CONSTRUCTION OF OBJECT IMAGES THROUGH TURBULENT ATMOSPHERE USING MONOSTATIC ILLUMINATION SCHEME

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We show in this paper that the generalized optical transfer function (OTF) of the turbulent atmosphere and receiving system, corresponding to the laws of linear filtration, can be constructed for the case of diffuse object illumination with incoherent optical radiation along a correlated path. At diffuse object illumination with a coherent radiation along a correlated path, one fails to construct an OTF meeting the principles of linear filtration. It has been established that when transceiving apertures of a coherent source and a telescope are matched, the account for the correlation between the wave illuminating the object and that reflected from it improves the quality of short-exposure image.

A number of papers have been recently published that analyze images of objects observed in coherent light through random media.[1–5] In this connection, it is interesting to study the peculiarities in formation of coherent optical images when the wave illuminating the object and that reflecting from it come through the same inhomogeneities of a medium and the effect of backscattering amplification takes place. In this paper we analyze the optical transfer function of the turbulent atmosphere in the case of monostatic scheme of illumination of an object observed at different degree of coherence of the optical radiation of illumination under conditions of strong intensity fluctuations. Peculiarities in formation of an image of a coherently illuminated object in the short-exposure mode are considered.

OPTICAL TRANSFER FUNCTION OF THE TURBULENT ATMOSPHERE AT A SOUNDING PATH

Let us consider, as is shown in Fig. 1, an object, whose amplitude reflection coefficient is described by the function $O(\boldsymbol{\rho}', \mathbf{r})$. Here, $\boldsymbol{\rho}'$ and $\mathbf{r}$ are two-dimensional vectors. The object is illuminated with an optical source, the distribution of field over the plane of emitting aperture is described the function $U_0(\mathbf{t})$. The source is at the distance L from an object.

The mean intensity $\langle I_t(l, \boldsymbol{\rho}'')\rangle$ in the plane l behind the telescope receiving lens equals[6] to

$$\langle I_t(l, \boldsymbol{\rho}'')\rangle = \left(\frac{k}{2\pi l}\right)^2 \int d^2t_{1,2}\langle U_0(\mathbf{t}_1)U_0^*(\mathbf{t}_2)\rangle \int d^2\rho'_{1,2} \times \int d^2r_{1,2}\,\langle O(\boldsymbol{\rho}'_1, \mathbf{r}_1)O^*(\boldsymbol{\rho}'_2, \mathbf{r}_2)\rangle \int d^2\rho_{1,2}\, T(\boldsymbol{\rho}_1)\, T(\boldsymbol{\rho}_2) \times \langle G_{\rm rec}(x, x_0; \boldsymbol{\rho}'_1\, \mathbf{t}_1)\, G^*_{\rm rec}(x, x_0; \boldsymbol{\rho}'_2\, \mathbf{t}_2)\, G_{\rm obj}(x_0, x; \boldsymbol{\rho}_1, \mathbf{r}_1) \times G^*_{\rm obj}(x_0, x; \boldsymbol{\rho}_2, \mathbf{r}_2)\rangle \exp\left[\frac{ik}{2l}\left(1 - \frac{l}{F_t}\right)(\rho_1^2 - \rho_1^2) - \frac{ik}{l}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)\,\boldsymbol{\rho}''\right], \quad (1)$$

where $G_{\rm rec}(x, x_0; \boldsymbol{\rho}, \mathbf{t})$ and $G_{\rm obj}(x_0, x; \boldsymbol{\rho}, \mathbf{r})$ are the Green's functions along the paths from source to object and back from object to telescope, respectively; F_t is the focal length of the telescope receiving lens; $k = 2\pi/\lambda$ is the wave number; x_0 determines the position of the plane of the optical source and the telescope; x determines the position of the object plane.

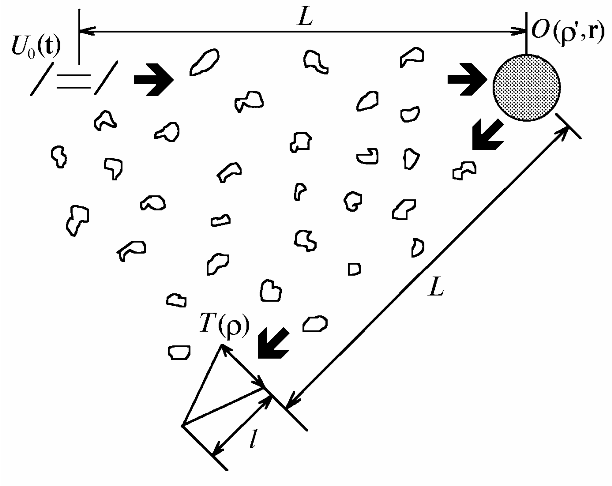

FIG. 1. Geometry of the image formation.

