1, \lambda_2)} - 1 \right\} . \qquad (14)$$

The behaviour of $\langle \tilde{K}_{11}(h) \rangle$ is well known[2]: fast decrease down to the value $\langle K_{11}(h_{min}) \rangle = \langle \tilde{K}_{11}(h_{min}) \rangle$ (transient regime) and subsequently far slower increase with asymptotic approach to $\langle K_{11}(h) \rangle = 1$. Analytical study of Eq. (5) gives $h_{min} - h_0 \simeq L/[4Q(h_0)]$, i.e., for $Q(h_0) \gg 1$ the altitude range of the transient region is much shorter.

It can be seen from Table III that for the pair 291.6 and 308 nm and $L = 300$ m the standard deviation

$$\delta_K = \left[\langle \tilde{K}_{11}(h) \rangle - \langle K_{11}(h) \rangle \right] / \langle \tilde{K}_{11}(h) \rangle$$

does not exceed 1% in the altitude range 6 km $\le h \le$ 12 km, in which $Q < 10^3$. For $Q \ge 10^3$ δ_K is even smaller for $h < 6$ km except the range $[h_0, h_{min}]$. Thus to determine the accuracy of optimal retrieving $N(h)$ in Eq. (13), $\langle K_{11}(h) \rangle$ can be substituted for $\langle \tilde{K}_{11}(h) \rangle$.

TABLE III. Dependence of the relative error δ_K on $h - h_0$.

$h - h_0$, km	6	7	8	9	10	11	12
δ_K,%	0.71	0.83	–0.25	0.08	0.89	0.42	0.21

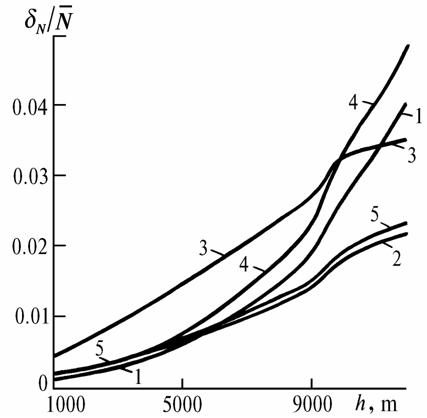

FIG. 2. Profiles of relative errors in optimum retrieving the profiles of ozone concentration $N(h)$ at the same pairs of wavelengths as in Fig. 1.

Figure 2 shows the profiles $\tilde{\delta}_N(h)$ when sounding is performed in the troposphere at various wavelengths λ_1 and λ_2 of the Hartley and Huggins bands. It can be seen that below 12 km sounding is efficient for the pairs of wavelengths 291.6 and 308 nm and 291.6 and 353 nm, because the corresponding values $\tilde{\delta}_N(h)$ are minimum practically at all altitudes. For the pairs of wavelengths 308 and 353 nm (below 10 km) and 282 and 291.6 nm (above 10 km) the values of $\tilde{\delta}_N(h)$ reach their maximum. However, in all the above–considered variants of sounding the error in optimal retrieving the profiles $N(h)$ below 12 km does not exceed 5% for the preset 10% level of fluctuations of $N(h)$.

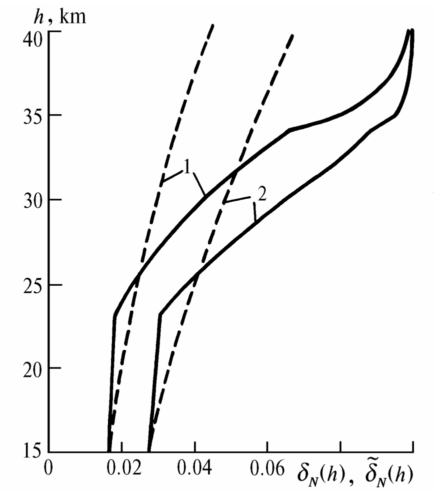

FIG. 3. Profiles of relative errors $\delta_N(h)$ *(solid line) and* $\tilde\delta_N(h)$ *(dashed line) in optimal retrieving the profiles of stratospheric ozone concentration for* $L = 2000$ *(1) and* 1000 m *(2).*

