CHARACTERISTIC FEATURES OF LIGHT ABSORPTION BY A FRACTAL CLUSTER

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Two new mechanisms of light absorption by a fractal cluster are considered. Both mechanisms are born out by specific features of the photon trajectory topology in the system of correlated scatterers. This trajectory, under certain circumstances, can become null-dimensional Antuan's set, the so-called Antuan's necklace. The first absorption mechanism results from the topological prohibition for the Antuan's null-dimensional photon to escape to space with a traditional topological dimensionality $d = 1,2,3$. *The second mechanism relies upon a sort of "mechanical" rigidity of the Antuan's trajectory. This rigidity is related to the singularity of Antuan's photon energy density resulting from the transformation of an actual three-dimensional photon to a null-dimensional object. Photon cannot leave a cluster because of the difficulties with breaking up the necklaced virtual Antuan's chains. Absorption cross sections for fractal clusters are calculated for both mechanisms.*

1. INTRODUCTION

By fractal cluster (FC) is meant an agglomerate of a micron size, consisting of nanometer solid particles held together by Van-der-Vaals forces. Fractal clusters are formed either as a result of extremely non-equilibrium condensation of evaporated solids with subsequent aggregation of nanometer particles-monomers, or at the initial stage of crystallization from solutions and melts. Such objects are ordinary coproducts of many technological and natural processes. Finely dispersed aerosols and aerogels in the Earth's atmosphere also have fractal structure.

We consider the following two mechanisms of light absorption by a fractal cluster:

1) strict localization of radiation within the cluster,

2) absorption caused by formation of the so-called virtual photons or bonded states of incident or scattered photons.

These features well manifest themselves in the following cases:

a) particles that make up a FC are weakly absorbing,

b) a photon incident on the cluster is "large", i.e. its wavelength λ is comparable with the characteristic cluster size and is much larger than the size of an individual particle-monomer.

2. SPECIFIC FEATURES OF STRICT LIGHT LOCALIZATION WITHIN THE FRACTAL CLUSTER

Long-range correlations in the FC location (or in other words, large number of holes with power-law size distribution) favor the process of renormalization of the wavelength of external radiation λ as radiation penetrates into the cluster: photon wavelength in the cluster, λ_{int}, becomes much shorter than λ (in so doing, photon frequency ω remains unchanged). The explanation is as follows. Let an incident photon, whose wavelength λ is of the order of the cluster size L, be trapped by a certain, sufficiently large FC hole. This trapping leads to the growth of effective dielectric constant of the cluster $\bar{\varepsilon}$. In its turn, the growth of $\bar{\varepsilon}$ triggers a decrease of the photon wavelength as $\lambda_{int} = \lambda/\sqrt{\bar{\varepsilon}}$. The photon with a renormalized wavelength λ_{int} finds another, and smaller hole. Again, trapping stimulates further decrease in ε and, hence, a decrease in λ_{int}.

Under certain conditions, the trajectory of renormalized photon between a pair of correlated cluster particles becomes the so-called Antuan set or Antuan's necklace.[1] This set is arranged like a closed chain consisting of necklaced links. Each loop of the chain, in turn, consists of similar, necklaced loops of smaller size, each being enclosed into the original loop, and so forth. Common part of all the necklaced loops of various scales constitutes the Antuan's necklace (Fig. 1).

Most remarkable property of such a formation is its zero topological dimension, that is this object is invisible in our real world, like a point or a finite set of points. At the same time, this object can have macroscopic extents and, moreover, the zero-dimensional Antuan necklace preserves certain properties of one-dimensional line or trajectory that we normally use in physics. For instance, while we can remove a circle, "threaded" through a usual, zero-

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dimensional set such as finite set of points in the three-dimensional space not crossing this set, we cannot treat similarly an Antuan set (see Fig. 1); neither can we unlink two interthreaded Antuan necklaces.

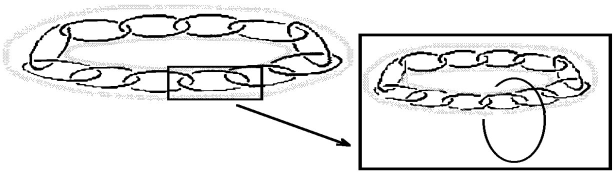

FIG. 1. Steps in constructing the Antuan set.