It is known[6] that under strong intensity fluctuations the following expression is valid:

$$\langle G_{rec}(x,x_0;\rho'_1,\mathbf{t}_1)\,G^*_{rec}(x,x_0;\rho'_2,\mathbf{t}_2)\,G_{obj}(x_0,x;\rho_1,\mathbf{r}_1)\,G^*_{obj}(x_0,x;\rho_2,\mathbf{r}_2)\rangle =$$

$$= \langle G_{rec}(x, x_0; \rho'_1, \mathbf{t}_1)\; G^*_{rec}(x, x_0; \rho'_2, \mathbf{t}_2)\; G_{rec}(x, x_0; \mathbf{r}_1, \rho_1)\; G^*_{rec}(x, x_0; \mathbf{r}_2, \rho_2)\rangle =$$

$$= \langle G_{rec}(x, x_0; \rho'_1, \mathbf{t}_1)G^*_{rec}(x, x_0; \rho'_2, \mathbf{t}_2)\rangle\,\langle G_{rec}(x, x_0; \mathbf{r}_1, \rho_1)\; G^*_{rec}(x, x_0; \mathbf{r}_2, \rho_2)\rangle +$$

$$+ \langle G_{rec}(x, x_0; \rho'_1, \mathbf{t}_1)G^*_{rec}(x, x_0; \mathbf{r}_2, \rho_2)\rangle\,\langle G^*_{rec}(x, x_0; \rho'_2, \mathbf{t}_2)G_{rec}(x, x_0; \mathbf{r}_1, \rho_1)\rangle. \qquad (2)$$

The first term in the expression (2) describes the influence of a medium on optical wave propagation, when the wave illuminating the object and that reflected from it come through different medium inhomogeneities, i.e. they are uncorrelated. The second term in Eq. (2) is responsible for correlation of these waves.

It can be shown that[6]:

$$\langle G_{rec}(x,x_0;\rho'_1,\mathbf{t}_1)\,G^*_{rec}(x,x_0;\rho'_2,\mathbf{t}_2)\rangle = \left(\frac{k}{2\pi(x-x_0)}\right)^2 \times$$

$$\times \exp\left\{\frac{ik}{2(x-x_0)}(\rho_1-\rho_2-\mathbf{t}_1+\mathbf{t}_2)(\rho_1+\rho_2-\mathbf{t}_1-\mathbf{t}_2) -\right.$$

$$\left. -\frac{\pi k^2(x-x_0)}{2}\int\limits_0^1 \mathrm{d}\xi\; N\left[(1-\xi)(\mathbf{t}_1-\mathbf{t}_2)+\xi(\rho_1-\rho_2)\right]\right\}, \qquad (3)$$

where

$$N(\rho) = 2\int \mathrm{d}^2\kappa\,\Phi_\varepsilon(\kappa)[1-\exp(i\kappa\rho)];$$

$\Phi_\varepsilon(\kappa)$ is the spectrum of turbulence.

Let us consider the peculiarities in formation of the image of an object with diffusely scattering surface, the reflection coefficient of which is described by the following expression[6]

$$\langle O(\rho'_1, \mathbf{r}_1)\; O^*(\rho'_2, \mathbf{r}_2)\rangle =$$

$$= \frac{4\pi}{k^2}\langle A(\mathbf{r}_1)\; A^*(\mathbf{r}_2)\rangle\,\delta(\mathbf{r}_1-\mathbf{r}_2)\,\delta(\rho'_1-\mathbf{r}_1)\,\delta(\rho'_2-\mathbf{r}_2),$$

where $\delta(\rho)$ is the Dirac delta function.

