EFFECT OF FLUCTUATIONS OF AEROSOL PARTICLES CONCENTRATION ON THE OPTICAL TRANSFER FUNCTION OF THE ATMOSPHERE

I.P. Lukin

Institute of Atmospheric Optics,
Siberian Branch of the Academy of Sciences of the USSR, Tomsk
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The optical transfer function in the case of observation through the atmosphere is studied, taking into account the effect of fluctuations of the concentration of the scattering particles owing to atmospheric turbulence. The conditions under which the distorting effect of these fluctuations on optical image quality is strongest are determined.

The random field of the dielectric constant of air and the discrete scatterers (aerosol, hydrometeors, etc.) randomly distributed in space cause the amplitude a,nd phase of an optical wave propagating in the atmosphere to fluctuate. These fluctuations in turn give rise to fluctuations in the spatial image produced by an optical receiving system. The combined effect of the atmosphere and the optical system is usually regarded as random linear filtering and the system atmosphere-optical system is characterized by the optical transfer function (OTF).[1] The optical transfer function is the Fourier transform of the intensity distribution in the space of the image of a point source located in the space of the object. The optical transfer function of a clear turbulent atmosphere is well known.[2–4] The optical transfer function of scattering media is studied in many works.[5–9] In all of these works, however, it was assumed that the atmospheric turbulence and the discrete scatterers make independent contributions to the distortion of the optical wave. It has nonetheless been established recently[10] that the characteristics of the spatiotemporal distribution of the concentration of scattering particles in the .atmosphere are related with the random wind-velocity field, i.e., they are caused by coherence function of the field of a spherical wave. In this paper the optical transfer function of the atmosphere is calculated taking into account the effect of fluctuations of the concentration of scattering particles for the case when the earth's surface is viewed from space.

The optical transfer function of the atmosphere ($M(p)$) can be approximately represented as the product of the OTF of a clear turbulent atmosphere ($M_t(p)$) and the OTF of the scattering medium ($M_s(p)$):[11]

$$M(p) \simeq M_t(p)\cdot M_s(p),$$

where p is the spatial scale, which is inversely proportional to the spatial frequency. As shown in Refs. 12 and 13, when the earth's surface is viewed from space there occur quite often situations in which the refraction turbulence of the atmosphere does not appreciably affect the image quality ($M_t(p)) \simeq 1$), i.e., all distortions are associated with the effect of discrete scattering particles (aerosol, etc.) ($M(p) \simeq M_s(p)$).

The optical transfer function of the atmosphere is related with the second-order coherence function of the optical field[1,2]

$$M(p) = \Gamma_2(x,\ p),$$

where $\Gamma_2(x, p) = \langle U(x, \rho_0)\cdot U^*(x, \rho_0 + \rho)\rangle$ is the second-order coherence function of the optical field $U(r)$. We shall first study the scattering of optical radiation by particles which are large compared with the wavelength of the radiation. In this case, the solution of the radiation transfer equation in the small-angle approximation[14] or the method of effective randomly nonuniform medium[15–17] can be used to find the second-order coherence function. The second-order coherence function of a spherical optical wave in a scattering medium with $\bar{a}\gg\lambda$ has the form

$$\Gamma_2(x,\ \rho) = \frac{U_0^2}{x^2}\exp\left\{-\,2\pi\chi\int_0^1 d\xi\, m_0(\chi\xi)\int_0^\infty da\,p(a)\,\times\right.$$
$$\left.\times\int_0^{\pi/2} d\theta\,\sin\theta\,[1 - J_0(\xi k\rho\,\sin\theta)]\,f_0(\theta k)\,f_0^*(\theta k)\right\}, \qquad (1)$$

where U_0 is the amplitude of an optical wave that has passed through a layer of the scattering medium; x is the distance from the receiving optical system to the earth's surface; $m_0(\xi)$ is the concentration of scattering particles in an elementary averaging volume V and depends on the altitude above the earth's surface; $p(a)$ is the size distribution function of the scattering particles; a is the radius of a scattering particle; $\bar{a}$ is the average radius of the scattering particles; $k = 2\pi/\lambda$, where λ is the wavelength of the radiation in vacuum; $f_0(\kappa)$ the scattering amplitude of an optical wave scattered by a separate particle; and, $J_0(x)$ is the zeroth-order Bessel function of the first kind. The formula (1) pertains to

