# Derivation of the system of material equations for the case of interaction between radiation and nanoparticles

**N.R. Sadykov**

*Russian Federal Nuclear Center,*
*E.I. Zababakhin Scientific Research Institute of Technical Physics, Snezhinsk*

Received March 15, 2007

A way to generate an active medium by the use of a stationary electric field is proposed to amplify ultra-high frequency radiation in the range of $\lambda \sim 10$ cm. The way is based on dispersing of lengthened conducting nanoparticles. The volume concentration of nanoparticles and the necessary value of the stationary electric field for pumping the medium are estimated.

At present, dust particles in a neutral gas medium or in an ionized gas are under active study. This direction is promising for numerous technological applications: combustion processes, plasma technologies, atmosphere physics, controlled thermonuclear fusion. In plasma technologies, main attention is paid to crystallization of dust particles in gas-discharge plasma, i.e., to formation of ordered structures.[1,2] Of certain interest are processes connected with interaction of radiation with nanoparticles and nanostructures.[3,4] In this paper a system of material equations is derived, which describes the amplification process in a resonator of ultra-high frequency radiation $\lambda \sim 10$ cm in the presence of conducting nanoparticles. The medium is pumped by a stationary electric field. The mass concentration of nanoparticles and the magnitude of the pumped field necessary for this process, are estimated

Let there be a non-bounded domain consisting of lengthened nanoparticles of concentration $n$. Electromagnetic radiation propagates through the domain. The electric field is denoted by $\mathbf{E}(t, \mathbf{r})$. To describe the mechanism of interaction of such particles with the electromagnetic radiation, we approximate the particles by two similar conducting balls of radius $R$ and mass $m$ (see Figure).

We suppose that the balls are connected by a conducting thin bar of length $L$, with elastic coefficient $\tilde{k}$ and electric resistance $r_0$. Let $g_1(t)$ and $g_2(t)$ be charges on the first and second balls, respectively. We suppose that the dipoles are parallel to the electric field of the wave.

In the general case, polarization consists of the linear $\mathbf{P}_0$ and non-linear $\mathbf{P}_1$ polarizations:

$$P_0 = \frac{1}{2} nL(g_2 - g_1),$$

$$P_1 = \frac{1}{2} nx(g_2 - g_1) - \frac{1}{2} nx^{(0)}\left(g_2^{(0)} - g_1^{(0)}\right);$$

$$x = x(t) = x_2(t) - x_1(t), \quad x^{(0)} = x_2^{(0)} - x_1^{(0)}, \qquad (1)$$

where $P_0$, $P_1$ are amplitudes of vectors $\mathbf{P}_0$, $\mathbf{P}_1$; $n$ is the concentration of nanoparticles; $x(t)$ is the quickly-oscillating in the scale of charge difference quantity, which characterizes the variation of the distance between the balls due to elastic forces; $x \ll L$; $g_1 + g_2 = g_1^{(0)} + g_2^{(0)}$; $x^{(0)}$ denotes an additional deformation due to elastic and Coulomb forces of two preliminarily charged balls with the charges $g_1^{(0)}$, $g_2^{(0)}$ in the presence of $E$:

$$\tilde{k}\left(x_2^{(0)} - x_1^{(0)}\right) = \left(g_2^{(0)} - g_1^{(0)}\right) E / 2. \qquad (2)$$

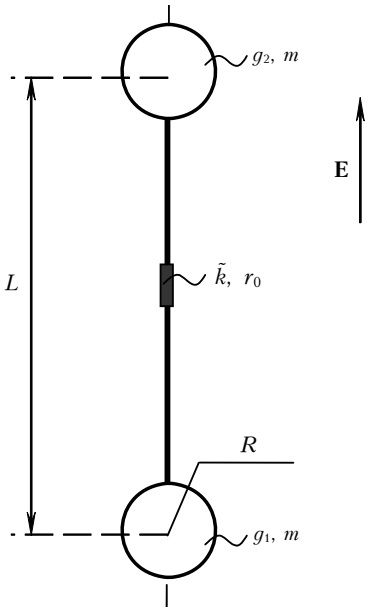


**Fig.** Approximation of lengthened nanoparticles by a dumbbell.

It is easy to see that $P_1$ is the variation of the non-linear polarization under the corresponding variation of charges. The linearity of $P_0$ in Eq. (2) follows from the fact that $(g_2 - g_1)/R = LE$ for $R \ll L$, i.e.,

$$P_0 = \frac{1}{2} nRL^2 E = \chi E, \qquad (3)$$

where the electrical susceptibility $\chi = nRL^2/2 = \text{const}$.

