A DISPERSION TECHNIQUE FOR DETERMINING THE ZENITH ANGLE OF AN OBJECT MOVING IN THE ATMOSPHERE

A.N. Shadrin

Received June 27, 1991

A dispersion technique for estimating the zenith angle of a target moving in the atmosphere is analyzed. It is based on measurements of two apparent zenith angles at two optical wavelengths and meteorological parameters at the observation point. Recommendations on how to choose proper pairs of wavelengths are given and the accuracy of this technique is estimated.

Optical refraction in the atmosphere bends the trajectories of light propagation thus hindering the determination of a true position of a moving target. To account for the effect of refraction one has to know the vertical profile of the refractive index of air along the beam propagation path, what could be possible either from operative measurements or from some model of the atmosphere.

Operative measurements of vertical profiles of the refractive index of the atmosphere are too problematic because of considerable technical difficulties, especially for objects moving along slant paths and unknown trajectories. An alternative approach could be based on the account for refraction using some statistical models of the atmosphere.

The techniques based on the use of statistical models of the atmosphere are used quite rarely, because of inherently large errors and the need for a computer with large operative memory.

There exist methods using analytical models of the profiles of the refractive index of the atmosphere that do not require big computers but their accuracy is very poor, especially at large zenith angles ($z > 750°$) of a target, because of deviations of the actual profiles of the refractive index from the model ones.

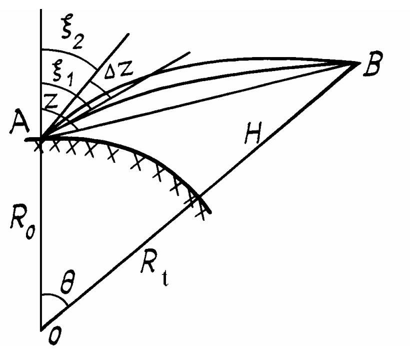

FIG. 1.

The accuracy of determining the zenith angle of a target can be improved by using a dispersion technique that is based on the use of the dependence of the angle at which the beam arrives at the receiver on wavelength. Let us consider the diagram presented in Fig. 1, where ξ_1 and ξ_2 denote the apparent zenith angles of a target measured at the wavelengths λ_1 and λ_2, Δr is the angle of differential refraction; z is the true zenith angle of a target, R_0 is the radius of the Earth; $R_t = R_0 + H$, H is the height of a target above the sea level.

It can be seen from Fig. 1, that the geocentric angle θ is independent of the wavelength, at which the target is observed. Therefore, one may write, according to the theory of refraction for a spherically stratified medium[1]

$$\theta = A_{01}\int\limits_{R_0}^{R_t}\frac{\mathrm{d}R}{R\sqrt{R^2n_1^2 - A_{01}^2}} = A_{02}\int\limits_{R_0}^{R_t}\frac{\mathrm{d}R}{R\sqrt{R^2n_2^2 - A_{02}^2}}\,, \qquad (1)$$

where $A_{01} = n_{01}R_0 \sin \xi_1$ and $A_{02} = n_{02}R_0 \sin \xi_2$, n_{01} and n_{02} are the refractive indices of air at the observation point for radiation with the wavelengths λ_1 and λ_2, respectively; h_1 and h_2 are the refractive index values at a point running along the light trajectory at corresponding wavelengths.

Under condition that the refractive index varies continuously along the beam path it can be replaced by its integral average, and, according to Ref. 2, the solution of equation (1) is as follows:

$$\theta = \arccos\frac{A_{01}}{R_t\langle n_1\rangle} - \arccos\frac{A_{01}}{R_0\langle n_1\rangle} =$$

$$= \arccos\frac{A_{02}}{R_t\langle n_2\rangle} - \arccos\frac{A_{02}}{R_0\langle n_2\rangle}\,. \qquad (2)$$

Using standard transformations of the inverse trigonometric functions[2] and denoting

$$A = \frac{n_{01}\sin\xi_1}{\langle n_1\rangle}\,,\quad B = \frac{n_{02}\sin\xi_2}{\langle n_2\rangle}\,,\quad G = \frac{R_0}{R_0 + H}\,,$$

after quite simple algebraic transformations of relation (2) we obtain

$$(A^2 - B^2)^2(1 - G^2)^2 = 0\,. \qquad (3)$$

It follows from relation (3) that only the first term of the product can become zero, since for slant beam paths $G < 1$ and, therefore, one obtains the equality

$$\frac{n_{01}\sin\xi_1}{\langle n_1\rangle} = \frac{n_{02}\sin\xi_2}{\langle n_2\rangle}\,. \qquad (4)$$

By substituting the Gladstone–Dahl formula for the refractive index of air into Eq. (4), after simple transformations we obtain the relation for estimating integral average of the refractive index of air at the wavelength

0235-6880/91/10 769-03 $02.00

$$\langle n\rangle = \frac{(C_{\lambda_2} - C_{\lambda_1})n_{01}\sin\xi_1}{C_{\lambda_2}n_{01}\sin\xi_1 - C_{\lambda_1}n_{02}\sin\xi_2}\,, \tag{5}$$

where C_{λ_1} and C_{λ_2} are the coefficients dependent on the wavelength.