For the uncorrelated paths, when the first term remains in Eq. (2), and for incoherent illumination

$$\langle U_0(\mathbf{t}_1)U^*_0(\mathbf{t}_2)\rangle = \frac{4\pi}{k^2}I_0(\mathbf{t}_1)\,\delta(\mathbf{t}_1-\mathbf{t}_2),$$

the distribution of the mean intensity of the object's image in the plane of sharp image $\left(1+\frac{L}{l}-\frac{L}{F_t}=0\right)$ is as follows:

$$\langle I_t(l,\rho'')\rangle_{inc} = \left(\frac{4\pi}{k^2}\right)^2\left(\frac{k}{2\pi L}\right)^4\left(\frac{k}{2\pi l}\right)^2 I_{inc}\int \mathrm{d}^2 r\langle A(\mathbf{r})A^*(\mathbf{r})\rangle \times$$

$$\times \int \mathrm{d}^2\rho_{1,2}\; T(\rho_1)\; T(\rho_2)\; H(x, 0; 0, \rho_1-\rho_2) \times$$

$$\times \exp\left[-\frac{ik}{L}(\rho_1-\rho_2)\left(\mathbf{r}-\frac{L}{l}\hat{\rho}''\right)\right], \qquad (4)$$

where $\hat{\rho}'' = -\rho''$, $I_{inc} = \int \mathrm{d}^2 t\; I_0(\mathbf{t})$,

$$H(x,0;0,\rho_1-\rho_2) = \exp\left[-\frac{\pi k^2 L}{2}\int\limits_0^L \mathrm{d}\xi N(\xi(\rho_1-\rho_2))\right].$$

One can see from Eq. (4) that the mean intensity of the object's image is expressed as a convolution integral of the object intensity and the function

$$f\left(\mathbf{r}-\frac{L}{l}\hat{\rho}''\right) = \int \mathrm{d}^2\rho_{1,2}\,T(\rho_1)\,T(\rho_2)\,H(x,0;0,\rho_1-\rho_2)\times$$

$$\times \exp\left[-\frac{ik}{L}(\rho_1-\rho_2)\left(\mathbf{r}-\frac{L}{l}\hat{\rho}''\right)\right], \qquad (5)$$

that has a meaning of the averaged point-spread function of the turbulent atmosphere and an optical system. It follows therefrom, according to the convolution theorem, that the spatial spectrum of the object's image is the product of the object spatial spectrum and the Fourier transform of the point-spread function of the turbulent atmosphere and the optical system (5).[7]

Really, having taken Fourier transforms of both sides of the expression (4) and after simple transformations, we obtain

$$\langle\tilde{I}_t(l,\omega)\rangle_{inc} =$$

$$= \left(\frac{4\pi}{k^2}\right)^2\left(\frac{k}{2\pi L}\right)^4 I_{inc}\,\tilde{I}_{obj}\left(\frac{l}{L}\omega\right)H_0\left(\frac{l}{k}\omega\right)H\left(x,0;0,\frac{l}{k}\omega\right), \qquad (6)$$

where $\tilde{I}_{obj}\left(\frac{l}{L}\omega\right) = \int \mathrm{d}^2 r\langle A(\mathbf{r})A^*(\mathbf{r})\rangle\exp\left(-\frac{il}{L}\omega\mathbf{r}\right)$ and $\langle\tilde{I}_t(l,\omega)\rangle_{inc}$ are the spatial spectra of the object and its image; $H_0\left(\frac{l}{k}\omega\right) = \int \mathrm{d}^2\rho\; T(\rho)\; T\left(\rho-\frac{l}{k}\omega\right)$ and $H\left(x, 0; 0, \frac{l}{k}\omega\right)$ are the optical transfer functions of the optical system and the turbulent atmosphere,[7] respectively. The point-spread function (5) is spatially invariant and only depends on the difference in coordinates $\mathbf{r}$ and $\boldsymbol{\rho}''$, hence the incoherent image of the diffuse object is isoplanatic.