There are two mechanisms of retaining light between a pair of correlated particles. Both are caused by the topology of photon trajectory belonging to the Antuan set. The first mechanism is associated with the fact that there is the topological prohibition for "zero-dimensional" Antuan photon to escape to a space with the traditional dimensionality $d = 1,2,3$ (it is this prohibition that prevents us from exiting a closed three-dimensional room). The second mechanism is associated with a sort of "mechanical" rigidity of the Antuan photon trajectory. The reason for such rigidity is singularity of energy density of zero-dimensional Antuan trajectory caused by the conversion of an actual three-dimensional photon to zero-dimensional object. We shall consider both mechanisms, the former one in the present section.

The absorption due to the first mechanism is expressed in terms of the imaginary part of the effective dielectric constant of a cluster $\bar{\varepsilon}$. The latter is calculated by jointly solving Dieson equation for averaged photon propagator in the cluster, and by summing up the perturbation theory series for mass photon operator. Figure 2 shows the perturbation theory series for the averaged t-matrix of photon scattering on the cluster particle-monomer.

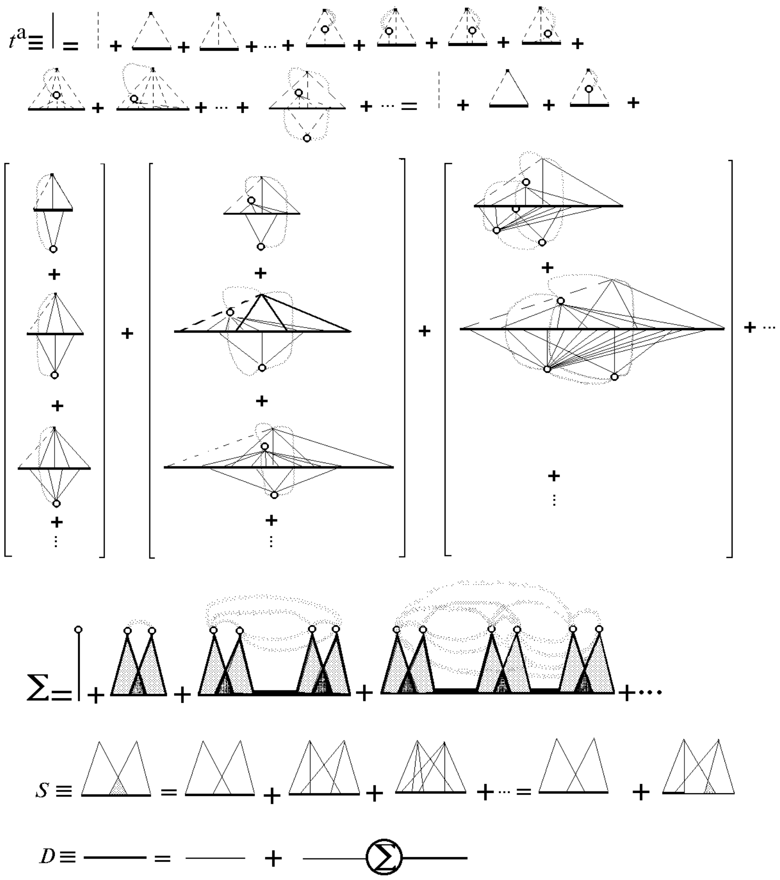

FIG. 2. Perturbation theory series for the t-matrix of photon scattering on a cluster particle-monomer; for mass photon operator Σ; for auxiliary, two-headed block S describing cycling up of a photon on a pair of particles; for an averaged one-photon propagator in the cluster D.