0235-6880/08/10 743-03 $02.00 

Taking into account the obtained relations, the equation for a field in a resonator with dispersed nanoparticles can be written in the form[5]

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} + \frac{1}{\tau}\frac{\partial \mathbf{E}}{\partial t} + \omega^2 \mathbf{E} = -\frac{4\pi}{\varepsilon}\frac{\partial^2 \mathbf{P}_1}{\partial t^2}, \qquad (4)$$

where $\varepsilon = 1 + 4\pi\chi$ is the dielectric capacity; $\tau$ is the time of resonator attenuation. In Eq. (4), spatial distribution of the polarization $\mathbf{P}_1$, which depends on the resonator field, is supposed to be the same as for the normal mode of the resonator field.

In the absence of radiation, the energy of the system for $R \ll L$ is

$$W_1 = n\left[(g_1^{(0)})^2 + (g_2^{(0)})^2\right] / 2R, \qquad (5)$$

where $g_1^{(0)}$, $g_2^{(0)}$ are charges on the balls at the start moment.

In the presence of the radiation field at an arbitrary time

$$W_2 = n\left[g_1^2 + g_2^2\right] / 2R, \qquad (6)$$

where

$$g_1 + g_2 = g_1^{(0)} + g_2^{(0)} = 0.$$

From Eqs. (5) and (6) we obtain that the energy of the system in the presence of radiation is

$$\Delta W = W_2 - W_1 = N - N_0,$$

$$N = \frac{n(g_2 - g_1)^2}{4R}, \quad N_0 = \frac{n\left(g_2^{(0)} - g_1^{(0)}\right)^2}{4R}. \qquad (7)$$

It was taken into account in the derivation of Eq. (7) that the following relation takes place:

$$g_1^2 + g_2^2 - (g_1^{(0)})^2 - (g_2^{(0)})^2 = 2g_1^{(0)}g_2^{(0)} - 2g_1 g_2.$$

By the use of the introduced variables $N$, $P$, the law of conservation of energy in the presence of a field can be written in the form

$$\frac{\partial N}{\partial t} + \frac{1}{T_1}(N - N_0) = E\frac{\partial P_1}{\partial t}, \qquad (8)$$

where $T_1$ is the relaxation time for the value $N - N_0$ in the absence of a field. Here we neglect the linear polarization.

The equation (8) is one of the two material equations.[5] Now derive the second equation. Relative to the generalized coordinate $x = x_2 - x_1$, where $x_1$, $x_2$ are respectively the coordinates of the balls' centers, the following motion equation takes place:

$$\mu\ddot{x} + \tilde{k}x = -\frac{1}{2}(g_2 - g_1)E. \qquad (9)$$

Here $\mu = m/2$ is the reduced mass; $\ddot{x} = \mathrm{d}^2x/\mathrm{d}t^2$. Equation (9) was derived with allowance for the fact that the charges $q_1$ and $q_2$ change places with amplification of the field (see the Figure).

Taking into account the forces of friction and the relation (1), obtain from Eq. (9)

$$\frac{\partial^2 P_1}{\partial t^2} + \frac{2}{T_2}\frac{\partial P_1}{\partial t} + \Omega^2 P_1 =$$

$$= -n\frac{(g_2 - g_1)^2}{4\mu}E + n\Omega^2\frac{g_2^{(0)} - g_1^{(0)}}{2}\left(x_2^{(0)} - x_1^{(0)}\right), \qquad (10)$$

where $\Omega^2 = \tilde{k}/\mu$; $T_2$ is the relaxation time of the non-linear polarization. The relation (10) was derived with allowance for the inequality

$$\left|\frac{\partial \ln|g_1|}{\partial t}\right| \sim \left|\frac{\partial \ln|g_2|}{\partial t}\right| \ll \left|\frac{\partial \ln|x|}{\partial t}\right|.$$

Taking into account Eq. (2), equation (10) can be written in the form

$$\frac{\partial^2 P_1}{\partial t^2} + \frac{2}{T_2}\frac{\partial P_1}{\partial t} + \Omega^2 P_1 = -\frac{R}{\mu}(N - N_0)E, \qquad (11)$$

where $N$, $N_0$ are defined in Eq. (7); the second summand in Eq. (11) characterizes the dissipative processes, for instance, forces of friction of the balls due to the presence of air.

