STATISTICAL MODEL OF THE TRANSFER OF AN OPTICAL IMAGE THROUGH THE EARTH'S ATMOSPHERE

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A statistical theory of the transfer of an optical image through the earth's atmosphere is developed. The distribution functions of the following characteristics are found: the brightness of atmospheric haze, the optical transfer functions, and the brightness of radiation reflected by the ground – atmosphere system.

An entire series of works devoted to the analysis of the transfer of an optical image through the earth's atmosphere has now been published (see, for example, Refs. 1 and 2). All of these works employed a determinate approach, in which the earth's atmosphere is regarded as a linear imaging system with determinate scattering parameters. Such a model is a quite rough approximation to reality, since it neglects the actually observed random fluctuations of the parameters of the atmosphere in time and space.[3,4] As a result of fluctuations of this type the characteristics of the atmosphere which determine the conditions of optical image transfer through it also fluctuate in a random manner. This paper is devoted to a study of the statistical characteristics of these fluctuations.

We shall study the following characteristics which are required in order to predict the quality of an optical image: the brightness of atmospheric haze – D; the diffuse transmittance – T; the optical transfer function (OTF) of the atmosphere – $\tau(\omega)$; the OTF of the system atmosphere-underlying ground – $\tau_p(\omega)$; and, the brightness of the radiation reflected by the system atmosphere-ground – I. We shall use the following statistical model of the atmosphere: the scattering phase function $i(\cos\gamma)$ and the photon survival probability Λ in a single scattering event are independent of the altitude and they are determinate functions, and the random fluctuations of the scattering coefficient $\sigma(z)$ of the atmosphere as a function of the altitude have an average value $\langle\sigma(z)\rangle$ and are characterized by the correlation function $R_{\sigma\sigma}(z_1;\ z_2)$. It is obvious that on the basis of this model it is assumed that the fluctuations of the scattering parameters as a function of altitude are determined solely by the change in the concentration of scattering particles. We shall neglect the horizontal fluctuations of the scattering parameters of the atmosphere.

The random realizations of the characteristics studied can be found based on Refs. 5 and 6 and they have the following forms

$$D = Bm/\alpha \left[1 - \exp\left(-\alpha\tau_\sigma/m\right)\right]; \tag{1}$$

$$T(\mu) = \exp\left(-\alpha\tau_\sigma/\mu\right); \tag{2}$$

$$\tau(\omega) = \exp\left[-(1-\Phi)\tau_f(\omega)/\mu\right]; \tag{3}$$

$$\tau_p(\omega) = \tau(\omega)/\left[1 + \left(\pi D/\beta_0 E_0 T(\mu_0)T(\mu)\right)\right]; \tag{4}$$

$$I = D + \frac{\beta_0}{\pi}\frac{E_0}{1 - \dfrac{\beta_0 A}{\pi}}T(\mu_0)T(\mu), \tag{5}$$

where

$$B = \frac{\Lambda i(\Omega\cdot\Omega_0)E_0}{4\pi\mu_0\mu}, \quad \alpha = 1 - \Lambda^*, \quad \Lambda^* = \Lambda(1-\Phi),$$

$$\tau_\sigma = \int_0^{z_s}\sigma(u)\,du, \quad \tau_f(\omega) = \int_0^{z_s}\sigma(u)F(\omega u)\,du,$$

$$F(\omega u) = 1 - \frac{1}{2}\int_0^{\pi/2} i(\gamma)\gamma J_0\left(\frac{\omega u}{\mu}\gamma\right)d\gamma, \quad m^{-1} = \mu_0^{-1} + \mu^{-1},$$

Φ is the relative fraction of the light that is scattered backwards in a single scattering act; $\beta_0 = \beta_0(\Omega;\ \Omega_0)$ is the luminance factor of the ground; Ω and Ω_0 are unit vectors, which determine the directions of observation and illumination; A is the albedo of the layer of atmosphere; E_0 is the solar illuminance at the top of the atmosphere; μ and μ_0 are the direction cosines of the vectors Ω and Ω_0 with vertical axis z; $J_0(x)$ is a Bessel function; and, z_s is the thickness of the layer of atmosphere.