Here, the dashed line shows the potential P of the photon-particle interaction; dots refer to the particle packing factor in the cluster or to the particle-occupied fraction of the entire cluster volume; thin horizontal line indicates free-photon propagator in the calibration with zero scalar potential D^0; thick horizontal line is the averaged photon propagator in the cluster $\mathbf{D}$; arch-shaped line is the pair correlator g. This series describes photon stucking in the Antuan set. Diagrams in the first (left) square bracket describe multiple photon passage of the priming or the thickest link of the Antuan necklace adjacent to a given particle. The farther to the right the column is in the brackets, the larger is the number of smaller links or photon loops. Also shown in the figure is the approximation used for the mass photon operator Σ. Two-headed block S, entering it, describes photon travel across the Antuan chain situated between the pair of particle-monomers of the cluster. Also presented is the Dieson equation for $\mathbf{D}$. For P, D^0, and g we use the following expressions:

$$P^a_{\alpha\beta}(\mathbf{r}, \mathbf{r}') = \frac{\varepsilon(\omega) - 1}{4\pi} \frac{\omega^2}{c^2} \delta_{\alpha\beta}\, \delta(\mathbf{r} - \mathbf{r}')\, \Theta\, (R - |\mathbf{a} - \mathbf{r}|);$$

$$D^0_{\alpha\beta}(\mathbf{r}, \mathbf{r}') = \left(\delta_{\alpha\beta} - \frac{c^2}{\omega^2} \nabla_\alpha \nabla'_\beta\right) \frac{\exp\,(-\, i\omega|\mathbf{r} - \mathbf{r}'|/c)}{|\mathbf{r} - \mathbf{r}'|};$$

$$g(r) = -\frac{1}{4\pi n_0\, L_c^D\, \Gamma(D)}\, r^{\,D-3} \exp\left(-\frac{r}{L_c}\right),$$

where R is the radius of the particle-monomer, $\mathbf{a}$ is the coordinate of its center, ε is the dielectric constant of the particulate matter, n_0 is the mean density of particles in the cluster, D is the fractal dimension, L_c is the correlation length, $\Theta(x)$ is the unit Heaviside step function, $\Gamma(x)$ is the Euler gamma function, $\delta(x)$ is the Dirac delta function, c is the light speed in vacuum, ω is the light frequency.

For simplicity, we will consider a cluster in which the number N of particles in the correlation block (in the volume $\sim L_c^3$) is equal to the number of correlation blocks in the cluster, so that $N \gg 1$.

The calculated results are as follows. At moderately high fractal dimensions, the cluster is characterized by the set N of $\bar{\varepsilon}$ values. Values of $\bar{\varepsilon}_m$ ($m = 1,2, \ldots N$) are uniformly distributed over the circle of radius $N^{[(3-2D)N]/(4D)}|\varepsilon - 1|$ in the complex plane Re $\bar{\varepsilon}$, Im $\bar{\varepsilon}$. Each of them describes a particular degree of renormalization of the wavelength in accordance with the relation $\lambda_{\text{int}.m} = \lambda/\sqrt{\bar{\varepsilon}_m}$. For the absorption cross section associated with this mechanism, we have

$$\sigma_a = \frac{\omega R^3}{c} \left(\frac{L}{D}\right)^D \text{Im}\ \varepsilon, \quad (1)$$

where L is the characteristic cluster size.

The absorption considered is possible in any prespecified wavelength range provided that (a) intrinsic absorption of particulate substance is small in this wavelength range and (b) the wavelength of incident radiation simultaneously covers a large number of particle-monomers of the cluster. The role of this specific “absorption” increases when moving toward long waves of the spectrum: e.g., for a metal in the far-IR range, $|\text{Im}\ \varepsilon| \sim 10^5$; and at the same time, strong Thomas-Fermi screening acts to decrease the role of intrinsic (“true”) absorption by the particulate matter. The latter is necessary in order that the photon to have enough time to “run” within the cluster. However, if the particulate matter is perfectly nonabsorbing (Im $\varepsilon = 0$), then, as is easily seen from equation (1), the localization disappears. This is a somewhat paradoxical feature of the phenomenon considered.

