Effect of electric field on orientation of ice cloud particles

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I consider orientation of water ice cloud particles under simultaneous actions of the aerodynamic force due to gravity and vertical gradient of the electric field. These processes compete. The former one orients particles by their large diameters horizontally while the latter one does the same along vertical. It is shown that a field with the strength about 10^4 V/m is capable of compensating for the action of the aerodynamic forces in case of particles with sizes of a few microns. To orient submicron particles, the electric field strength of 10^5 V/m is required, because at lower values the energy of electric interaction turns out to be less than kT. Besides, the field of such strength is capable to overcome the action of the aerodynamic forces and orient micron particles by their large diameters vertically. To orient all particles with sizes between 0.1 and 1000 μm following the electric pattern, the electric field has to have the strength about $2 \cdot 10^6$ V/m.

Introduction

Spatial orientation of non-spherical particles of water ice clouds shows noticeable effect on the values of the transmission directed scattering coefficients. This must be taken into account in calculating the solar radiative fluxes.

Orientation depends on the combined effect of several physical factors. Among them there are the following: aerodynamic force moments appearing at particles falling down and at wind velocity pulsations. Orientation under the action of electric field is also referred to these factors. The factors destroying orientation are interaction of particles with small-scale turbulent motions of air and Brownian motion. The last factor is only essential for particles of submicron and micron size.

Orientation under the effect of aerodynamic factors was considered earlier in previous papers[1,2] as the moments of aerodynamic forces for particle size of submicron and micron range are comparable with the random moments of Brownian motion, such particles were considered as not oriented. Actually, particles with the size greater than 20 μm are subject to aerodynamic orientation. Brownian motion is inessential for such particles, and the main destructive factor is their interaction with turbulent cells of the energy dissipation interval. Possible effect of electric field on orientation of water ice cloud particles was not taken into account.[1,2] Joint effect of the aerodynamic force moment appearing as the particle falls down and the electric force moment appearing in the presence of vertical gradient of the electric field is considered in this paper.

Potential energy of non-spherical water ice particles in the electric field

Orientation of particles in the electric field can appear due to anisotropy of dielectric properties of the particulate matter and due to the difference in polarization of non-spherical particles with isotropic dielectric constant along different directions. Let us assume that the second case corresponds to water ice cloud particles, and let us ignore the insignificant birefringence of hexagonal water ice crystals. It is assumed that electric conductivity is equal to zero. Then the relative dielectric susceptibility

$$\chi = \varepsilon - 1,$$

where ε is the dielectric constant. For pure water ice in a static field $\chi = 72$.[3]

Then let us approximate hexagonal ice columns and plates by elongated and oblate ellipsoids of revolution. Let us introduce a mobile coordinate system, which is denoted by the set of three mutually orthogonal unit vectors $\mathbf{n}_1 \times \mathbf{n}_2 = \mathbf{n}_3$, directed along the axes of the ellipsoid of revolution. The direction $\mathbf{n}_3$ is considered as the axis of rotation. In an immobile coordinate system $\mathbf{e}_x \times \mathbf{e}_y = \mathbf{e}_z$ the vector of the static field strength $\mathbf{E}$ is directed along z-axis. Transformation of the vector components from the immobile coordinate system to the mobile one is made by the matrix operator $\mathbf{M}(\varphi, \theta, \gamma)$, where φ, θ, and γ are the Euler angles.

The potential energy of a particle is equal to the scalar product of the vector of induced dipole moment to the vector of the field strength taken with the opposite sign

$$U = -\mathbf{pE}. \quad (1)$$

The components of the dipole moment of the homogeneous isotropic ellipsoid in the mobile coordinate system are determined by the formula[4]:

$$p_n = \frac{\varepsilon_0 \chi V}{1 + \kappa_n \chi} E_n, \; n = 1, 2, 3, \quad (2)$$

where $\varepsilon_0 = (4\pi \cdot 9 \cdot 10^9)^{-1}$ C/(V · m) is the electric constant,[5] $V = 4\pi a_1 a_2 a_3 / 3$ is the volume of the