When illuminating the object with a coherent light

$$U_0(\mathbf{t}_1) = U_0 \exp\left(-\frac{t^2}{2a^2} - \frac{ik}{2F}t^2\right), \qquad (7)$$

where a and F are respectively the radius and the distance, at which the optical beam is focused, in the plane of sharp image $1 + \frac{L}{l} - \frac{L}{F_t} = 0$ we have the following expression for the mean intensity distribution

$$\langle I_t(l, \rho'')\rangle_{\rm coh} = \frac{4\pi}{k^2}\left(\frac{k}{2\pi L}\right)^4\left(\frac{k}{2\pi l}\right)^2 \int d^2r\langle A(\mathbf{r})A^*(\mathbf{r})\rangle I_{\rm coh}(\mathbf{r})\times$$

$$\times \int d^2\rho_{1,2}T(\rho_1)T(\rho_2)H(x,0;0,\rho_1-\rho_2)\times$$

$$\times \exp\left[-\frac{ik}{L}(\rho_1-\rho_2)\left(\mathbf{r}-\frac{L}{l}\hat{\rho}''\right)\right],$$

where

$$I_{\rm coh}(\mathbf{r}) =$$
$$= \pi a^2 \int d^2t \exp\left\{-\frac{t^2}{4a^2}\left[1+\Omega^2\left(1-\frac{L}{F}\right)^2\right] - \frac{ik}{L}\mathbf{tr}\right\} H(x,0;0,\mathbf{t}),$$

By applying the Fourier transform, we obtain

$$\langle \tilde{I}_t(l, \omega)\rangle_{\rm coh} =$$
$$= \frac{4\pi}{k^2}\left(\frac{k}{2\pi L}\right)^4 \tilde{I}_{\rm obj,coh}\left(\frac{l}{L}\omega\right) H_0\left(\frac{l}{k}\omega\right) H\left(x,0;0,\frac{l}{k}\omega\right), \qquad (8)$$

where

$$\tilde{I}_{\rm obj,coh}\left(\frac{l}{L}\omega\right) = \int d^2r\langle A(\mathbf{r})A^*(\mathbf{r})\rangle I_{\rm coh}(\mathbf{r}) \exp\left(-\frac{il}{L}\omega\mathbf{r}\right)$$

is the spatial spectrum of the illuminated part of the object. Comparing the expressions (6) and (8), we find that, in the case of coherent illumination, the image of the illuminated part of the diffuse object proves to be isoplanatic.[8] At $\Omega \to \infty$ (mode of infinite plane wave), the expressions (6) and (8) coincide accurate to a constant value.

Thus, in the case of incoherent illumination along an uncorrelated path the isoplanatic image of the whole object is formed while at a coherent illumination only of its illuminated part.

Taking into account the correlation between the counter directed waves at incoherent illumination, the expression for mean intensity of the image of the diffuse object in the plane $1+\frac{L}{l}-\frac{L}{F_t}=0$ comprises two terms; one of them is determined by Eq. (4) whereas the second one is

$$\langle I_t(l, \rho'')\rangle_{2,\rm inc} = \left(\frac{4\pi}{k^2}\right)^2\left(\frac{k}{2\pi L}\right)^4\left(\frac{k}{2\pi l}\right)^2\times$$

$$\times \int d^2r\langle A(\mathbf{r})A^*(\mathbf{r})\rangle \int d^2t I_0(\mathbf{t})\times$$

$$\times \int d^2\rho_{1,2}T(\rho_1)T(\rho_2)H(x,0;0,\mathbf{t}-\rho_2)H(x,0;0,\rho_1-\mathbf{t})\times$$

$$\times \exp\left\{-\frac{ik}{L}(\rho_1-\rho_2)\left(\mathbf{r}-\frac{L}{l}\hat{\rho}''\right)\right\}. \qquad (9)$$

For the spectral region we obtain

$$\langle \tilde{I}_t(l,\omega)\rangle_{2,\rm inc} = \left(\frac{4\pi}{k^2}\right)^2\left(\frac{k}{2\pi L}\right)^4 \tilde{I}_{\rm obj}\left(\frac{l}{L}\omega\right)\Phi(\omega),$$

where

$$\Phi(\omega) = \int d^2\rho\, T(\rho)\, T\left(\rho - \frac{l}{k}\omega\right) H_{\rm cor}\left(\rho, \frac{l}{k}\omega\right),$$

$$H_{\rm cor}\left(\rho, \frac{l}{k}\omega\right) =$$
$$= \int d^2t I_0(\mathbf{t})\, H\left(x,0;0,\mathbf{t}-\rho+\frac{l}{k}\omega\right) H(x,0;0,\rho-\mathbf{t}). \qquad (10)$$

By analogy with Eqs. (4)–(6) we can consider the function $\Phi(\omega)$ as a joint optical transfer function of the turbulent atmosphere and the optical system, caused by the correlation of the counter waves. In this case it is impossible to separate the contributions from the medium and the optical system.

