SOUNDING OF REFRACTION CHANNELS

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A structure of a sounding beam of optical radiation passing through the refraction channel at an angle of 90° to the channel optical axis is treated theoretically in the paper. An analytical solution has been obtained for the second–order mutual–coherence function of the sounding radiation field based on the Huygen––Kirchhoff method. The conditions have been determined when the maximum sensitivity of characteristics of a sounding optical beam to the parameters of refraction channel realizes. The wave–front bend of the sounding beam is shown to be large enough to ensure an acceptable accuracy when measuring the parameters of the refraction channel. The conditions of small aberration distortions of a sounding beam are given.

It has been known that at thermal blooming[1,2] or resonance self–action[3,4] of intense optical radiation, there occur zones with regular variation of the medium refractive index, namely, refraction channels.[1–4] The demand for the information on characteristics of refraction channels, that is, the diameter d_c and variation of relative dielectric constant of the medium at the axis of refraction channel ε_2, can appear in non–contact metrology of intense optical radiation,[5–7] calorimetric spectroscopy of substances,[8,9] and adaptive correction of distortions of intense optical radiation.[10]

In recent years the optical methods have been widely used when sounding the refraction channels. The refraction channel is generally sounded by radiation along the optical axis.[1–7] In this case the refraction channel is considered either as the nonaberrational lenticular medium[1,2,5–7] or as the medium with small aberrations.[11] However, such a scheme of measurement is not always realized in practice. This is connected both with the need for overcoming large technical difficulties concerning the input and output of sounding radiation in the channel of intense optical radiation and out of it and with strong influence of aberrations of the refraction channel on the measured characteristics of sounding radiation.

To overcome the above–mentioned difficulties the methods of side sounding of refraction channels are used.[12,13] This scheme was used for measuring the concentration and rate of motion of an absorbing matter from the variation of intensity of sounding radiation behind the point diaphragm (method of thermolens) or the position of center of gravity of a sounding laser beam (mirage–effect). This paper describes the theoretical study of the structure of a sounding pulse of optical radiation passing through the refraction channel perpendicular to its optical axis.

Without limiting the commonness of statement of the problem we consider the following geometric diagram of a meter. Let the beam source of sounding optical radiation be in the origin of coordinates and emit in the positive direction of the axis OX. The field of the sounding radiation source in the plane $x = 0$ is given as the Gaussian beam

$$E(0,\ \rho) = E_0 \exp\left\{-\frac{\rho^2}{2\,a_0^2} - \frac{i\,\kappa}{2\,R_0}\,\rho^2\right\}, \qquad (1)$$

where E_0 is the initial amplitude of the sounding radiation; a_0 is the initial radius of a sounding beam; R_0 is the radius of curvature of the wave front in the center of emitting aperture; $k = 2\pi/\lambda$; λ is the wavelength of sounding radiation in vacuum; $\rho = \{y,\ z\}$ is the coordinate transverse to the direction of propagation of sounding radiation.

We consider the case of a cylindrical refraction channel with an arbitrary profile of variation of dielectric constant of the medium $\varepsilon_2(x,\ \rho)$ in the limited region close to the optical axis of the channel, from x_1 to x_2 ($|x_1 - x_2| \simeq d_c \ll x_0$) and with the optical axis parallel to the axis OY and intersecting the axis OX at the point $x = x_0$ and the axis OZ at the point $z = z_0$. The field from the source of sounding radiation (1) close to the channel ($x \simeq x_0$) is of the form:

$$E(x_0,\ \rho) = \frac{E_0 \exp\,(i\,\kappa\,x_0)}{1 - \dfrac{x_0}{R_0} + i\,\dfrac{x_0}{\kappa\,a_0^2}} \exp\,\{-\,g(x_0)\,\rho^2\}, \qquad (2)$$

where

$$g(x) = (1 + i\,\kappa a_0^2\,/\,R_0)\,/\,[2\,a_0^2\,(1 - x\,/\,R_0 + i\,x\,/\,\kappa\,a_0^2)].$$

To find the sounding radiation field at $x > x_0$, that is, passed through the refraction channel, we use the Huygen––Kirchhoff formula:

$$E(x,\ \rho) = \frac{\kappa \exp\,[i\,\kappa(x - x_0)]}{2\pi\,i(x - x_0)} \int\!\!\int_{-\infty}^{\infty} \mathrm{d}\,\rho'\,E(x_0,\ \rho')\times$$

$$\times \exp\left\{\frac{i\,\kappa}{2\,(x - x_0)}\,(\rho - \rho')^2 + \frac{i\,\kappa}{2}\int_{x_1}^{x_2} \mathrm{d}\,x'\,\varepsilon_2(x',\ \rho')\right\}.$$