3. FORMATION OF THE BONDED STATE AS A PAIR OF VIRTUAL PHOTONS OR "MECHANICAL" TENSION OF NECKLACED ANTUAN CHAINS

The second mechanism of light absorption by FC is more complicated, being, at the same time, more interesting. In section 2, we calculated localization cross section by using only the imaginary part of the photon mass operator, and all the results we obtained are actually quite understandable from the optical theorem view point. In contrast, the phenomenon we shall address here is difficult to treat within the optical theorem framework only, and we have to use averaged two-photon propagator in the cluster. Usually, this quantity is used to calculate cross-section of the elastic light scattering. This is not strange because the absorption cross-section we are interested in is found to be expressed in terms of the imaginary part of the elastic scattering cross section.

Differential cross section of the photon elastic scattering by a cluster, $d\sigma/(d\mathbf{n}_f)$ ($\mathbf{n}_f$ is the unit vector in the direction of scattered photon) is obtained after averaging of the square of the modulus of scattering amplitude $\Im$ for which the perturbation theory series exists[2]

$$\Im = \mathbf{e}_{i\alpha} \int \exp\,(-\, i\mathbf{k}_i\, \mathbf{r}) \sum_{\mathbf{a}} P^{\mathbf{a}}_{\alpha\beta}(\mathbf{r}, \mathbf{r}')\, \mathbf{e}_{t\beta} \times$$

$$\times \exp\,(i\mathbf{k}_f\, \mathbf{r}')\, d\mathbf{r}\, d\mathbf{r}' + \mathbf{e}_{i\alpha} \int \exp\,(-\, i\mathbf{k}_i\, \mathbf{r}) \times$$

$$\times \sum_{\mathbf{a}} P^{\mathbf{a}}_{\alpha\beta}(\mathbf{r}, \mathbf{r}_1)\, D^0_{\beta\gamma}(\mathbf{r}_1, \mathbf{r}_2) \sum_{\mathbf{b}} P^{\mathbf{b}}_{\gamma\nu}(\mathbf{r}_2, \mathbf{r}') \times$$

$$\times\, \mathbf{e}_{f\nu} \exp\,(i\mathbf{k}_f\, \mathbf{r}')\, d\mathbf{r}\, d\mathbf{r}'\, d\mathbf{r}_1\, d\mathbf{r}_2 + \ldots ,$$

where $\mathbf{k}_i$ and $\mathbf{k}_f$ are the wave vectors of the incident and scattered photons; $\mathbf{e}$ are their respective unit polarization vectors; and $\mathbf{a}$, $\mathbf{b}$,... are the coordinates of the particle centers. The structure of the PT series for $|\Im|^2$ or $d\sigma/(d\mathbf{n}_f)$ is presented in Fig. 3. The series has the same terms as those in Fig. 2.

The diagrams in Fig. 3 are very much like the standard diagrams of the field theory that describe the interaction of a pair of particles. We, however, deal with a single photon. Where does the pair interaction come from? The answer is simple. In situation with $\lambda \sim L$, we cannot distinguish between the incident (ith) and elastically scattered (fth) photons while they interact with the cluster particles. It is meaningful to envision that a pair of virtual photons is present in the cluster rather than a single actual photon. The diagrams in Fig. 3 just describe the pair interaction of these virtual photons that arises due to their scattering on the same (or different, but correlated) particles. This interaction, of course, disappears once it is possible in principle to distinguish between these photons, i.e., if the virtual photons revert to actual ones. This occurs if polarization type changes during the scattering process, i.e., when a *p*-polarized photon converts into a *s*-polarized photon.

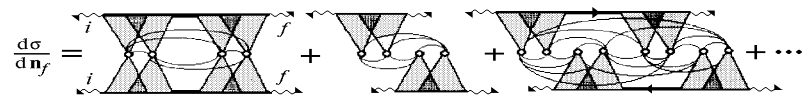

FIG. 3. Structure of the perturbation theory series for differential scattering cross section.

The photon identity also disappears upon *p*-scattering at non-zero angle (except at π): the simplest test for polarization is sufficient for a distinction. Of course, for a qualitative understanding of physics of diagrams in Fig. 3, we could restrict ourselves to their quasi-classical interpretation within which they describe photon interaction with itself in the course of its travel in the loops with characteristic size of the order of wavelength (i.e., photon collides with itself after turns). However, such an interpretation is insufficient, and hempers understanding of the polarization features.