According to the convolution theorem, it follows from Eq. (10) that

$$\tilde{H}_{\rm cor}\left(\nu, \frac{l}{k}\omega\right) = \tilde{I}_0(\nu)\,\tilde{M}(\nu, \omega),$$

where

$$\tilde{M}(\nu, \omega) =$$
$$= \int d^2\rho H\left(x, 0; 0, \rho + \frac{l}{k}\omega\right) H(x, 0; 0, \rho) \exp(-i\nu\rho).$$

From that it follows that

$$\Phi(\omega) = \int d^2\nu \tilde{I}_0(\nu)\tilde{M}(\nu,\omega)\int d^2\rho\, T(\rho)\, T\left(\rho-\frac{l}{k}\omega\right)\exp(i\nu\rho).$$

Thus, when illuminating a diffuse object with incoherent light through a correlated path, then the spatial spectrum of its image can be represented as a product of the object spectrum and the generalized optical transfer function of the following form

$$H^{\rm dif}_{\rm inc}(\omega) = H_0\left(\frac{l}{k}\omega\right) H\left(x, 0; 0, \frac{l}{k}\omega\right) + I^{-1}_{\rm inc}\Phi(\omega).$$

For the case when the diffuse object is illuminated with coherent light through a correlated path, our attempt to present the spatial spectrum of its image as a product of the spatial spectrum of the object's illuminated part and the generalized optical transfer function failed.

SHORT EXPOSURE IMAGE OF A COHERENTLY ILLUMINATED OBJECT

It is well known that the image of an object observed through the turbulent atmosphere is blurred due to diffraction on small-scale inhomogeneities of a medium and is shifted as a whole due to random refraction on large-scale inhomogeneities. There are two modes of image formation: long-exposure and short-exposure. In the first case the object's image is distorted due to both random diffraction and random refraction. In the second mode the image is distorted mainly due to diffraction on random inhomogeneities of a medium. The presence of an additional distorting factor namely, random shifts of the image as a whole, results in deterioration of the quality of long-exposure images as compared to short-exposure ones.

Different approaches to description of short-exposure images are known. Their main idea is to exclude distorting factors due to random shifts of the image as a whole from the general expressions for long-exposure images. Fried[9] was first who proposed to do this by correcting the tilted components of a wave front. In Refs. 10 and 11 the same task is achieved by using a moving coordinate system in the image plane. This system is related to instantaneous centroid of the image. Last and Tur[12] solved this problem using filtration of large-scale inhomogeneities of a medium in comparison with the telescope aperture. Note that in Refs. 9–12 short-exposure images were considered within the scope of the theory of incoherent images on the basis of the optical transfer function.

Let us assume[12] that short-exposure images are distorted due to spatial inhomogeneities whose size does not exceed the size of the telescope receiving lens. That means that the effective spectrum of turbulence in this case can be written as

$$\Phi_\varepsilon(\kappa)=A_0C_\varepsilon^2\kappa^{-11/3}\left[1-\exp\left(-\alpha a_t^2\kappa^2\right)\right]\exp(\kappa^2/\kappa_m^2)\,. \tag{11}$$

In this expression C_ε^2 is the structure constant of the medium dielectric constant fluctuations; α is the constant; $\kappa_m^{-1} \sim l_0$; l_0 is the inner scale of turbulence; $A_0 = 0.033$; a_t is the effective radius of the receiving lens of the telescope, whose amplitude transmission coefficient we approximate with the Gaussian function. The presentation of the turbulence spectrum in the form (11) is analogous to introduction of the effective outer scale of turbulence comparable with the radius of the telescope receiving lens. Detailed analysis of the outer scale of turbulence influence on the statistical characteristics of the image can be found in Ref. 13.

Taking into account Eq. (11) we have for the function $N(\rho)$ at $\rho\kappa_m \gg 1$:

$$N(\rho) = 2A_0C_\varepsilon^2\pi\,\frac{6}{5}\,\Gamma(1/6)\,\times$$

$$\times\left\{\Gamma^{-1}(11/6)2^{-5/3}\rho^{5/3} - \frac{5}{6}\,\frac{\rho^2\kappa_m^{1/3}}{4(1+\alpha a_t^2\,\kappa_m^2)^{1/6}}\right\}, \tag{12}$$

where $\Gamma(\gamma)$ is the gamma-function. Assuming that $a_t \to \infty$ in Eq. (12), we find that in the case of long-exposure image only the first term remains in braces in Eq. (12) for the function $N(\rho)$.