Analysis of Eqs. (1)–(5) shows that fluctuations of all of the characteristics studied are caused by fluctuations of the two quantities, τ_σ and τ_f, which depend on the profile $\sigma(z)$. Assuming that the distribution

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function (probability density) of these quantities $f(\tau_\sigma,\ \tau_f)$ is known, from Eqs. (1)–(3) we obtain the following expressions for the distribution functions of the brightness of atmospheric haze $f_d(D)$, the diffuse transmittance $f_t(T)$, and the OTF $f_\tau(\tau;\ \omega)$:

$$f_d(D) = \int_0^\infty \frac{d\tau_f}{B - \frac{\alpha D}{m}} f\left[-\frac{m}{\alpha}\ln\left(1 - \frac{\alpha D}{Bm}\right);\ \tau_f\right]; \tag{6}$$

$$f_t(T) = \int_0^\infty \frac{\mu d\tau_f}{\alpha T} f\left(-\frac{\mu}{\alpha}\ln T;\ \tau_f\right); \tag{7}$$

$$f_\tau(\tau;\ \omega) = \int_0^\infty \frac{\mu\ d\tau_\sigma}{(1-\Phi)\tau_\omega} f\left[\tau_\sigma;\ -\frac{\mu\ln\tau_\omega}{1-\Phi}\right], \tag{8}$$

where $\tau_\omega = \tau(\omega)$.

Neglecting the very weak effect of fluctuations of the albedo of the atmosphere A on the OTF $\tau_p(\omega)$, from Eq. (4) we obtain the following expression for the distribution function of the OTF $\tau_p(\omega)$ with determinate values of the luminance factor of the ground:

$$f_{\tau_p}(\tau;\ \omega) = \int_0^\infty \frac{\mu d\tau_\sigma}{\tau_{p\omega}} \times$$

$$\times f\left[\tau_\sigma;\ -\mu\ln\tau_{p\omega}\frac{1 + \exp(-\alpha\tau_\sigma/m)(\beta_0 C - 1)}{\beta_0 C\exp(-\alpha\tau_\sigma/m)}\right], \tag{9}$$

where

$$\tau_{p\omega} = \tau_p(\omega),\quad C = \frac{4\mu_0\mu\alpha}{\Lambda i(\Omega\cdot\Omega_0)m\left[1 - \frac{\beta_0\langle A\rangle}{\pi}\right]},$$

and the angular brackets denote averaging.

The distribution function of the brightness of the radiation reflected by the system atmosphere-ground $f_1(I)$ can be found in the more general case when the luminance factor of the ground β_0 is a random quantity with the distribution $f_1(\beta_0)$. Then

$$f_1(I) = \int_0^\infty\int_0^\infty \frac{m\ d\beta_0 d\tau_f}{|Bm - \alpha I|} f_1(\beta_0) f\left(\frac{m}{\alpha}\ln\frac{G\beta_0\alpha - Bm}{\alpha I - Bm};\ \tau_f\right), \tag{10}$$

where $G = \dfrac{E_0}{\pi - \langle\beta_0\rangle\langle A\rangle}$. In deriving Eq. (10), analogously to Eq. (9), we neglected in Eq. (5) the insignificant effect of the fluctuations of the term $1 - \dfrac{\beta_0 A}{\pi}$ on the brightness of the reflected radiation.

