

# Analysis of a possibility for increasing the energy characteristics of cw LFM lidars

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The lidar equation for cw lidars with linear frequency modulation (LFM) of the sensing beam intensity is under analysis. Photo-resistor operation under radio-heterodyning regime (RHR) is analyzed. It is shown that RHR provides for multiple reduction of the received signal threshold power and correspondingly increase energy characteristics of infrared cw LFM lidars. Principles of operation of the coherent LFM lidar with reference beam phase conservation are considered. It is shown that the artificial phase conservation (with the help of optical fibers) provides for multiple lengthening (by about  $10^5$  times) of the sensed path at conservation of mutual coherence of the reference and received beams. This allows application of laser diodes in the systems with the operation range up to 10 km.

Continuous-wave lidars with linear frequency modulation of optical signals allow detection of various pollutants with a sufficient spatial resolution at a low power of the sensing radiation.<sup>1,2</sup> The operation principle of the LFM lidars is based on the possibility of keeping invariable the energy characteristics of radar systems<sup>3</sup> when going from pulsed operation mode to cw mode at a large accumulation time. Analysis performed for linear mode of the received signal detection<sup>3</sup> can be immediately applied to coherent LFM lidars with heterodyne reception of the optical radiation.<sup>2</sup>

In such systems, the photomixer operates in a linear mode, and the photodetector self-noises can be neglected as compared to external noises.<sup>4</sup> However, such an operation mode needs in lasers with a high temporal radiation coherence. This narrows the range of application of the coherent LFM lidars. The photoreceiver in the LFM lidars operates in the direct photodetection mode, which is characterized by quadratic parameters.<sup>4</sup> The detectability  $D^*$  of the receiver in this case is determined by its self-noises, which complicates a direct application of the above results<sup>3</sup> to such systems.

In this paper, the analysis of the lidar equation for the lidars with linear frequency modulation of optical radiation intensity is presented. A possibility of raising the energy characteristics of such systems is considered, as well as a possibility of applying the low radiation coherence lasers in the coherent LFM lidars with the LFM of the optical carrier.

## 1. Analysis of the lidar equation for LFM lidars

Consider the lidar equation for the pulsed operation mode<sup>5</sup>:

$$P_{ri}(R) = P_1(c\tau_i/2)\beta(R)A_rR^{-2} \exp\left[-2\int_0^R\alpha(r)dr\right], \quad (1)$$

where  $P_{ri}(R)$  is the instant value of the power received at the moment  $t$ ;  $P_1$  is the power sent at the moment  $t_0$ ;  $c$  is the light speed;  $\tau_i$  is the pulse length;  $\beta(R)$  is the volume backscattering coefficient;  $A_r$  is the effective area of the receiver;  $\alpha(r)$  is the volume extinction coefficient.

The range resolution is

$$\Delta R = c\tau_i/2. \quad (2)$$

Having set the range resolution of the cw LFM lidar to be  $\Delta R$  [Eq. (2)], which is provided by the choice of frequency deviation,<sup>6</sup> we obtain the lidar equation for the continuous mode:

$$P_{rc}(R) = P_0\Delta R\beta(R)A_rR^{-2} \exp\left[-2\int_0^R\alpha(r)dr\right], \quad (3)$$

where  $P_{rc}(R)$  is the power of signal received from the layer of  $\Delta R$  thickness located at  $R$  distance from the receiver;  $P_0$  is the cw sensing radiation power.

When comparing sensing radiation powers required for cw and pulsed lidars, all other conditions being equal, it must be taken into account that the use of a photodetector operating in the radio-heterodyne (RH) mode (considered in Ref. 7 for photomultipliers) for reception of the LFM signals is most effective. The LFM voltage following the sensing signal modulation law<sup>1</sup> is applied to the photomultiplier modulator; in this case the output current contains components of difference frequency  $f_R$  between the base voltage frequency and the modulation frequency of the received radiation.<sup>1,6</sup> The  $f_R$  magnitude (range frequency) is determined by

the distance to the sensed path segment. The RH mode is characterized by the conversion factor  $K_C$ , which makes 0.3 for photomultiplier.<sup>7</sup> The detectability  $D_{RH}^*$  in the RH mode is determined through detectability  $D^*$  in the direct photodetection mode:

$$D_{RH}^* = K_C D^*. \quad (4)$$

When the photodetector sensitivity is confined by its self-noises, the threshold power of the received optical signal is determined as<sup>4</sup>:

$$P_t = \sqrt{S\Delta F} / D^*, \quad (5)$$

where  $S$  stands for the area of photosensitive detector surface;  $\Delta F$  is the transmission band of the receiving channel. For the pulsed mode,  $\Delta F$  is in inverse proportion to  $\tau_i$  (of the order of  $10^{-8}$  s). For the cw lidar,  $\Delta F$  is in inverse proportion to the received signal accumulation time  $T_S$ , which is determined by the rate of changes of atmospheric conditions. At  $\Delta R = 10$  m and wind speed of 100 m/s,  $T_S = 0.1$  s.