Since the scales of variation of the refraction channel parameters along its optical axis (OY axis) are large as compared with the sounding radiation beam radius, then the values of dielectric constant of the medium can be considered independent of the coordinate y. In this case the second–order mutual–coherence function of the sounding radiation beam is written as

$$\Gamma_2(x, \rho_1, \rho_2) = E(x, \rho_1)\, E^*(x, \rho_2) =$$

$$= \frac{\kappa^2}{4\pi^2(x - x_0)^2} \int\limits_{-\infty}^{\infty}\!\!\int \mathrm{d}\,\rho' \int\limits_{-\infty}^{\infty}\!\!\int \mathrm{d}\,\rho''\, E(x_0, \rho')\, E^*(x_0, \rho'') \times$$

$$\times \exp\left\{\frac{i\,\kappa}{2\,(x - x_0)}(\rho_1 - \rho')^2 - \frac{i\,\kappa}{2\,(x - x_0)}(\rho_2 - \rho'')^2 + \right.$$

$$\left. + \frac{i\,\kappa}{2}\int\limits_{x_1}^{x_2} \mathrm{d}\,x'\,[\varepsilon_2(x', 0, z') - \varepsilon_2(x', 0, z'')]\right\}. \qquad (3)$$

Having substituted (2) into (3) we calculate the integrals over the variables y' and y''

$$\Gamma_2(x, y_1, y_2, z_1, z_2) = \frac{E_0^2\,\kappa\,a_0^2}{2\pi(x - x_0)\, a_y(x)\, a(x_0)} \times$$

$$\times \exp\left\{-\frac{y_1^2 + y_2^2}{2\,a_y^2(x)} - \frac{i\,\kappa}{2}\, S_y(x)\,(y_1^2 - y_2^2)\right\} \int\limits_{-\infty}^{\infty} \mathrm{d}\,z' \int\limits_{-\infty}^{\infty} \mathrm{d}\,z'' \times$$

$$\times \exp\left\{- g(x_0)\frac{(z')^2}{2\,a_0^2} - g^*(x_0)\frac{(z'')^2}{2\,a_0^2} + \frac{i\,\kappa(z_1 - z')^2}{2\,(x - x_0)} - \frac{i\,\kappa(z_2 - z'')^2}{2\,(x - x_0)} + \right.$$

$$\left. + \frac{i\,\kappa}{2}\int\limits_{x_1}^{x_2} \mathrm{d}\,x'\,[\varepsilon_2(x', 0, z') - \varepsilon_2(x', 0, z'')]\right\}, \qquad (4)$$

where $a(x_0) = a_0\sqrt{(1 - \frac{x_0}{x}\,\mu)^2 + (x_0\,/\,x)^2\,\Omega_0^{-2}}$ is the sounding radiation beam radius at the input of refraction channel; $a_y(x) = a_0\sqrt{(1 - \mu)^2 + \Omega_0^{-2}}$ is the ranning radius of the sounding radiation beam along the axis OY; $S_y(x) = (1\,/\,x)\,[(1 - \mu)\,\mu - \Omega_0^{-2}]\,/\,[(1 - \mu)^2 + \Omega_0^{-2}]$ is the running curvature of the wave front of sounding radiation along the axis OY; $\mu = x/R_0$ is the parameter of the beam initial focussing; and, $\Omega_0 = \kappa\,a_0^2/x$ is the Fresnel number of transmitting aperture.

Representing the remainder of dielectric constants of the medium in (4) as the Taylor expansion, that is possible when fulfilling the conditions $a_0 \ll d_c$ and $z_0 \ll d_c$,

$$\varepsilon_2(x', 0, z') - \varepsilon_2(x', 0, z'') \simeq \left.\frac{\partial\varepsilon(x', z)}{\partial z}\right|_{z=0} (z' - z'') +$$

$$+ \frac{1}{2}\left.\frac{\partial^2\varepsilon(x', z)}{\partial z^2}\right|_{z=0} (z'^2 - z''^2) + \frac{1}{6}\left.\frac{\partial^3\varepsilon(x', z)}{\partial z^3}\right|_{z=0} (z'^3 - z''^3) +$$

$$+ \frac{1}{24}\left.\frac{\partial^4\varepsilon(x', z)}{\partial z^4}\right|_{z=0} (z'^4 - z''^4) + \dots , \qquad (5)$$

and having limited by two terms of the expansion (5), one can calculate the integrals over z' and z'' in Eq. (4). As a result, for the second–order mutual–coherence function of the sounding radiation beam passed through the refraction channel we obtain the simple analytical expression:

$$\Gamma_2(x, y_1, y_2, z_1, z_2) = E(x, y_1, z_1)\, E^*(x, y_2, z_2) = \frac{E_0^2\, a_0^2}{a_y(x)\, a_z(x)} \times$$