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ellipsoid; κ_n is the shape factor. The components of the static field strength in Eq. (2) should be presented in the mobile coordinate system. For this purpose, the transformation $\mathbf{E}_n = \mathbf{M}\mathbf{E}_i$ $(i = x, y, z)$ is applied. To present the dipole moment vector in the immobile coordinate system, one has to perform the inverse transformation $\mathbf{p} = \mathbf{M}^{-1}\mathbf{p}_{(n)}$. The matrix $\mathbf{M}$ is orthogonal so that $\mathbf{M}^{-1} = \mathbf{M}^{\mathrm{T}}$. The following expression for the dipole moment should be substituted to Eq. (1):

$$\mathbf{p} = \varepsilon_0 V \mathbf{M}^{-1}\hat{\chi}\mathbf{M}\mathbf{E}, \qquad (3)$$

where $\hat{\chi}$ is the diagonal matrix with the components $\chi/(1+\kappa_n\chi)$. The shape factors κ_n determining the view of the particle polarization tensor are calculated using the half-lengths of the ellipsoid axes[4]:

$$\kappa_n = \int_0^\infty \frac{a_1 a_2 a_3 \mathrm{d}x}{2\left(a_n^2 + x\right)\left[\left(a_1^2 + x\right)\left(a_2^2 + x\right)\left(a_3^2 + x\right)\right]^{1/2}}. \qquad (4)$$

Let us denote the half-axes of the ellipsoid of revolution $a_1 = a_2 = a_\perp$, $a_3 = a_\parallel$, i.e. perpendicularly and parallel to the axis of rotation. Hence, $\kappa_1 = \kappa_2 = \kappa_\perp$, $\kappa_3 = \kappa_\parallel$. Dependence of the shape factors on the relationship between the half-axes β calculated by Eq. (4) is shown in Fig. 1.

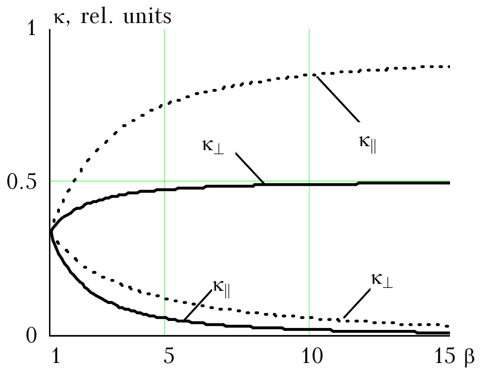

Fig. 1. Shape factor, κ: solid lines for elongated ellipsoids; dotted lines for spheroids.

Substitution of Eq. (3) into the expression (1) after corresponding matrix transformations gives the following formula for the potential energy:

$$U_\mathrm{e}(\theta) = -\frac{1}{2}\varepsilon_0 V E^2\left(\alpha_\perp \sin^2\theta + \alpha_\parallel \cos^2\theta\right), \qquad (5)$$

where

$$\alpha_\perp = \chi/(1+\kappa_\perp\chi), \quad \alpha_\parallel = \chi/(1+\kappa_\parallel\chi).$$

Ignoring the effect of all other forces except for the electric one, the particle will take such a position, at which the potential energy takes its minimum, and the affecting force moment M_e reduces to zero. The second derivative here should be positive:

$$M_\mathrm{e}(\theta) = \frac{\partial U_\mathrm{e}}{\partial\theta} = -\frac{1}{2}\varepsilon_0 V E^2\left(\alpha_\perp - \alpha_\parallel\right)\sin 2\theta = 0;$$
$$U''_\theta = -\varepsilon_0 V E^2\left(\alpha_\perp - \alpha_\parallel\right)\cos 2\theta > 0. \qquad (6)$$

If $\alpha_\parallel > \alpha_\perp$ (the case of elongated ellipsoids), the conditions (6) are fulfilled at $\theta = 0$. The rotation axis is parallel to the field vector $\mathbf{E}$. If $\alpha_\parallel < \alpha_\perp$, the rotation axis becomes perpendicular to the vector $\mathbf{E}$. In any case, the big diameter of the particle takes its position parallel to the field strength vector. If the direction $\mathbf{E}$ coincided with vertical, this tendency is opposite to the action of aerodynamic forces, acting on the falling particles and making their big diameters to take a horizontal position.