Thus, the expression for the short-exposure image can be derived from Eq. (1) with regard for Eq. (2) if in the expressions of the type (3) the function $N(\rho)$ is used in the form (12).

One can see from Eq. (12) that the spatial spectrum of mean intensity of the image can be presented in the form

$$S(l,\omega) = S_1(l,\omega) + S_2(l,\omega). \tag{13}$$

Then for a point object, for which

$$O(\rho,\,\mathbf{r}) = \frac{4\pi}{k^2}\,\delta(\rho)\,\delta(\rho - \mathbf{r})\,,$$

we have

$$S_1(l,\,\omega) = \mathrm{const}\,(g^2 + 2p_1 - 2p_2)^{-1}\,\times$$

$$\times\exp\left[-\frac{\omega^2}{\omega_0^2}\left(1+\Omega_t^2Q^2+2p_1\Omega_t/\Omega-2p_2\Omega_t/\Omega\right)\right], \tag{14}$$

$$S_2(l,\,\omega) = \mathrm{const}\cdot A\cdot B\exp\left(-C\,\frac{\omega^2}{\omega_0^2}\right). \tag{15}$$

In Eqs. (14) and (15) we introduced the following designations: $\omega_0=2a_tk/l$, $g^2=1+\Omega^2(1-L/F)^2$, $Q = 1 + L(1/l - 1/F_t)$, $p_1 = 2\gamma_1^{6/5}\Omega\,\beta_0^{12/5}$, $p_2 = 2\theta\gamma_2\kappa_m^{1/3}a^{1/3}\Omega^{6/5}\beta_0^2/(1+\alpha a_t^2\kappa_m^2)^{1/6}$, $A = [b_1^2 + \Omega^2(1 - L/F)^2]^{-1}$, $B = b_2 - \Omega_t[1 - A\,\Omega^2(1 - L/F)^2]\times(p_1 - p_2)^2/(\Omega b_1)$, $C = b_2 - A\Omega_tb_1(p_1 - p_2)^2/\Omega + {}$ $+\,\Omega_t^2\,[Q - A(1 - L/F)^2(p_1 - p_2)^2]/B$, $b_1 = 1 + p_1 - p_2$, $b_2 = 1 + \Omega_t(p_1 - p_2)/\Omega$, $\gamma_1 = 0.442$, $\gamma_2 = 0.244$, $\Omega = ka^2/L$ and $\Omega_t = ka_t^2/L$ are the Fresnel numbers of the emitting aperture and the telescope, respectively; $\beta_0^2 = 0.31\,C_\varepsilon^2\,k^{7/6}L^{11/6}$ is the parameter characterizing the turbulent conditions of propagation along the path and in the case of strong fluctuations under consideration it takes the values $\beta_0^2 \gg 1$. The parameter θ equals zero at short exposure and unity at long exposure. The analysis of coherent images in the long-exposure mode ($\theta = 0$) was performed earlier in Ref. 14.

Let us estimate the enhancement in the image quality when using short-exposure images instead of long-exposure ones. Using Eqs. (14) and (15) we calculate the functional of the image quality[15]

$$\theta(l) = \frac{\int d^2\omega\omega^2 \mid N(l,\omega) \mid^2}{\int d^2\omega \mid N(l,\omega) \mid^2},$$

where $N(l, \omega)$ is the normalized spatial spectrum of the mean intensity of an optical wave in the plane l behind the telescope receiving lens:

$$N(l, \omega) = S(l,\omega)/S(l, 0),$$

$S(l, \omega) = \int d^2\rho'' \langle I(l, \rho'') \rangle \exp(i\omega\rho'')$ is the spatial spectrum of the mean intensity. Let us introduce the ratio $M = \theta_{short}(l)/\theta_{long}(l)$ of the image quality functional at a short exposure θ_{short} to that at a long exposure without correlation of counter waves $\theta_{long}(l)$ as a measure of quality of a short-exposure image.