Therefore, taking into account Eqs. (1), (3), and (4), the sensing radiation power required for attainment of threshold conditions in the cw mode changes in inverse proportion to the square root of the accumulation time:

$$P_0 = (P_t / K_C) \sqrt{\tau_i / T_S}. \quad (6)$$

For typical operating modes of cw and pulsed lidars, reduction of the required power (6) at conversion from pulsed to cw mode makes  $10^3$ , which allows application of low-power lasers (a few milliwatt) at a sensed path length of several hundred meters.<sup>1</sup>

## 2. Analysis of photoresistor operation in the radio-heterodyne mode

As is shown above, the use of photodetector in the RH mode provides a considerable reduction of the sensing signal power needed for the LFM lidars. However, this mode has been studied only for photomultipliers.<sup>1,7</sup> This does not allow one to realise properly the advantages of the LFM lidars in the IR region, which is of great interest for analysis of gas mixture composition.<sup>6,8</sup>

The analysis of photodetector features shows that the main property that makes possible its operation in the RH mode is a pronounced dependence of photodetector characteristics on the applied voltage. Photoresistors are capable of functioning in the IR region.<sup>9</sup> The volt-ampere characteristic (VAC) of a photoresistor is linear<sup>9</sup>:

$$I_{ph} = Q\Phi U, \quad (7)$$

where  $Q$  is the coefficient of proportionality determined by the electro-optical and geometrical parameters of the photoresistor;  $\Phi$  is the light flux (the incident radiation power);  $U$  is the applied voltage.

In the direct photodetection mode, photoresistor detectability is  $D^*$ , and the coefficient of proportionality between the photocurrent and the light flux  $Q_1$  is  $QU$ . To find the photoresistor detectability  $D_{RH}^*$  and its conversion coefficient  $K_C$  in the RH mode, it is necessary to determine the dependence of the photocurrent amplitude at the difference frequency  $f_R = |f_H - f_S|$  on the modulated light flux amplitude  $\Phi_m$  at the signal frequency  $f_S$  and the heterodyne voltage of frequency  $f_H$  applied to the photoresistor. Assume that the light flux and the heterodyne voltage  $u_H$  change by the laws

$$\Phi = \Phi_0 + \Phi_m \cos(2\pi f_S t + \varphi_S); \quad (8)$$

$$u_H = U_0 + U_H \cos(2\pi f_H t + \varphi_H),$$

where  $\Phi_0$  and  $\Phi_m$  are the constant and the amplitude of modulated light flux;  $f_S$  and  $\varphi_S$  are its frequency and phase;  $U_0$  is the constant shift;  $U_H$ ,  $f_H$  and  $\varphi_H$  are amplitude, frequency, and phase of the reference voltage. Of practical interest is the mode of weak signal reception, when the reaction to the photoresistor work at a low output voltage can be neglected. In this case, substitution of Eq. (8) to Eq. (7) ( $U$  is changed for  $u_H$ ) determines the expression for the difference frequency photocurrent, wherefrom its amplitude is  $I_{IR} = 0.5QU_H\Phi_m$ .

When the working point is in the VAC center,  $U_H$  must be half as small as the applied voltage in the direct photodetection mode:  $U_H = 0.5U$ . Therefore,  $K_C = 0.25$  and  $D_{RH}^* = 0.25D^*$ . At a zero shift,  $U_H$  can be increased to  $U$ , the functioning is possible both in the positive and negative VAC branches, and  $K_C$  grows up to 0.5.

Note that the use of heterodyne voltage with a large amplitude may cause a nonlinearity of the photoresistor VAC. This is equivalent to the case of the linear VAC and the heterodyne LFM voltage containing higher-harmonic components. However, the analysis similar to consideration of square pulsed signals in the LFM lidars<sup>10</sup> shows that in case of harmonic modulation of the LFM radiation intensity, the output photoresistor current contains only the difference frequency component  $|f_H - f_S|$ , because the difference frequency components determined by heterodyne voltage harmonics vary with time and are not accumulated at a resonance load.