Equations of particle motion

Equation of rotation of a solid body in generalized coordinates has the following form

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L(q_i,\dot{q}_i)}{\partial \dot{q}_i} - \frac{\partial L(q_i,\dot{q}_i)}{\partial q_i} = -\xi\dot{q}_i + N(t), \qquad (7)$$

where L is the Lagrange function, and the Euler angles and their derivatives with respect to time are the generalized coordinates and velocities. If the right-hand parts of these equations equal zero, the equations describe the motion in the field of conservative forces. In this case the viscous friction forces and the random moment of forces appearing due to fluctuation of the number of collisions with air molecules are in the right-hand part. The moment of force has the following properties

$$\langle N(t) = 0\rangle, \quad \langle N(t)N(t')\rangle = F(N)\delta(t - t'), \qquad (8)$$

where N is the random value with dimension of the moment of force.

The Lagrange function for a symmetric rotor has the following form:

$$L = \frac{I_\perp}{2}\left(\dot{\varphi}^2\sin^2\theta + \dot{\theta}^2\right) + \frac{I_\parallel}{2}\left(\dot{\varphi}\cos\theta + \dot{\gamma}\right)^2 -$$
$$-U_\mathrm{e}(\theta) - U_\mathrm{a}(\theta), \qquad (9)$$

where $I_\parallel$ is the inertia moment at rotation around the rotation axis of the ellipsoid – the axis $\mathbf{n}_3$ of the mobile coordinate system. The moment of inertia at rotation about any direction perpendicular to the aforementioned one, is denoted as $I_\perp$. The moment of electric forces is defined by Eq. (6). The derivative $\partial U_\mathrm{a}/\partial\theta$ of the potential energy of interaction with airflow incident on the falling particle determine the acting moment of aerodynamic forces[1]:

$$M_\mathrm{a}(\theta) = \partial U_\mathrm{a}/\partial\theta = \lambda u^2 \rho V \sin 2\theta/2, \qquad (10)$$

where u is the velocity of particle falling down; ρ is the density of air; λ is the aerodynamic shape-factor expressed through the ellipsoid eccentricity.[1] If the ratio of the big half-axis to small has been changed from 1.5 to 5, λ changes from 0.53 to 0.6 for spheroids and from 0.36 to 0.12 for elongated ellipsoids of revolution.

Substitution of formula (9) into Eq. (7) taking into account Eqs. (6) and (10) gives the system of three equations of motion. The equations are quite difficult for analysis, because they involve the terms containing the products of generalized angular velocities. Moreover, the components of the random moment of force are in the right-hand parts. At the same time, it is clear that rotations by the angles φ and γ are caused only by Brownian motion, because the moments M_a and M_e depend only on the angle θ. Let us consider the Brownian motion in the mobile coordinate system, where the equations of motion have the following form:

$$\begin{aligned} I_1\frac{d\omega_1}{dt} + (I_3 - I_2)\omega_2\omega_3 &= -\xi\omega_1 + N_1(t), \\ I_2\frac{d\omega_2}{dt} + (I_1 - I_3)\omega_1\omega_3 &= -\xi\omega_2 + N_2(t), \\ I_3\frac{d\omega_3}{dt} + (I_2 - I_1)\omega_2\omega_3 &= -\xi\omega_3 + N_3(t), \end{aligned} \qquad (11)$$

and $I_1 = I_2 = I_\perp$, $I_3 = I_\parallel$. The angular velocities directed along the respective axes of the mobile coordinate system are denoted as ω_i. The system of equations (11) with zero right parts describes precession of the symmetric top, rotating with constant velocity around the direction $\mathbf{n}_3$, on which outside forces do not affect.

Formal solution of the third of Eqs. (11) has the following form[7]:

$$\omega_3(t) = Ce^{-\xi t/I_3} + e^{-\xi t/I_3}\int N_3(t)e^{\xi t/I_3}dt. \qquad (12)$$

According to Eq. (8), the function $N(t)$ is the sequence of noncorrelated pulses with alternating signs, zero mean value, and random time of arrival, amplitudes and durations. So, one can say only about some statistical characteristics averaged over time. It is clear that $\langle\omega_3\rangle = 0$, because the function with alternating sign and zero mean value is under the integral, and the transitional term, which is the solution of homogeneous equations, also becomes equal to zero after averaging. The autocorrelation function has the form

$$f(\tau) = \langle\omega_3(t)\omega_3(t-\tau)\rangle = F_3\exp(-\xi\tau/I_3). \qquad (13)$$

The second property of Eq. (8) and the theorem of ergodicity are used for its determination. To determine F, one can use the fact that the correlation function at $\tau = 0$ takes the value of the variance of the random value $\langle\omega_3^2\rangle$ and the known principle of the kinetic theory, that the kinetic energy $I\langle\omega^2\rangle/2$, related to one degree of freedom is equal to $kT/2$. Taking into account this fact, we finally obtain

$$f(\tau) = kT\exp(-\xi\tau/I_3)/I_3. \qquad (14)$$

One can call the value I/ξ the time of "forgetting" the current state.