$$\times \exp\left\{-\frac{y_1^2 + y_2^2}{2\,a_y^2(x)} - \frac{i\,\kappa}{2}\, S_y(x)\,(y_1^2 - y_2^2) - \frac{(z_1 - \alpha)^2 + (z_2 - \alpha)^2}{2\,a_z^2(x)} - \right.$$

$$\left. - \frac{i\,\kappa}{2}\, S_z(x)\,(z_1^2 - z_2^2) + i\,\kappa\varphi(x)\frac{\alpha}{(x - x_0)}(z_1 + z_2)\right\}, \qquad (6)$$

where $a_z(x) = a_0\sqrt{[(1 - \mu) - (1 - \frac{x_0}{x}\,\mu)\,\mu_c]^2 + \Omega_0^{-2}\,[1 - \frac{x_0}{x}\,\mu_c]^2}$ is the running radius of the sounding radiation beam along the OZ axis; $S_z(x) = -\dfrac{1}{a_z(x)}\dfrac{\mathrm{d}\,a_z(x)}{\mathrm{d}\,x}$ is the running curvature of the wave–front of sounding radiation along the axis OZ; $\varphi(x) = -\dfrac{1}{a_z(x)}\dfrac{\delta\,a_z(x)}{\delta\,\mu_c}$ is the running regular wave–front tilt due to linear inhomogeneity of the profile of refraction channel; $\mu_c = (x - x_0)/R_c$ is the parameter of the refraction channel focussing; $R_c = -\,2\,\Big/\displaystyle\int\limits_{x_1}^{x_2} \mathrm{d}\,x' \left.\frac{\partial^2\varepsilon(x', z)}{\partial z^2}\right|_{z=0}$ is the focal length of refraction channel in the direction of the axis OZ; and, $\alpha = \dfrac{1}{2}\displaystyle\int\limits_{x_1}^{x_2} \mathrm{d}\,x' \left.\frac{\partial\,\varepsilon(x', z)}{\partial\,z}\right|_{z=0} (x - x_0)$ is the linear shift of coordinate of the gravity center of a sounding beam.

Analysis of the second–order mutual–coherence function of the sounding radiation field (6) shows that passing through a limited region with a varying value of dielectric constant of the medium results in the additional, as compared with propagation in a homogeneous medium, curvature of wave–front of sounding radiation along the axis OZ (the axis perpendicular to the direction of propagation of the sounding radiation beam and optical axis of refraction channel). The wave–front deformation, in its turn, causes additional change of the radius value of the sounding radiation beam along the axis OZ, which increases as it propagates in the homogeneous medium after intersecting the refraction channel. The characteristics of a sounding beam in the direction of the axis OY (parallel to the optical axis) do not differ from the corresponding characteristics of the beam propagating through the free space. The above–mentioned effects manifest themselves only at $\left.\dfrac{\partial^2\varepsilon(x', z)}{\partial\,z^2}\right|_{z=0} \neq 0$, that is, in this case the refraction channel possesses the qualities of a cylindrical lens. If $\left.\dfrac{\partial\varepsilon(x', z)}{\partial\,z}\right|_{z=0} \neq 0$, then except the phenomena mentioned, there appears an additional wave–front tilt and connected with it shift of center of gravity of a sounding beam, that is similar to the action of an optical wedge.

For a wide beam, in a diffraction sense ($\Omega_0 \gg 1$), when the change of characteristics of the sounding beam because of diffraction effect can be neglected, the running values of the radius and curvature of the beam wave front of sounding radiation along the axis OZ are equal, respectively

$$a_z(x) \simeq a_0 \; |(1-\mu) - (1 - \frac{\mu x_0}{x})\, \mu_c|,$$

$$S_z(x) \simeq -(1/R_0 + (1 - x_0/R_0)/R_c)/((1-\mu) - (1 - \frac{\mu x_0}{x})\, \mu_c).$$

It should be noted that for a sounding beam focused at a point close to the optical axis of the refraction channel ($R_0 = x_0$), its influence is minimum: $a_z(x) \simeq a_0 |1 - x/x_0|$ and $S_z(x) \simeq -1/x_0(1 - x/x_0)^{-1}$. In the limit of $x \gg x_0$ the transition to a spherical wave $a_z(x) \gg a_0$, $S_z(x) \simeq 1/x$ occurs. In the case of a collimated beam ($R_0 = \infty$, that is, $\mu = 0$) the parameters of a sounding beam have the following form:

$$a_z(x) \simeq a_0 \; |1 - \mu_c| = a_0 \left| 1 - \frac{x - x_0}{R_c} \right|, \tag{7}$$

$$S_z(x) \simeq -\frac{1}{R_c} \frac{1}{1 - \mu_c} = -\frac{1}{R_c - (x - x_0)} \,. \tag{8}$$