The equations (4), (8), and (11) form the system of material equations in a resonator. Lengthened nanoparticles play the part of the medium. In this paper, lengthened nanoparticles are approximated by lengthened electroconducting dumbbells. The variables $N$ and $P_1$ in Eqs. (8) and (11) play the part of population difference between the levels and polarization, respectively (see Refs. 5 or 6).

Let us consider a coherent radiation in a resonator:

$$P_1 = \frac{1}{2}\tilde{P}_1\exp(-i\omega t) + \text{ c. c.};$$

$$E = \frac{1}{2}\tilde{E}\exp(-i\omega t) + \text{ c. c.} \qquad (12)$$

Suppose that the resonator is tuned so that $\omega = \Omega = \omega_0$. Then, it follows from Eqs. (11) and (12) that

$$\tilde{P}_1 = -i\frac{T_2 R(N - N_0)}{2\mu\omega}\tilde{E} = \chi_1\tilde{E}. \qquad (13)$$

Substituting Eq. (13) into Eqs. (4) and (8), we obtain

$$\frac{\partial N}{\partial t} + \frac{1}{T_1}(N - N_0) = -\frac{T_2 R(N - N_0)}{4\mu}\left|\tilde{E}\right|^2;$$

$$\frac{\partial \tilde{E}}{\partial t} + \frac{1}{2\tau}\tilde{E} = i\frac{2\pi\omega}{\varepsilon}\tilde{P}. \qquad (14)$$

It follows from Eqs. (13) and (14) that

$$\frac{\partial\left|\tilde{E}\right|^2}{\partial t} + \frac{1}{\tau}\left|\tilde{E}\right|^2 = \frac{2\pi T_2 R(N - N_0)}{\mu\varepsilon}\left|\tilde{E}\right|^2. \qquad (15)$$

Let us estimate the coefficient at $|\tilde{E}|^2$ in the right side of Eq. 15. It has the dimension of the inverse value of time:

$$\frac{1}{T} = \frac{4\pi T_2 R(N - N_0)}{m\varepsilon}, \qquad (16)$$

where $m = 2\mu$.

Let $\omega T_2 = 10^3$, $\omega = 2 \cdot 10^{10}$ sec$^{-1}$ (the wavelength in vacuum $\lambda \approx 10$ cm); $\rho = 2 \cdot 10^3$ kg/m$^3$ is the density of the balls of radius $R$; $L = 50R$; $R = 10^{-8}$ m; $E_0^2/8\pi = 10$ J/m$^3$ is the energy density of the electrostatic field, by which the system is pumped; $\varepsilon = 1$; $c_0 = 8\pi R^3 n/3 = 10^{-3}$ is the volume concentration of the dumbbells. Taking into account the relation $m = 4\pi R^3/3$, we obtain from Eq. (16) for $N = 0$:

$$(N - N_0)\big|_{t=0} = \frac{n(g_2^{(0)} - g_1^{(0)})^2}{4R} = \frac{nRL^2}{4}|E_0|^2;$$

$$\frac{1}{T} = \frac{9 \cdot 10^3 L^2 c_0}{4\omega\rho R^4}\frac{|E_0|^2}{8\pi} \approx 1.4 \cdot 10^7 \text{ sec}^{-1}, \qquad (17)$$

where equation (17) was derived with allowance for $(g_2^{(0)} - g_1^{(0)})/R = LE_0$.

It is seen from Eq. (17) that $T \approx 7.1 \cdot 10^{-8}$ sec, while the period of electromagnetic oscillations is $2\pi/\omega \approx 3.2 \cdot 10^{-10}$ sec$^{-1}$.