Summing the first and second equations of the system (11), one can obtain similar result for rotation in the plane containing the rotation axis of the ellipsoid. The formula will be different by the factor 2 in front of kT, because two degrees of freedom take part in the motion, and one should substitute $I_1 = I_2 = I_\perp$ instead of I_3.

The moments of inertia at rotation around the direction perpendicular to the ellipsoid axis are equal to $I_1 = I_2 = M(a_\perp^2 + a_\parallel^2)/5$, and at rotation around the axis $I_3 = 2Ma_\perp^2/5$, where $M = 4\pi a_\perp^2 a_\parallel\rho/3$ is the particle mass. The viscous friction coefficient ξ [$\mathrm{kg\cdot m^2\cdot s^{-1}}$] is expressed through the dynamic viscosity η by the following equations[8]: for a sphere $\xi = 8\pi\eta R^3$; for strongly oblate spheroid rotating around the big axis $32\eta R^3/3$; for elongated ellipsoid with small half-axis $a_\perp$ and the half-axes ratio $a_\parallel/a_\perp = \beta$ rotating around the small axis $\xi = 16\pi\Lambda\eta a_\perp{}^3/3$, where

$$\Lambda = (\beta^4 - 1)\left[(2\beta^2 - 1)(\beta^2 - 1)^{-0.5}\ln(\beta + \sqrt{\beta^2 - 1}) - \beta\right]^{-1}. \qquad (15)$$

The value $B_\omega = 1/\xi$ is called "rotation mobility."[8] The mean angle <θ> by which the particle turns during the characteristic time I/ξ is equal to $\sqrt{2kTI_\perp}/\xi$ (see Table).

Mean turn angle <θ> of an ellipsoid particle with the big half-axis length *l* during the correlation time $\tau = I/\xi$

l, μm	0.1	1	5	10	50	100
<θ>°	1.6	0.5	0.22	0.16	0.07	0.05

It is seen from the data presented that the essential turns are possible for particles of submicron size. The Brownian motion of larger particles of crystal clouds is nothing but disorderly turns within the tenths of a degree. One can show that the time, during which orientation of a particle, for example, of a spheroid with the ratio of the big half-axis to small one equal to β, occurs is determined by the formula

$$\tau_0 = 8\eta(1 + \kappa_\perp\chi)(1 + \kappa_\parallel\chi)\left[\varepsilon_0\pi\beta|\kappa_\perp - \kappa_\parallel|\chi^2E^2\right]^{-1}.$$

At $E = 10^4$ V/m it is equal to $2.88\cdot10^{-2}$ s for particles of all sizes. During this time, due to rotational diffusion, the particle can deviate by the angle $\langle\theta\rangle \simeq \sqrt{2kTB_\omega\tau_0}$. Estimates show that only particles of submicron size can deviate by the angle greater than 1 rad. Particle of the size of 10 μm can deviate by approximately 2°. If $E = 10^6$ V/m, then $\tau_0 = 2.88\cdot10^{-6}$, and even submicron particles have no time to perform essential angular drift. Let us assume that, at least for particles of the size greater that several micrometers, Brownian motion is a small disturbance superposed on the motion under the effect of conservative forces and friction force. Then the equation of motion can be written in the following from:

$$I_{\perp}\frac{d^2\theta}{dt^2}+\xi\frac{d\theta}{dt}+\frac{1}{2}m\sin 2\theta=0, \tag{16}$$

where

$$m=\frac{1}{2}V\left[\lambda u^2\rho-\varepsilon_0\left(\alpha_{\perp}-\alpha_{\parallel}\right)E^2\right]. \tag{17}$$

The substitution $2\theta = \Theta$ transforms Eq. (16) to the equation of motion of a pendulum with damping. It is known that in the case of small oscillations Eq. (16) admits three kinds of motion depending on the sign of $s=\left((\xi/I_{\perp})^2-|m|/I_{\perp}\right)$. Non-periodic motion toward the equilibrium state occurs at $s > 0$ and $s = 0$, and at $s < 0$ — damped oscillations about this position. Dynamics of the process is essential in studying the electrooptical phenomena. In our case, it is out of our interest, the same as non-linearity of Eq. (16), which under particular conditions admits rotation of a particle with subsequent transition to the regime of damped oscillations. In any case, the particle finally will take the position, at which the acting force moment is equal to zero. This fact is interesting from the viewpoint of the effect of electric field on orientation. So, it is sufficient to consider the behavior of the sum of the moments of aerodynamic and electric interactions.