In many cases, to calculate the distribution functions it is sufficient to know instead of the distribution function $f(\tau_\sigma;\ \tau_f)$ the distribution function of the random quantity τ_σ

$$f_\sigma(\tau_\sigma) = \int_0^\infty d\tau_f f(\tau_\sigma;\ \tau_f)$$

and the distribution function of the random quantity τ_f

$$f_\omega(\tau_f) = \int_0^\infty d\tau_\sigma f(\tau_\sigma;\ \tau_f).$$

Then

$$f_d(D) = \frac{f_\sigma\left[-\frac{m}{\alpha}\ln\left(1 - \frac{\alpha D}{Bm}\right)\right]}{B\left(1 - \frac{\alpha D}{Bm}\right)};$$

$$f_t(T) = \frac{\mu}{\alpha T} f_\sigma\left(-\frac{\mu}{\alpha}\ln T\right); \tag{11}$$

$$f_\tau(\tau;\ \omega) = \frac{\mu}{(1-\Phi)\tau_\omega} f_\omega\left(-\frac{\mu}{1-\Phi}\ln\tau_\omega\right).$$

In the case when the fluctuations of the scattering parameters are not too large the normal distribution function can be used to calculate the fluctuations of the characteristics of light fields in the atmosphere:

$$f(\tau_\sigma;\ \tau_f) = (2\pi)^{-1}\left(\sigma_\tau^2\sigma_f^2 - R_{\sigma f}\right)^{-1/2} \times$$

$$\times \exp\left\{-2^{-1}\left(\sigma_\tau\sigma_f - R_{\sigma f}\right)^{-1}\left[\sigma_f^2(\tau_\sigma - \langle\tau_\sigma\rangle)^2 - \right.\right.$$

$$\left.\left. - 2R_{\sigma f}(\tau_\sigma - \langle\tau_\sigma\rangle)(\tau_f - \langle\tau_f\rangle) + \sigma_\tau^2(\tau_f - \langle\tau_f\rangle)^2\right]\right\},$$

where $\langle\tau_\sigma\rangle$, σ_τ^2 are the average value and the variance, respectively, of τ_σ and $\langle\tau_f\rangle$ and σ_f^2 are the average value and variance of τ_f; $R_{\sigma f}$ is the correlation coefficient of τ_σ and τ_f. All these quantities can be found if the average value $\langle\sigma(z)\rangle$ and the correlation function $R_{\sigma\sigma}(z_1;\ z_2)$ are known:

$$\langle\tau_\sigma\rangle = \int_0^{z_s}\langle\sigma(z)\rangle\ dz,$$

$$\langle\tau_f\rangle = \int_0^{z_s}\langle\sigma(z)\rangle F(\omega z)\ dz,$$

$$\sigma_\tau^2 = \int_0^{z_s}\int_0^{z_s} R_{\sigma\sigma}(z_1;\ z_2)\ dz_1 dz_2, \tag{13}$$

$$\sigma_f^2 = \int_0^{z_s}\int_0^{z_s} R_{\sigma\sigma}(z_1;\ z_2) F(\omega z_1) F(\omega z_2)\ dz_1 dz_2,$$

$$R_{\sigma f} = \int_0^{z_s}\int_0^{z_s} R_{\sigma\sigma}(z_1;\ z_2) F(\omega z_1)\ dz_1 dz_2.$$

We shall find the average values and the variance of the random quantities studied. It is obvious that in this case the average value of I is

$$\langle I\rangle = \langle D\rangle + \langle\beta_0\rangle G\langle T(m)\rangle,$$

and the variance is

$$\sigma_i^2 = \sigma_d^2 + 2\langle\beta_0\rangle GR_{td} + $$
$$+ G^2\left[\sigma_\beta^2\sigma_t^2 + \sigma_\beta^2\langle T(m)\rangle^2 + \langle\beta_0\rangle^2\sigma_t^2\right],$$

where $T(m) = T(\mu_0)T(\mu)$; $\langle D\rangle$ and $\langle T(m)\rangle$ are the average values of D and $I(m)$; σ_t^2, σ_d^2, and σ_β^2 are the variances of $T(m)$, D, and β_0; and R_{td} is the correlation coefficient of the random quantities $T(m)$ and D. Denoting by $\Phi_\sigma(v)$ the characteristic function of the random quantity τ_σ, we easily obtain

$$\langle D\rangle = \frac{Bm}{\alpha}\left[1 - \Phi_\sigma\left(i\frac{\alpha}{m}\right)\right], \quad \langle T(m)\rangle = \Phi_\sigma\left(i\frac{\alpha}{m}\right),$$