Now estimate the value of oscillation frequency of the lengthened particles, considering cylinder-shaped particles instead of dumbbells. Suppose that the mass of a cylindrical bar of length $L$ is localized at the ends of the bar. Then the reduced mass of the oscillator is

$$\mu = m/2 = \pi R^2 L\rho/4,$$

where $\rho$ is the density of the bar mass. The relation

$$\tilde{k} = W\pi R^2/L,$$

where $W$ is Young's modulus, takes place for the elasticity coefficient in the case of lengthened nanoparticles. Then the oscillation frequency is

$$\Omega = \sqrt{\tilde{k}/\mu} = \sqrt{4W/(L^2\rho)}.$$

For majority of metals, Young modulus is in the range of $W \sim 10^9 \div 10^{12}$ J/m$^3$. Putting $\rho = 2 \cdot 10^3$ kg/m$^3$, obtain $\Omega \approx 3 \cdot 10^9 \div 9 \cdot 10^{10}$ sec$^{-1}$, what does not contradict to the above-mentioned supposition $\omega = 2 \cdot 10^{10}$ sec$^{-1}$.

Now estimate the time $T_1$ in Eq. (8). In the International System, when the field $E = 0$ is absent, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{g_2 - g_1}{2}\right] = -\frac{g_2 - g_1}{4\pi\varepsilon_0 R r_0}, \qquad (18)$$

where $\varepsilon_0$ is the electric constant. Taking into account Eqs. (7) and (8), one can write the equation (18) in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}N = -\frac{1}{\pi\varepsilon_0 R r_0}N. \qquad (19)$$

It follows from Eq. (19) that $T_1 = \pi\varepsilon_0 R r_0$. To estimate the magnitude $T_1$, we need the Ohm law. Justify the possibility of applying this law in our problem. For continuous media, the length of a free path of an electron is $\lambda_0 \sim 10^{-10}$ cm, whereas the size of balls in our case does not exceed the value $R \sim 10^{-8}$ m ($L \sim 10^{-7}$ m). It means that, under such scales, the motion of electrons occurs in correspondence with the macroscopic motion of electrons in conducting media. The depth of field penetration in our calculations, due to the skin-effect, is of order of $\delta = c/\sqrt{2\pi\sigma\omega} = c\sqrt{\varepsilon_0/(2\sigma\omega)} \sim 10^{-6} \div 10^{-5}$ m $\gg R$, where $\sigma$ is the electrical conductance; $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m. For the resistance, there is a relation $r_0 = \rho_0 L/S = 20\rho_0/(\pi R)$ implying $T_1 = 20\varepsilon_0\rho_0$, where $\rho_0 = 1/\sigma$ is the specific resistance. For the graphite, $\rho_0 \sim 10^{-5}$ $\Omega \cdot$ m, and, therefore, $T_1 \sim 10^{-15}$ sec. It is seen that the first summand $\partial N/\partial t$ in Eq. (8) may be neglected for $\omega = 2 \cdot 10^{10}$ sec$^{-1}$.

Thus, the obtained system of material equations describes the process of amplification of ultra-high frequency radiation at $\lambda \sim 10$ cm in a resonator. For $\omega = 2 \cdot 10^{10}$ sec$^{-1}$ and volume concentration $c_0 = 10^{-3}$ for lengthened nanoparticles $L = 50R$ the characteristic time of amplification is $T_1 \sim 10^{-8} \div 10^{-7}$ sec. The theoretical approach, which was presented in this paper, also makes it possible to consider the amplification process for a running wave, i.e., amplification of ultra-high radiation.

## References

1. V.E. Fortov, A.P. Nefedov, O.S. Vaulina, A.M. Lipaev, V.I. Molotkov, A.A. Samaryan, V.P. Nikitskii, A.I. Ivanov, S.F. Savin, A.V. Kalmykov, A.Ya. Solov'ev, and P.V. Vinogradov, Zh. Eksp. Teor. Fiz. **114**, Is. 6 (12), 2004–2021 (1998).
2. A.P. Nefedov, O.F. Petrov, and V.E. Fortov, Usp. Fiz. Nauk **167**, No. 11, 1215–1226 (1997).
3. V.A. Zatsepin, N.R. Sadykov, M.O. Sadykova, V.K. Filippov, and A.N. Shcherbina, Atmos. Oceanic Opt. **20**, No. 4, 340–341 (2007).
4. V.A. Zatsepin, V.P. Smyslov, N.R. Sadykov, M.O. Sadykova, V.K. Filippov, and A.N. Shcherbina, Atmos. Oceanic Opt. **17**, Nos. 2–3, 146–148 (2004).
5. R.H. Pantel and H.E. Puthoff, *Fundamentals of Quantum Electronics* (Wiley, New York, 1969).
6. P.G. Kryukov and V.S. Letokhov, Usp. Fiz. Nauk **99**, No. 2, 169–225 (1969).