Similarly to Ref. 10, for the heterodyne voltage, it is possible to use square pulses (of meander type) with linearly varying repetition frequency. In this case, intensity modulation of optical signal must be harmonic. Then the amplitude of the first harmonic of the reference voltage  $4/\pi = 1.27$  times exceeds the pulse amplitude,<sup>10</sup> which allows an additional raise of the transformation coefficient  $K_C$  to 0.636 and  $D_{RH}^*$  to  $0.636D^*$ .

Consider application of this mode to remote control for ammonia leakages in indoor conditions. The LFM lidar designed for such aims,<sup>8</sup> has a photoresistor operating in the direct photodetection

mode, which converts the LFM backscattering signal into electric one. After that, the signal comes to the mixer, to which the reference LFM voltage was applied as well. At the mixer output, the difference signal is selected. The photodetector transmission band corresponds to the band of modulating frequencies  $\Delta F = 10$  MHz required to provide for the resolution  $\Delta R$  of several meters.

When making use of the photoresistor (e.g., based on InAs) with  $\lambda = 1.5 \dots 3.0 \mu\text{m}$   $D^* = 10^{11} \text{ cm} \cdot \text{Hz}^{1/2} \cdot \text{W}^{-1}$ , and the photosensitive area  $S = 0.25 \times 0.25 \text{ mm}^2$ , the threshold receiver sensitivity<sup>8</sup> is  $P_t = 8 \cdot 10^{-10} \text{ W}$ . Switching photoresistor to the RH mode makes it possible to reduce the transmission band to 10 Hz (at the accumulation time of 0.1 s), which, in the conditions described in Ref. 8 and accounting for Eq. (4) and  $K_C = 0.636$ , allows a three orders of magnitude raise in the threshold sensitivity, i.e.,  $P_t = 1.24 \cdot 10^{-12} \text{ W}$ . Thus, the use of photoresistor in the RH mode allows a multiple (by several orders of magnitude) increase of energy characteristics of the infrared LFM lidars. Further increase of energy characteristics is possible with the use of either the square pulsed sensing LFM signals<sup>10</sup> or the square pulsed LFM heterodyne voltage.

### 3. Coherent LFM lidars with conservation of the reference radiation phase

A full realization of continuous-wave sensing methods based on the equivalence of energy characteristics of cw and pulsed systems<sup>3</sup> is possible for coherent LFM lidars at the heterodyne reception of optical signals<sup>2</sup> providing for the linear detection mode. The principle of operation of such systems is based on the use of radiation with a large coherence time  $\tau$ . In this case, the radiation coherence length  $l$  must exceed the doubled length of the sensed path  $R_{\text{max}}$ :

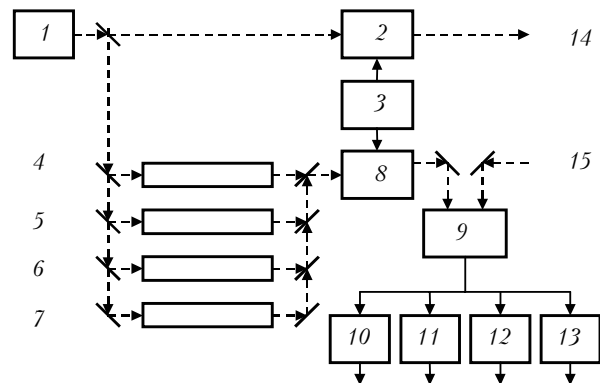
$$l = c\tau \geq 2R_{\text{max}}, \quad (9)$$

which makes impossible the application of the semiconductor lasers (with  $l$  of a few cm), which are promising in the IR region. Analysis of the operation principles of the coherent LFM lasers shows that the main feature, which supports their high performance characteristics is a conservation of the reference radiation phase with respect to the phase of the received optical signal. Therefore, for lasers with a small  $\tau$  such an artificial conservation is possible through making use of optical fibers with a low dispersion  $\sigma$ .

Figure 1 shows the scheme of the LFM lidar with conserved phase of the reference radiation.