Figure 3 shows the profiles $\tilde\delta_N(h)$ when sounding of the stratospheric ozone is performed at the wavelengths $\lambda_1 = 308$ nm and $\lambda_2 = 353$ nm by a bifrequency lidar. This lidar has a XeCl–laser transmitter generating radiation pulses with an increased energy of 0.4 J and the rest of the parameters indicated in Table I. The analysis of the altitude behaviour of $\tilde\delta_N(h)$ shows that for $L = 1000$ km and the number of sounding events $M = 10^4$ the filtering enables one to retrieve $N(h)$ with $\tilde\delta_N(h) \simeq 5\%$ at altitudes below $h \simeq 27$ km, $\tilde\delta_N(h) \simeq 7.5\%$ below $h = 31.5$ km, and $\tilde\delta_N(h) > 9.5\%$ at altitudes $h > 35$ km.

Thus in order to effectively retrieve the profiles $N(h)$ at altitudes up to 35 km and even higher with the preset 10% level of the fluctuations $\Delta N(h)$, it is necessary either to increase the energy potential of the lidar or to decrease the spatiotemporal resolutions.

Using Eqs. (4) and (14) and assuming $d\langle K_{22}(h)\rangle/dh = 0$, we write down the approximation for $\langle K_{22}(h)\rangle$ as

$$\langle\tilde K_{22}(h)\rangle = \sqrt{\frac{\langle\tilde K_{11}(h)\rangle}{Q(h;\lambda_1,\lambda_2)}}\,\bar\tau_{12}^2(0,h_L)\,. \qquad (15)$$

In its turn, substituting Eq. (15) into Eq. (12) we obtain the variance of the estimate of total ozone content

$$D[J^*] \simeq (h)\,L^2\sqrt{\frac{\langle\tilde K_{11}(h)\rangle}{Q(h;\lambda_1,\lambda_2)}}\,. \qquad (16)$$

Let us characterize the efficiency of filtering of concrete realization of the profile $J(h)$ by the function

$$E_J(h) = D[J^*(h)]/D[J(h)]$$

representing the ratio of its *a posteriori* and *a priori* variances. Using the proper approximation $\tilde E_J(h)$ of the function $E_J(h)$ for $h - h_0 \gg L$ we can represent the efficiency figure $\tilde\delta_J = \sqrt{\tilde E_j}$ in the form

$$\tilde\delta_J(h) \simeq \sqrt{\frac{\langle\tilde K_{11}(h)\rangle L}{2(h-h_0)}\sqrt{\frac{\langle\tilde K_{11}(h)\rangle}{Q(h;\lambda_1,\lambda_2)}}}\,. \qquad (17)$$

Figure 4 shows the dependences of $\tilde\delta_J(h)$ on the altitude difference $h - h_0$ for various pairs λ_1 and λ_2 when sounding is performed in the troposphere and stratosphere.

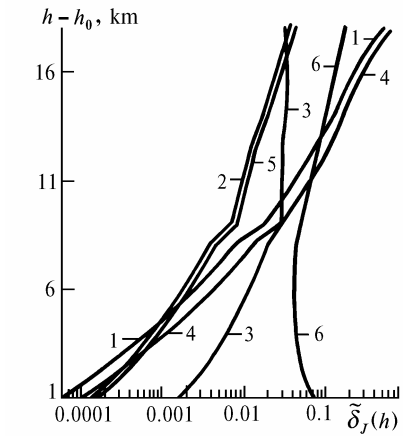

FIG. 4. Profiles of the efficiency figure of filtering of the total ozone content $\tilde\delta_J(h)$ *in the troposphere (curves 1–5, see Fig. 1) and in the stratosphere (curve 6, see curve 2 in Fig. 3).*

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REFERENCES

1. G.N. Glazov and G.M. Igonin, Atm. and Oceanic Opt. **5**, No. 6, 417–422 (1992).

2. G.M. Igonin, in: *Opto–Meteorological Studies of the Earth's Atmosphere* (Nauka, Novosibirsk, 1987), pp. 150–162.