Relationship between aerodynamic and electric forces

Let us first make a remark concerning the ability of the electric field to orient water ice particles of different size, assuming that no other forces affect them, except for the Brownian motion. Let us take the beginning of exceeding of the level kT by potential energy of electric interaction (5) as a criterion of the beginning of orientation. Then it occurs that the field strength of the order of 10^5 V/m is required for orientation of particles of 0.5-μm size; the field of $E = 10^4$ V/m is capable of orienting particles of the size greater than 2 μm, and so on: $E = 10^3$ V/m, $l > 10$ μm; $E = 10^2$ V/m, $l > 35$ μm; $E = 10$ V/m, $l > 200$ μm. It is clear that such a behavior is caused by the dependence of electric force moment on the particle volume.

The above said means that the weighted particles of the size of several tens of micrometers and more could be oriented in electric field of order 10^2 V/m that is comparable with the field near the Earth's surface. The particles of submicron and micron range can be considered as practically suspended, but the particles of the size more than 10 μm have noticeable sedimentation velocity. Aerodynamic forces act on them, which, as was mentioned above, are directed opposite to the action of the electric forces.

Let us note that in the definition of the moment (10) accepted in Ref. 1, the angle θ was read from the vertical direction to the small axis of the ellipsoid, and in this paper it is read from the vertical direction to the rotation axis. In the coordinate system accepted here θ for spheroids has the same meaning as in Ref. 1 and in the case of elongated ellipsoids one should substitute the term $(\theta + \pi/2)$ in Eq. (10) instead of the angle θ, that means the change of sign in the right-hand part of Eq. (10).

Taking into account the above said concerning the total moments, the following should be fulfilled in the equilibrium position:

for spheroids

$$M_{\text{sph}}(\theta)=\frac{1}{2}V\left[\lambda u^2\rho-\varepsilon_0\left(\alpha_{\perp}-\alpha_{\parallel}\right)E^2\right]\sin 2\theta=0, \tag{18}$$

for ellipsoids

$$M_{\text{el}}(\theta)=\frac{1}{2}V\left[-\lambda u^2\rho-\varepsilon_0\left(\alpha_{\perp}-\alpha_{\parallel}\right)E^2\right]\sin 2\theta=0, \tag{19}$$

the condition of minimum being

$$\partial^2 U/\partial^2\theta=\partial M/\partial\theta>0. \tag{20}$$

Let us note that the values V, u, λ, and α should be marked, respectively, by the scripts "sph" and "el", because they are determined by different formulas for spheroids and ellipsoids.

For spheroids $\alpha_{\perp} > \alpha_{\parallel}$ and it follows from Eqs. (18) and (20) that the particle axis takes the position $\theta = 0$ if $\lambda u^2\rho > \varepsilon_0\left(\alpha_{\perp}-\alpha_{\parallel}\right)E^2$, but $\theta = \pi/2$ if $\lambda u^2\rho < \varepsilon_0\left(\alpha_{\perp}-\alpha_{\parallel}\right)E^2$. Besides, the moments become equal to zero at equality of expressions in brackets to zero. This means mutual compensation for aerodynamic and electric forces, i.e., the absence of orientation. Then it is interesting to consider the ratio between these forces for particles of different size.

Let us note that the particle volume is equally included into formulas for the moments of aerodynamic and electric forces. Hence, the ratio we are interested in depends only on the terms in brackets of Eqs. (18) and (19). The shape factor λ depends on the type of particle and on the ratio of the big axis to small. The values $\alpha_{\perp}$ and $\alpha_{\parallel}$ depend on the same ratio (see Eqs. (4) and (5) and Fig. 1). But the velocity of particle fall down depends on its size and is determined by the known empirical relation,[9] which, being written in main units of SI system, has the following form:

$$u=10^{3b-2}Al^b,\ \text{m/s}, \tag{21}$$

where l is the big diameter of particle, m. The values of the empirical constants A and b for particles with different ratios between big and small diameters are presented in Ref. 9. For illustration, let us consider particles with the diameter ratio 2.5/1 and define: for spheroids $A = 50$, $b = 0.75$; for elongated ellipsoids $A = 70$, $b = 0.92$. According to the results shown in Fig. 1, we have: for ellipsoids $\kappa_{\parallel} = 0.134$, $\kappa_{\perp} = 0.432$; for spheroids $\kappa_{\parallel} = 0.590$, $\kappa_{\perp} = 0.200$. The values of the shape factor λ are the following: for spheroids 0.57; for ellipsoids 0.23.