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observation at the nadir for $x > k\overline{a^2}$. The scattering amplitude of an optical wave scattered by a particle whose radius is larger than the wavelength of the optical radiation can be written as follows:[14]

$$f_0(\kappa) \simeq i\,\frac{ka^2}{2}\,\frac{2J_1(\kappa a)}{\kappa a} \approx i\,\frac{ka^2}{2}\exp\left[-\frac{\kappa^2 a^2}{8}\right],$$

where $J_1(x)$ is the first-order Bessel function of the first kind. The modulus of the complex degree of coherence of a spherical wave $\gamma_2(p) = \Gamma_2(p)/\Gamma_2(0)$ in this case has the form

$$\gamma_2(\rho) = \exp\left\{-\pi\chi\int_0^1 d\xi\, m_0(\chi\xi)\int_0^\infty da\, p(a)\, a^2 \times \left[1-\exp\left(-\xi^2\frac{\rho^2}{a^2}\right)\right]\right\}.$$

The results of the calculation of the modulus of the complex degree of coherence of a spherical wave propagating in a uniform ($m_0(\xi) = m_0$) coarsely ($\bar{a} \gg \lambda$) monodispersed ($p(a) = \delta(a - \bar{a})$) medium, are presented in Fig. 1 for different optical thicknesses of the scattering layer $\tau = \pi m_0 \overline{a^2} x = 0.1$ (curve 1), 1 (2), and 10 (3). For small optical thicknesses of the scattering layer ($\tau \lesssim 1$) the modulus of the complex degree of coherence of the spherical wave drops at spatial separations $\rho \sim \bar{a}$ from 1 to $\exp(-\tau)$, after which the coherence is constant as a function of ρ. In dense scattering media ($\tau > 1$) the length scale of $\gamma_2(\rho)$ is $\rho_k \sim (m_0 x)^{-1/2} \sim \bar{a}\sqrt{\tau} < \bar{a}$, at which distance the coherence of the wave decreases from 1 to 0.

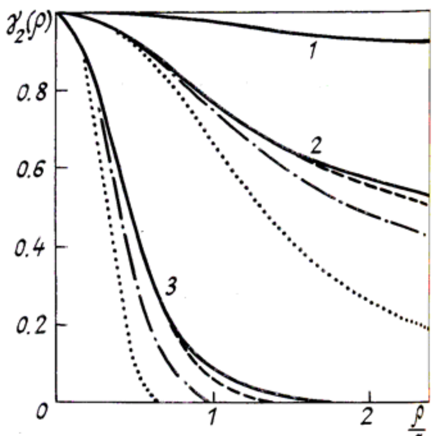

FIG. 1. The modulus of the complex degree of coherence of a spherical wave propagating in a coarsely monodispersed scattering medium. $\tau = 0.1$ *(1)*, 1 *(2)*, *and* 10 *(3)*. $\langle m_0' \rangle^2 = 0$ *(solid lines)*, $\langle m_0'^2 \rangle = 0.1\langle m_0' \rangle^2$ *(dashed lines)*, $\langle m_0'^2 \rangle = \langle m_0' \rangle^2$ *(dot-dashed lines), and* $\langle m_0'^2 \rangle = 5\langle m_0' \rangle^2$ *(dotted lines).*

This model of the optical coupling channel for systems for viewing the earth's surface from space, however, neglects the fact that the concentration of aerosol particles in the atmosphere fluctuates.[10] The concentration of scattering particles can be represented as a sum of the average concentration and a fluctuation correction, and in addition $m_0 = \langle m_0 \rangle + m_0'$, $\langle m_0' \rangle = 0$ while m_0' is a Gaussian random quantity. In Refs. 10 and 18 it is shown that fluctuations of the aerosol concentration are caused by atmospheric turbulence and the spectrum of these fluctuations is of the form

$$\Phi_{m_0} = \begin{cases} 0.033 C_{m_0}^2\, \varphi_0(\kappa)\kappa^{-11/3} & \text{for } \kappa < 1\ \text{m}^{-1}, \\ \dfrac{5\chi\nu^{1/2}}{4\pi\varepsilon^{1/2}}\kappa^{-3}(1+\kappa l_s)e^{-\kappa l_s} & \text{for } \kappa > 10\ \text{m}^{-1}, \end{cases}$$