Figure 2 shows M as a function of the Fresnel number of the coherent source illuminating the point object at different values of the Fresnel number of the telescope receiving lens. The parameter β_0^2 equals 49. First it is seen from the figure that, in both exposure modes (we analyzed the parameter M in the long-exposure mode in Ref. 14) the correlation between counter waves leads to the enhancement of the point object image quality at matched transceiving apertures ($\Omega = \Omega_t$) and has no influence on the image quality when apertures are not matched ($\Omega \ll \Omega_t$ or $\Omega \gg \Omega_t$). Secondly the increase in the image quality when going from the long-exposure mode to the short-exposure one (if both the first and second terms are considered in the spatial spectrum of the mean intensity of the image) is greater than in the case when only the first term is considered in the spatial spectrum.

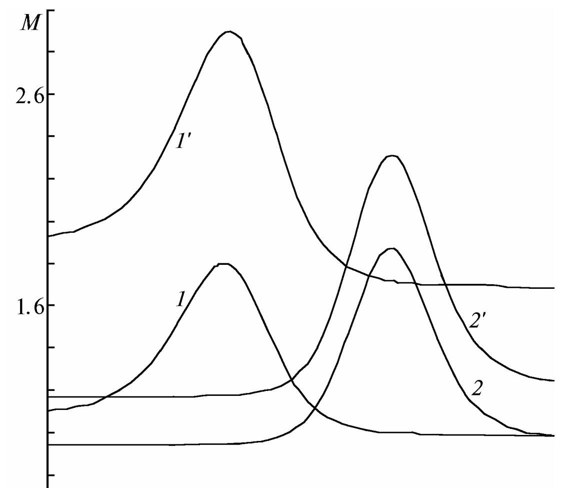

FIG. 2. The parameter $M = Q_{short}(l)/Q_{long}(l)$ for the point object versus the Fresnel number of the coherent source, $\beta_0^2 = 49$: long exposure (curves 1 and 2), short exposure (curves 1′ and 2′), $\Omega_t = 0.1$ (1 and 1′), $\Omega_t = 10$ (2 and 2′).

Let us consider how the effect of backscattering amplification manifests itself in the image of a two-point object. Let us write the reflection coefficient in the form

$$O(\rho, \mathbf{r}) = \frac{2\pi}{k^2} [\delta(\mathbf{r} - \mathbf{r}_0) + \delta(\mathbf{r} + \mathbf{r}_0)]\, \delta(\rho - \mathbf{r}), \qquad (16)$$

where $2\mathbf{r}_0$ is the distance between the two point objects. Using Eqs. (1)–(3), (11), (12), and (16), the expression for the mean intensity distribution in the image of such an object can be readily derived.

The mean intensity distribution in the image of two-point objects calculated at different Fresnel numbers of the emitting and receiving apertures is shown in Figs. 3 and 4. The x axis shows the distance in the plane normal to the telescope optical axis, normalized to l/ka_t.