$$\sigma_d^2 = \frac{B^2m^2}{\alpha^2}\sigma_t^2, \quad \sigma_t^2 = \Phi_\sigma\left(2i\frac{\alpha}{m}\right) - \Phi_\sigma^2\left(i\frac{\alpha}{m}\right),$$

$$R_{td} = -\frac{Bm}{\alpha}\sigma_t^2.$$

Expressions for the average value of the OTF of the atmosphere $\langle\tau(\omega)\rangle$ and its variance $\sigma_{\tau\omega}^2$ can be written down quite simply. According to Eq. (3), it is sufficient to know the characteristic function of the random quantity $\tau_f(\omega) - \Phi_\omega(v)$. It is obvious that

$$\langle\tau(\omega)\rangle = \Phi_\omega\left[\frac{i(1-\Phi)}{\mu}\right],$$

$$\sigma_{\tau\omega}^2 = \Phi_\omega\left[\frac{2i(1-\Phi)}{\mu}\right] - \Phi_\omega^2\left[\frac{i(1-\Phi)}{\mu}\right].$$

The higher-order moments of the random quantities studied can also be found analogously.

As an example we shall study the statistical parameters of the fluctuations of the: characteristics for a model atmosphere with the correlation function

$$R_{\sigma\sigma}(z_1;\ z_2) = D_\sigma^2\exp(-gz_1 - gz_2 - p|z_1 - z_2|), \quad (14)$$

where p and g are approximation parameters; D_σ^2 is the variance of the scattering coefficient at the ground; and, $D_\sigma^2\exp(-2gz)$ describes the decrease in the variance of the scattering coefficient as a function of the altitude. We shall describe the scattering phase function of the atmosphere by the small-angle Henyey-Greenstein approximation:[7]

$$i(\gamma) = 2\kappa/\left(\kappa^2 + \gamma^2\right)^{3/2}$$

where $\kappa = (1-\bar{\mu})\bar{\mu}^{1/2}$, and $\bar{\mu}$ is the cosine of the scattering angle. In this one can easily verify,

$$\sigma_\tau^2 = D_\sigma^2\ I(p;\ g;\ g),$$

$$\sigma_f^2 = D_\sigma^2\left[I(p;\ g;\ g) - I(p;\ g+\zeta;\ g) - I(p;\ g;\ g+\zeta) + I(p;\ g+\zeta;\ g+\zeta)\right], \quad (15)$$

$$R_{\sigma f} = D_\sigma^2\left[I(p;\ g;\ g) - I(p;\ g;\ g+\zeta)\right],$$

where $\zeta = \kappa\omega$, and

$$I(p;\ f_1;\ f_2) = -\frac{1-\exp(-(f_1+p)z_s)}{(f_2-p)(f_1+p)} - \frac{2p}{f_2^2 - p^2}\frac{1-\exp(-(f_1+f_2)z_s)}{f_1+f_2} - \exp(-(f_2+p)z_s)\frac{1-\exp(-(f_1-p)z_s)}{(f_2+p)(f_1-p)}.$$

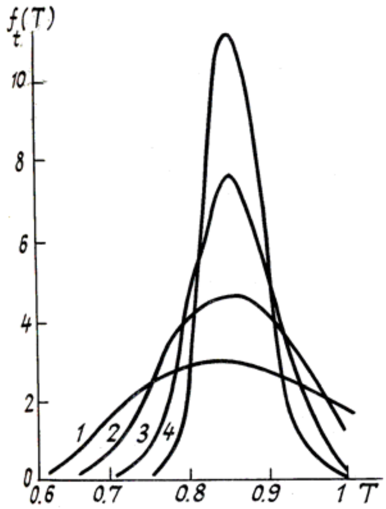

FIG. 1. The distribution function of the diffuse transmittance for $u = \mu_0 = 1.0$; $\langle\tau_\sigma\rangle = 0.5$; $p = 4.0\ \text{km}^{-1}$; $V_\sigma = 4.0$ *(1),* 2.5 *(2),* 1.5 *(3),* 1.0 *(4), and the horizontal meteorological visibility range* $S = 20$ km.