where $C_{m_0}^2$ is the structure constant of the fluctuations of the aerosol concentration; $\varphi_0(\kappa) = (1 + \kappa_0^2/\kappa^2)^{-11/6}$; $\kappa_0 = 2\pi/L_0$, where L_0 is the outer scale of turbulence; $l_s = 5.5(D^2\nu/\varepsilon)^{1/4}$; χ is the rate of dissipation of the temperature gradient; ν is the kinematic viscosity of the air; ε is the rate of dissipation of turbulent kinetic energy; and, D is the thermal diffusivity. The largest values of the spatial scales of the correlation of fluctuations of the aerosol concentration in the atmosphere, according to the experimental data of Refs. 19 and 20, are equal to hundreds of meters. For this reason, when the earth's surface is viewed from space and $x \gtrsim 100$ km, the propagation path of the optical radiation contains a large number of scattering volumes in which aerosol concentration fluctuations in different volumes are uncorrelated. Since the radius of the first Fresnel zone $\sim\sqrt{\lambda x}$ at optical and infrared wavelengths is less than the correlation length of the fluctuations of the atmospheric aerosol concentration, to calculate the second-order coherence function it can be assumed that the atmospheric aerosol concentration varies only along the optical ray, i.e., it depends only on x. Thus at any moment in time it can be assumed that the integration along the propagation path results in averaging of the fluctuations of the atmospheric aerosol concentration.

After the expression for the coherence function of a spherical wave is averaged over the fluctuations of the atmospheric aerosol concentration the following relation is obtained instead of the formula (1):

$$\Gamma_2(\chi,\rho) = \frac{U_0^2}{\chi^2}\exp\left\{-2\pi\langle m_0\rangle\chi\int_0^1 d\xi\int_0^\infty da\,p(a)\times\right.$$

$$\times\int_0^{\pi/2} d\theta\sin\theta\,|f_0(\theta k)|^2\left[1-J_0(\xi k\rho\sin\theta)\right] - 2\pi^2\langle m_0'^2\rangle\times$$

$$\times\left[\chi\int_0^1 d\xi\int_0^\infty da\,p(a)\int_0^{\pi/2} d\theta\,\sin\theta\,|f_0(\theta k)|^2\times\right.$$

$$\times \left[1 - J_0(\xi k\rho \sin\theta)\right]^2\Big\},$$

where

$$\langle m_0'^2 \rangle = \iiint_{-\infty}^{\infty} d\kappa \Phi_{m_0}(\kappa)$$

is the variance of the fluctuations of the atmospheric aerosol concentration. In Fig. 1 the dashed, dot-dashed, and dotted curves show the modulus of the complex degree of coherence of a spherical wave propagating in a scattering atmosphere in which the concentration of the scattering particles fluctuates. It is obvious that for small optical thicknesses ($\tau \lesssim 0.1$) the spatial structure of the field of the optical wave is virtually identical to that of a medium in which the atmospheric aerosol concentration is constant. For $\tau \sim 1$ and small spatial separations $\rho < \bar{a}$ no appreciable dependence of $\gamma_2(\rho)$ on the variance of fluctuations of the atmospheric aerosol concentration is observed, though for $\rho > \bar{a}$ ($\tau \sim 1$) in a coarsely dispersed scattering medium the degree of coherence is equal to $\exp\left\{-\tau\left[1+\frac{1}{2}\frac{\langle m_0'^2\rangle}{\langle m_0'\rangle^2}\tau\right]\right\}$ instead of $\simeq e^{-\tau}$, i.e., when $\langle m_0'^2\rangle \sim \langle m_0'\rangle^2$ the effect of fluctuations of the aerosol concentration on image quality becomes significant. As the optical thickness of the scattering layer increases ($\tau > 1$) the effect of fluctuations of the aerosol concentration will be manifested at increasingly smaller spatial scales.

An analogous analysis for a finely dispersal scattering medium (for $\tau < 1$) shows that the image quality when viewing through the atmosphere depends slightly on the fluctuations of the concentration of the scattering particles. In addition, the experimental data of Ref. 19 show that the concentration of the coarse ($a > 10^{-6}$ m) fraction of the atmospheric aerosol fluctuates significantly more strongly than that of the small fraction ($a \sim (0.15–1.0) \cdot 10^{-6}$ m). Thus when the earth's surface is viewed from space the largest distortions of the image caused by fluctuations of the aerosol concentration will be observed for wavelengths $\lambda < 10^{-6}$ m. For $\lambda > 10^{-6}$ m the contribution of fluctuations of atmospheric aerosol concentration to the degradation of image sharpness will decrease as the wavelength of the optical radiation increases.

In conclusion we note that the results of this work can be used to evaluate the image quality for optical systems in which the transverse dimensions of the input aperture do not exceed 10 m. For larger apertures and for $\tau \lesssim 1$ the spatial structure of the field of the received optical wave will have a two-scale character. The first scale ($\rho \sim \bar{a}$), which was studied in this paper, is determined by scattering by aerosol particles. The second scale is connected with fluctuations of the concentration of the atmospheric aerosol caused by entrainment of aerosol particles by atmospheric turbulence, and according to the data of Ref. 20 this scale is ~ 100 m.

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