Summation of the PT series for $d\sigma/(d\mathbf{n}_f)$ reduces to solving Bete-Solpiter equation (Fig. 4) for two-photon propagator K. Key element of this equation are diagrams belonging to the same correlation block.

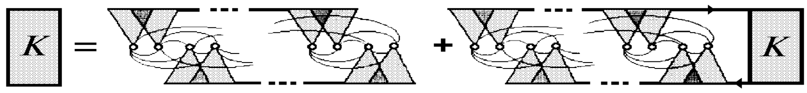

FIG. 4. Bete-Solpiter equation for averaged, two-photon propagator in a cluster.

Alternatively, diagrams interesting to us can be interpreted as follows. The *i*th incident photon, converting to the scattered *f*th photon, passes through a sequence of Antuan chains localized between the cluster particles belonging to a given correlation block – top of the diagrams. These small correlated chains can be regarded as a single large Antuan chain encompassing entire correlation block. Simultaneously, the *f*th photon reverts to the *i*th photon (bottom of the diagrams).

With the polarization restrictions mentioned above, as the photon travels in the loop, these processes cannot be distinguished when $\lambda \sim L$. The diagrams in Fig. 3 describe the interference of the probability amplitudes corresponding to these processes. Having finished its travel along Antuan chain belonging to this correlation block, photon then transits to another correlation block, to another, “larger”, Antuan chain, and so on. These chains are in no way correlated with each other. Since each particle of a cluster can be considered to belong simultaneously to several correlation blocks, these Antuan chains are necklaced.

One remarkable feature of the Antuan chain we already mentioned in section 2 is as follows. Despite their zero-dimensionality, two necklaced chains cannot be "pulled" through one another. Any given photon simultaneously belongs to a ball of necklaced virtual Antuan chains each related to its own correlation block. And if there is a special mechanism of keeping photon inside a cluster: non-local photon cannot escape from a cluster for this would require a work to break up the ball? Physically, the arising rigidity of the photon trajectory is caused by the transformation of an actual, three-dimensional photon to a zero-dimensional object, and the inevitably associated singularity of the energy density of Antuan photon or Antuan chain.

Alternatively, this topological mechanism of photon retaining in the cluster can be explained in more conventional terms we started with, in terms of the diagram methods of the field theory. Simply interpreted, there occurs bonded state of a pair of virtual photons due to infinitely multiple effective interaction of the pair of virtual photons, reflected in the step-like series shown in Fig. 4.

The imaginary part of the differential scattering cross section corresponding to the series from Fig. 4 describes the cross section of photon captured by the cluster due to mechanism considered in this section. The total absorption cross section for the s-polarized light is

$$\sigma \cong \frac{4\pi}{5}\,\frac{L\,L_c}{(3-D)}\,N^{[(3-2D)\,N]/(2D)}\,\times$$

$$\times\left(\frac{\omega\,L}{c}\right)^2\left(\frac{\omega\,L_c}{c}\right)^2\,|\varepsilon-1|^2. \qquad (2)$$

In the power of (2) we dropped the terms which do not contain $N \gg 1$. The thresholding fractal dimension $D = 2$, below which the cluster is apt to absorb is exclusively because of the fact that we are considering a cluster in which the number of particles contained in the correlation block is equal to the number of correlation blocks in the cluster. If this requirement is not met the critical *D* value, below which the cluster becomes an effective absorber, changes. Thus, for instance, when absolutely all particles of a cluster correlate the critical *D* value is 3/2.

CONCLUSION

The anomalous fractal properties are not restricted to absorbing properties, but also refer to an elastic scattering channel as well. In particular, "zero-dimensionality" of Antuan photons and, hence, the associated ability to concentrate many photons in a small volume, explain the existence, in fractal clusters, of strong, local electromagnetic fields. Maybe, that is why the anomalously high-rate catalytic processes take place in many heterogeneous nanosystems. For instance, strong local electromagnetic fields generated in fractal atmospheric clusters during their interaction with solar ultraviolet, may be responsible for the destruction of the Earth's ozone layer.

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