Based on the theorem of sines we obtain an expression for geocentric angle from the triangle AOB in Fig. 1:

$$\theta = z - \arcsin(G\sin z)$$

By substituting this equation into the right side of Eq. (2) and solving it relative to sin z by its twice squaring and reducing identical terms, we obtain

$$(\sin^2 z - A^2)^2(1 - G^2)^2 = 0\,. \tag{6}$$

The analysis of Eq. (6) shows that this equality holds only when the first term vanishes, thus resulting in the following equality:

$$\sin z = (n_{01}\sin\xi_1)/\langle n_1\rangle$$

or in

$$z = \arcsin\frac{C_{\lambda_2}n_{01}\sin\xi_1 - C_{\lambda_1}n_{02}\sin\xi_2}{C_{\lambda_2} - C_{\lambda_1}}\,. \tag{7}$$

Thus, according to the theory of refraction in a spherically stratified atmosphere and assuming that the target is observed at two optical wavelengths, we obtain an expression which makes it possible to estimate the true value of the zenith angle of a target moving in the atmosphere. As can be seen from Eq. (7) to calculate the true value of the zenith angle one needs to measure the refractive index of air (or meteorological parameters) at the observation point together with the apparent zenith angle at corresponding wavelengths.

It should be noted that the two apparent zenith angles may be measured by both an active or a passive technique, that provides for isolating spectral intervals at two wavelengths from the optical signal using some filters.

It is obvious that the differential refraction will be the largest if one wavelength lies within the IR, and the other one within the UV range. However, strong extinction of the UV radiation prevents from using it at large zenith angle. At the same time, if two close wavelengths in the IR range are used, then the differential refractive angle cannot be measured at all. Thus, the problem on the proper choice of a pair of wavelengths for such measurements arises.

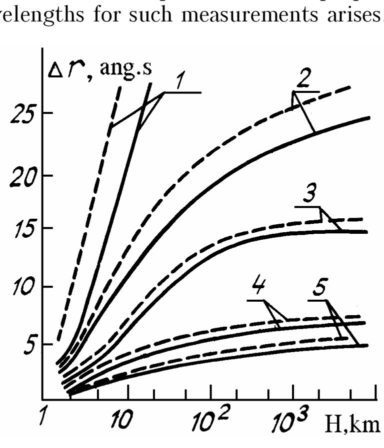

FIG. 2.

To demonstrate this we computed the angles of differential refraction for several pairs of wavelengths as a function of the height and the zenith angle of a target, using a model of a local atmosphere. Some results of such computations are presented in Figs. 2 – 5. It can be seen from this figures that within the whole range of heights and zenith angles the largest angles of differential refraction are obtained for the pair of wavelengths of $\lambda_1 = 0.6943$ μm and $\lambda_2 = 0.3472$ μm (Fig. 2).

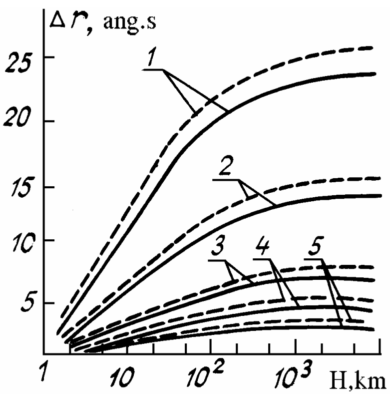

FIG. 3.

However, the use of this pair of wavelengths is limited because of significant extinction of radiation at the second wavelength for $z > 75°$.

Figures 3 and 4 show the dependences of the angles of differential refraction on the zenith angles and target heights for two pairs of wavelengths: $\lambda_1 = 0.4416$ and $\lambda_2 = 0.3472$ and $\lambda_1 = 1.06$ and $\lambda_2 = 0.84$ μm, respectively. It can be seen from these figures, that despite a significant steepness of the corresponding characteristic the first pair cannot be used, because of strong absorption of radiation by the atmosphere (see Fig. 6). The second pair of wavelengths lying in the near–IR suffers weaker atmospheric absorption. However, even at the zenith angles exceeding 75° its differential refraction remains within several seconds of arc, so that extremely strict restrictions would be imposed on the measurement system, that can hardly be attainable in practice. Therefore, the optimal pair of wavelengths could be $\lambda_1 = 0.6943$ and $\lambda_2 = 0.4416$ μm (Fig. 5), since the angle of differential refraction for it has quite a steep dependence on zenith angle, and the atmospheric extinction is not very strong at these wavelengths.