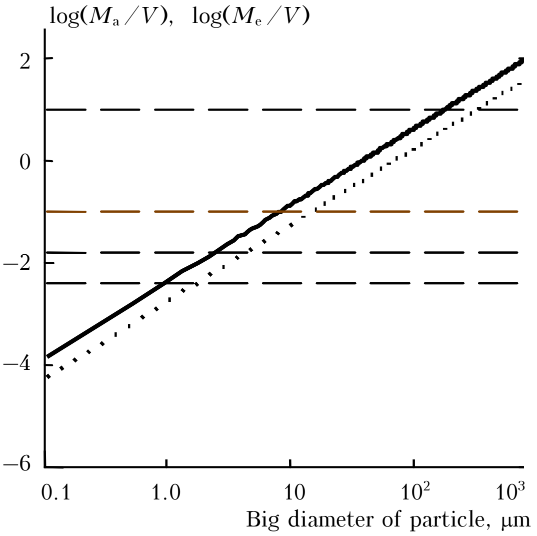

Fig. 2. Moments of aerodynamic force related to the particle volume: spheroids (solid line), elongated ellipsoids (dotted line); moments of electric force (horizontal dotted lines from bottom to top) calculated for spheroids and related to the particle volume at the strength of the electric field $E_i = 10^4$, $2 \cdot 10^4$, $1 \cdot 10^5$, $5 \cdot 10^5$ V/m. The values of the moments of the forces for elongated ellipsoids are close to that calculated for spheroids.

The comparison of the moments of aerodynamic and electric forces related to the particle volume is shown in Fig. 2. The points of crossing the lines $\log(M_a(l)/V)$, $\log(M_e(E_i)/V)$ with horizontal lines determine the big diameter of particle, at which mutual compensation for the moments of force occurs, if $E = E_i$. Particles smaller than this size undergo mainly the action of electric forces. Aerodynamic orientation is prevalent for larger particles.

Conclusion

Electric fields with the strength of 10^4 V/m and more are required for noticeable manifestation of electric orientation in water ice clouds occurring under conditions of its competition with orientation by aerodynamic force. If one assumes existence of such fields, joint effect of electric and aerodynamic forces will manifest itself as follows: as the strengths of the electric field increases, first small and then larger and larger particles will be oriented by big diameters along the direction of the electric field. Then the size range should follow, where mutual compensation occurs, and orientation of particles is not observed. Then the range of large particle follows, for which aerodynamic orientation is prevalent. If one sets the upper threshold of size by the value of 10^3 μm, the field with the strength of the order $2 \cdot 10^6$ V/m, i.e., close to electric breakdown in air will be required for complete prevalence of the electric type of orientation.

Possibility of strong electrostatic charging of water ice particles falling down does not raise doubts, the phenomenon of winter thunderstorms is known. But, they were observed in strong snowfalls and snowstorms. Author has no data, which confirm or disprove existence of the fields of the strength of 10^4–10^6 V/m in crystal clouds of the upper level.

The estimates obtained from the value of the current density and conductivity of air in nimbostratus clouds[10] show that the field strength can reach the values of the order of 10^4 V/m, but the concentration of particles in these clouds is much greater than in cirrus clouds. On the other hand, such kinds of halo as sun pillars and false Sun, as well as anomalous backscattering observed at laser sounding of crystal clouds are explained by horizontal position of big sides of crystals, which favors toward aerodynamic orientation of particles.

The conclusion follows from the above said, that, with rare exceptions, electric field does not essentially affect the orientation of particles of crystal clouds. One can draw more particular conclusions, as the data on electric fields in crystal clouds will be accumulated. If the values of the strength of the electric field have been known, all stated in this paper, together with materials of Ref. 1, make it possible to determine the distribution of particles over the orientation angle at joint effect of aerodynamic and electric orientation.

Acknowledgments

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