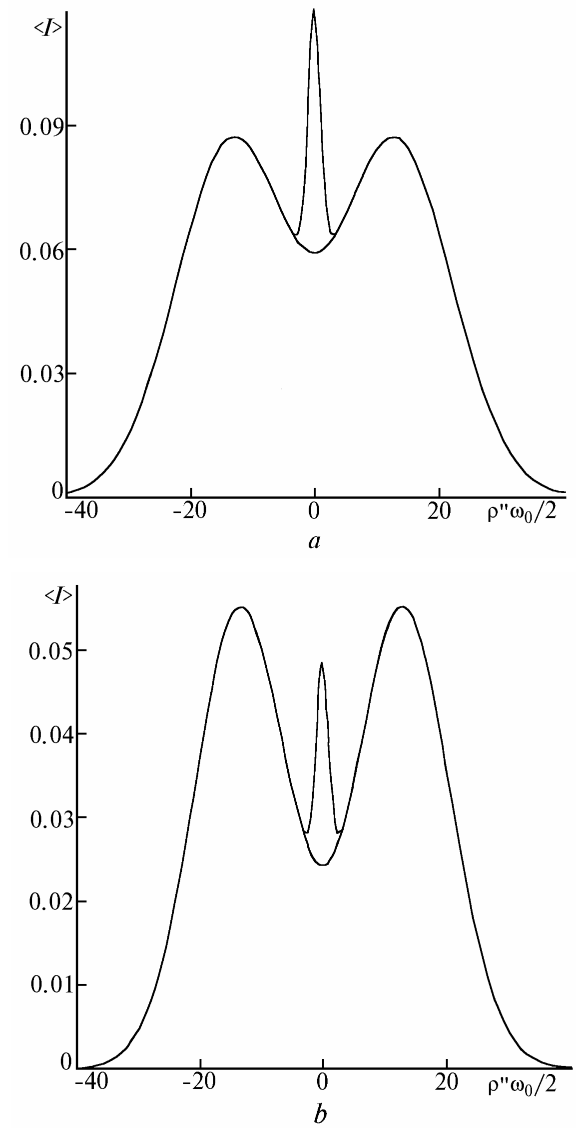

FIG. 3. Intensity distribution in the image of a two-point object at $r_0/\rho_n = 150$, $\Omega = \Omega_t = 1$, $\beta_0^2 = 49$: long exposure (a), short exposure (b).

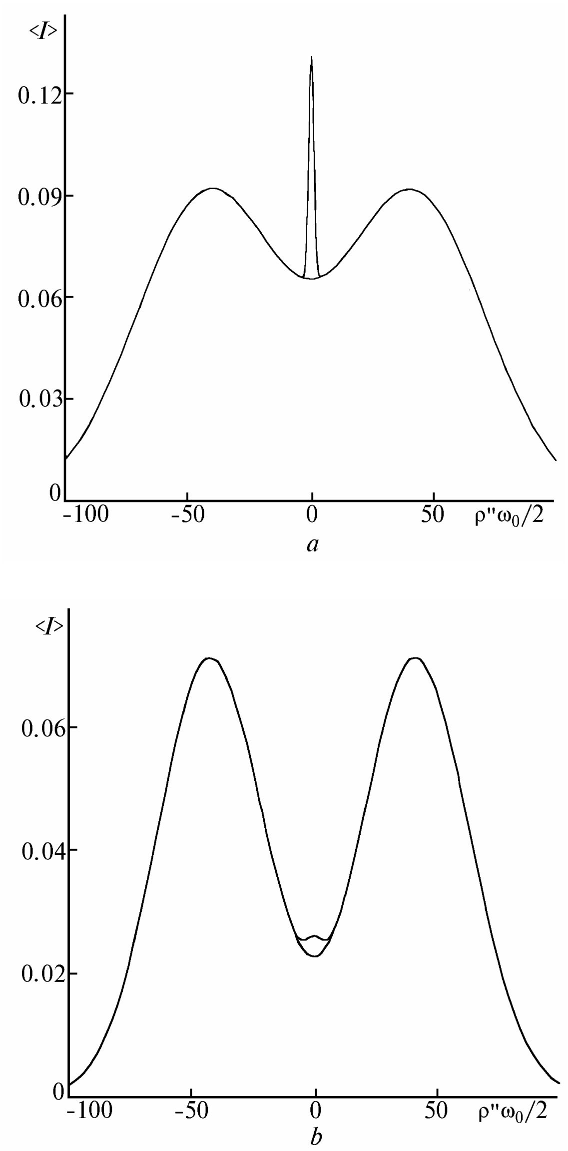

FIG. 4. Intensity distribution in the image of a two-point object at $r_0/\rho_n = 150$, $\Omega = \Omega_t = 10$, $\beta_0^2 = 49$: long exposure (a), short exposure (b).

One can see from the figures that the transition from the long exposures to the short ones, as one would expect, results in higher resolution in the image of a two-point object (depth of the dip in Figs. 3 and 4 increases). The account for correlation between the counter waves gives an additional peak located strictly at the optical axis of the telescope. This fact can be used for a fine target indication and object tracking.[16] In contrast to the long-exposure mode, in the case of short exposure the peak amplitude decreases fast depending on the Fresnel number of transceiving apertures. Thus, if at $\Omega = \Omega_t = 1$ the peak amplitude is maximum in both exposure modes, then at $\Omega = \Omega_t = 10$ the peak amplitude is practically zero in the short-exposure mode and comparable with the magnitude of the mean intensity of the point object image in the long-exposure mode.

The peak width is proportional to L/ka_t, while the width of the point object image is determined by diffraction on the receiving aperture of the telescope with turbulent broadening and proportional[14] to $\nu \sim (L/ka_t)\left(1 + \Omega_t^2 + 2p_1\Omega_t/\Omega\right)^{1/2}$. From the comparison of the effective widths of the peak and the image of the point object, the parameter β_0^2 can be determined that characterizes the turbulent conditions of propagation along the path.

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