For the sounding beam focused at a point of observation ($R_0 = x$) the influence of the refraction channel will be described by the formulas:

$$a_z(x) \simeq a_0 \, (x - x_0)^2 / x \, R_c \,, \tag{9}$$

$$S_z(x) \simeq [R_c + (x - x_0)] / (x - x_0)^2. \tag{10}$$

From Eqs. (7) and (9) the conclusion should be drawn that for measuring the parameters of refraction channels by the thermolens method[8,9] consisting in the recording of the sounding radiation intensity through a point diaphragm on the beam optical axis, either a collimated sounding beam at $(x - x_0) \gg R_c$, when a useful signal is proportional to

$$1 / \sqrt{a_z(x)} = \sqrt{R_c / a_0 \, (x - x_0)},$$

or the sounding beam, focused at a point of observation at $R_c < k\, a_0^2$, when

$$1 / \sqrt{a_z(x)} = \sqrt{x \, R_c / a_0 \, (x - x_0)^2},$$

can be used. The method of the mirage–effect[8,9,12,13] based on the measurement of the shift of coordinate of the center of gravity of a sounding beam

$$\alpha = \frac{1}{2} \int_{x_1}^{x_2} \mathrm{d}\, x' \left.\frac{\partial\, \varepsilon(x', z)}{\partial z}\right|_{z=0} (x - x_0)$$

depends weakly on the sounding beam parameters. As one can see from the above–mentioned expression, the sensitivity of these methods increases with the growth in x. In contrast to this the method of image overfocussing, consisting in recording of the wave–front deformation, is applicable when $(x - x_0) \lesssim R_c$. From Eqs. (8) and (10) it follows that under these conditions for the collimated beam $S_z(x) \simeq 1/R_c$, and for the focused one $S_z(x) = \dfrac{R_c}{(x - x_0)^2}$. It has been known[5–7] that the change of the wave–front curvature causes the shift of the plane of a sharp image relative to the focal plane of lens (overfocussing) by the value

$$\Delta = F_L^2 / S_z(z),$$

where F_L is the focal length of the receiving lens. Consequently, in the considered case of the wide collimated beam (8)

$$\Delta = -F_L^2 / R_c,$$

and for a beam focused at an observation point (10)

$$\Delta = \frac{F_L^2 \, R_c}{(x - x_0)^2} \,,$$

that is, when measuring the shift of plane of sharp image, the use of a collimated beam should be preferable.

If the refraction channel has the axiosymmetrical Gaussian profile of dielectric constant variation

$$\varepsilon(x, y, z) = \varepsilon_0 + \varepsilon_2 \exp\{-4\,[(x - x_0)^2 / d_c^2] - 4\,[(z - z_0)^2 / d_c^2]\},$$

it appears that the linear shift of the coordinate of center of gravity of the sounding beam equals

$$\alpha = \frac{1}{2} \int_{x_1}^{x_2} \mathrm{d}\, x' \left.\frac{\partial\, \varepsilon(x', z)}{\partial z}\right|_{z=0} (x - x_0) = 2\sqrt{\pi}\, \Phi(1)\, \varepsilon_2 \frac{z_0}{d_c} (x - x_0),$$

where $\Phi(1) = 0.8427$; and the focal length of the refraction channel is determined as follows:

$$R_c = \frac{1}{2\sqrt{\pi}\, \Phi(1)} \frac{d_c}{\varepsilon_2} \,.$$

Then for the refraction channel with $d_c = 10^{-2}$ m and $\varepsilon_2 = 10^{-5}$ at the focal length of the receiving lens $F_L = 10$ m the value of the parameter R_c will be about several hundred meters and the shift of the sharp image plane will be about some tens of centimeters. Measurements of overfocussing Δ of such a value can be performed with high precision. When using the expansion (5) it is not difficult to show that the influence of aberrations can be neglected if

$$\left| \int_{x_1}^{x_2} \mathrm{d}\, x' \left.\frac{\partial^3 \varepsilon(x', z)}{\partial z^3}\right|_{z=0} \right| \ll \frac{6}{\sqrt{2}} \frac{1}{\kappa\, a_0^3(x_0)} \,, \text{ and}$$

$$\int_{x_1}^{x_2} \mathrm{d}\, x' \left.\frac{\partial^4 \varepsilon(x', z)}{\partial z^4}\right|_{z=0} \ll 12 \frac{1}{\kappa\, a_0^4(x_0)} \,.$$

The estimates indicate, for the near axial region of the refraction channel of the Gaussian profile of dielectric constant of the medium, the aberration distortions are few in number for all conditions realized in practice.

Thus, it appears that at side sounding of the refraction channels the deformation of wave front of the sounding beam is sufficiently large to ensure the acceptable accuracy of measurement of the refraction channel parameters.

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