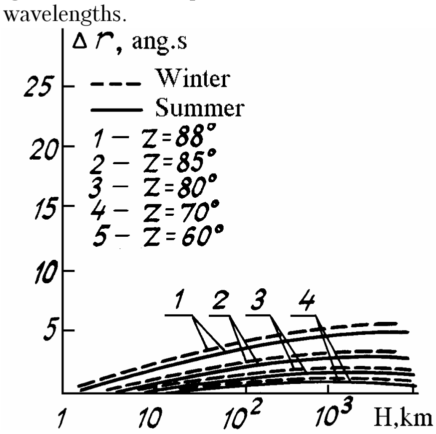

FIG. 4.

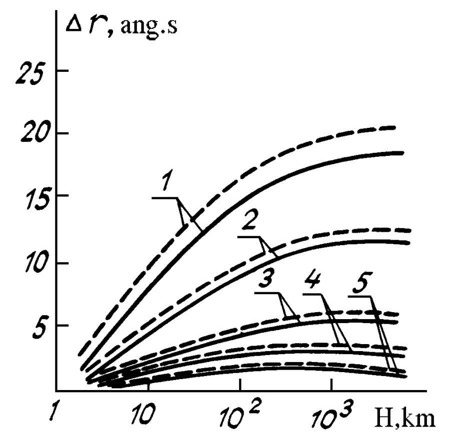

FIG. 5.

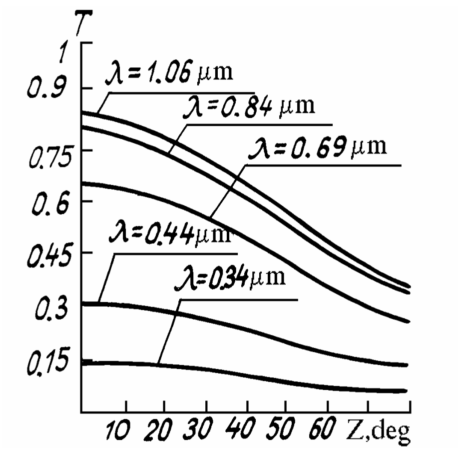

FIG. 6.

It is evident from the figures that the pair of wavelengths $\lambda_1 = 0.6943$ and $\lambda_2 = 0.3472$ μm is the most suitable for observations of objects at the zenith angles $z < 60°$ while the pair $\lambda_1 = 0.6943$ and $\lambda_2 = 0.4416$ μm better suites the observations at larger zenith angles.

To estimate the accuracy of the technique we have specially derived expressions, which showed that the error of calculations is mainly due to the errors in measurements of the zenith angle and differential refraction. The potential accuracy of the technique was analyzed for the following initial data: apparent zenith angle varied from 20° to 89°, target height changed from 2 to 1000 km, pressure and temperature at the observation point were 976.24 mb and 263.69 K, respectively, assuming the accuracy of their measurements to be 0.1 mb and 0.1 K.

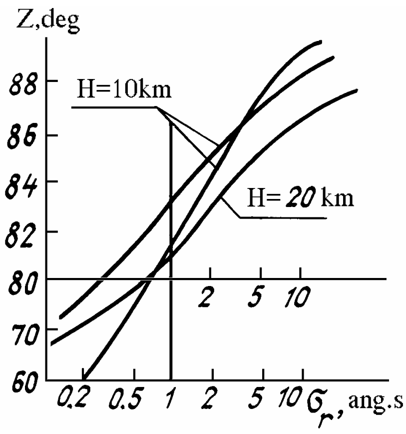

FIG. 7.

The results of this analysis are presented in Fig. 7. One can see that the error of the true zenith angle estimate is comparable to the rms error of the refractive angle retrieved from the results of aerological sounding. Note also that the proposed technique significantly reduces the effect of atmospheric turbulence, since the apparent zenith angles are measured during the time interval when the atmosphere can be considered to be "frozen".

The latter feature makes it possible to estimate the true direction to target on a real time scale, providing thus a possibility of accounting for inhomogeneities along the sensing path.

REFERENCES

1. V.P. Aksenov, V.A. Alekseev V.A. Banakh, et al, *Influence of the Atmosphere on Laser Beam Propagation* Tomsk, (Institute of Atmospheric Optics, Siberian Branch of the Academy of Sciences of the USSR, Tomsk, 1987).
2. I.N. Bronshtein, K.A. Semendyaev *Handbook on Mathematics* (Nauka, Moscow, 1986).
3. N.F. Nelyubin, Astrometriya Astrofiz., **50**, 66–75 (1983).