Figures 1 and 2 show the results of calculations, performed using Eqs. (11), (12), and (15), of the distribution functions of the diffuse transmittance and the OTF of the atmosphere as a function of the coefficient of variation of the scattering coefficient $V_\sigma = D_\sigma / \langle\sigma\rangle$. It was assumed that V_σ does not depend on the altitude. As one can see from the data presented, an increase in the fluctuations of the scattering coefficient results in an increase in the probability density for small and large values of T and $\tau(\omega)$.

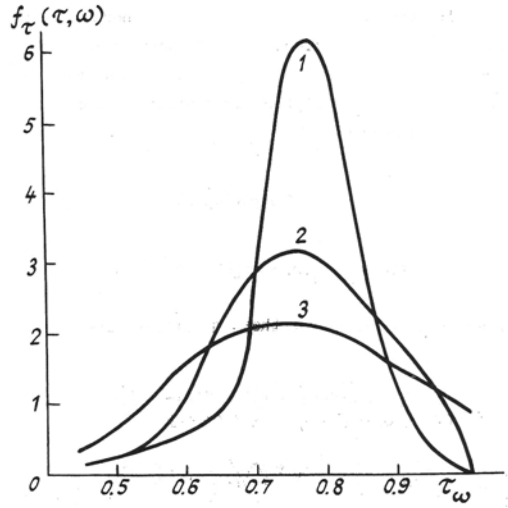

FIG. 2. The distribution *function of the OTF of the atmosphere calculated for* $\langle\tau_\sigma\rangle = 0.5$, $\mu = 1.0$; $\dfrac{\kappa\omega}{g} = 1.0$; $p/g = 1.0$; $V_\sigma = 0.5$ *(1),* 1.0 *(2), 1.5 (3).*

If the distribution of σ is normal, the distribution functions of T and $\tau(\omega)$ are, according to Eq. (11), log-normal. The average values of T and $\tau(\omega)$ for observation at the nadir in this case are

$$\langle T\rangle = T_0 \exp\left(\alpha^2\sigma_\tau^2/2\right),$$

$$\langle\tau(\omega)\rangle = \tau_0(\omega)\ \exp\left((1-\Phi)^2\sigma_f^2/2\right),$$

where the variances of these quantities are

$$\sigma_t^2 = T_0^2\left[\exp\left(2\alpha^2\sigma_\tau^2\right) - \exp\left(\alpha^2\sigma_\tau^2\right)\right],$$

$$\sigma_{\tau\omega}^2 = \tau_0^2(\omega)\left[\exp\left(2\alpha^2\sigma_f^2\right) - \exp\left(\alpha^2\sigma_f^2\right)\right].$$

Here T_0 and $\tau_0(\omega)$ are, respectively, the diffuse transmittance and the OTF of the atmosphere with average parameters. The relative fluctuations of T and $\tau(\omega)$ are, evidently, equal to

$$\delta_t = \sqrt{\exp\left(\alpha^2\sigma_\tau^2\right) - 1},$$

$$\delta_\omega = \sqrt{\exp\left((1-\Phi)^2\sigma_f^2\right) - 1}.$$

Estimates show that for cloud-free conditions the inequality $\alpha^2\sigma_\tau^2 \ll 1$, so that $\sigma_t \approx \alpha\sigma_\tau$. The value of δ_ω increases as the spatial frequency ω increases and it reaches a maximum in the limit $\omega \to \infty$, equal to $(1-\Phi)\ \sigma_\tau$ for $\sigma_f^2 \le 1$. For typical atmospheric conditions $\Lambda \simeq 0.8$, $\Phi \simeq 0.05$, $\alpha \simeq 0.24$, $1-\Phi \simeq 0.95$. This means that the magnitude of the maximum relative fluctuations of the OTP of the atmosphere is approximately equal to the rms variance of the fluctuations of the optical thickness σ_τ; the magnitude of the relative fluctuations of the diffuse transmittance is approximately four times smaller.

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