A part of laser radiation (the sensing signal) here undergoes the LFM and goes to the atmosphere. Another part (the reference radiation) is distributed among  $m$  conservation channels (optical fibers). The length of the  $m$ th fiber  $R_m$  corresponds to the

distance to the sensed path segment of the particular channel. The radiation having passed through the fiber is forwarded to the common modulator, where it undergoes LFM by the same law as the sensing signal. Further, the reference radiation is mixed with the received optical signal in the photoreceiver, where a mutual coherence between the reference radiation of the  $m$ th channel and the received signal corresponding to the range  $R_m$  is provided.



**Fig. 1.** Functional scheme of the coherent LFM lidar with conservation of reference radiation phase: optical quantum generator (1); optical modulators (2, 8); generator of LFM voltage (3); circuit of reference radiation delay based on optical fiber (4–7); photoreceiver operating in the photomixing mode (9); filters of difference (ranging) frequencies (10–13); sensing radiation (14); backscattering signal (15), optical signal - - - ->; electric signal ->.

Analysis of the principles of operation of the LFM lidar (see Fig. 1) shows that coherent reception is possible in each channel from atmospheric layer of  $l/2$  thickness.

Therefore, at  $l \leq 2\Delta R$ , there is no mutual influence of the reception channels, which holds true for the promising applications of this method, e.g., in the use of semiconductor lasers ( $\tau = 250 \text{ ps}$ ,  $l = 7.5 \text{ cm}$ ) and  $\Delta R$  of several meters. In this case, the reference and received radiations in neighboring channels are mutually incoherent, the spectral width of the difference frequency signal at photodetector output multiply exceeds the transmission band of the output narrow-band filters, which excludes the accumulation of noises from the neighboring channels.

To estimate the limiting characteristics of the lidars allowing phase conservation of the reference radiation, it is necessary to take into account the influence of optical fiber parameters on the coherence of radiation propagating through it. Time spread  $\Delta t$  in optical signal components appearing at the output of the optical fiber of  $l_c$  length and the dispersion  $\sigma$ , equals  $l_c\sigma$ , i.e., at the optical fiber output, components of incoming radiation entered during the interval  $\Delta t$  are present.

At  $\Delta t \leq \tau = l/c$ , the random phase incursion of the reference radiation with respect to the received signal phase does not exceed  $180^\circ$ , i.e., their mutual

coherence holds. This allows us to take this expression as a condition for the lidar's coherent mode of operation with conservation of the reference radiation phase. Since  $R$  is determined by the fiber length with the core refractive index  $n$  in the expression  $R = nl_C/2$ , then, taking into account the maximum  $l_C$ , at which  $\Delta t \leq \tau = l/c$ , we can find the range as  $R_{Cmax} \leq nl/(2c\sigma)$ .

Analysis of  $R_{Cmax}$  and Eq. (9) shows that lengthening of the sensed path is determined only by optical fiber parameters and thus makes  $\gamma = n(c\sigma)^{-1}$ . At  $\sigma = 10$  ps/km (Ref. 11) and  $n = 1.5$   $\gamma = 5 \cdot 10^5$ . For a single-frequency semiconductor laser ( $l = 7.5$  cm),  $R_{Cmax} = 18.7$  km, which is in accordance with capabilities of the known lidars based on highly-coherent gas lasers.

Mutual independence of the receiving channels allows application of uneven distribution of reference radiation power between the conservation channels with the purpose of advancing the system's metrological characteristics. Since the output photomixer current is determined by the quantity  $2\sqrt{P_S P_H}$  (where  $P_S$  is the signal power;  $P_H$  is the reference radiation power)<sup>4</sup>; the choice of the distribution coefficient obeying the law  $g_m \sim 1/m^2$  allows one to lessen the dynamic range of the output photodetector signals by the factor of  $(R_{Cmax}/R_{min})^2$  (where  $R_{min}$  is the minimal range) and compensate the extinction of output signals following the law  $R^{-2}$  described by Eq. (1). Correspondingly, the power of the reference radiation of the  $m$ th channel  $P_{Hm} = \theta P_H m^2$  (where  $\theta$  is constant). In the ideal case

$$\sum_{m=1}^M P_{Hm} = P_H,$$

$$\theta = 1 / \sum_{m=1}^M m^2 = 1 / \left[ \frac{1}{3}(M+1)^3 - \frac{1}{2}(M+1)^2 + \frac{1}{6}M + \frac{1}{6} \right].$$

This provides for a threefold rise of energy characteristics at irregular distribution of the reference radiation power among the conservation channels as compared to the regular one:

$$P_{HM} = P_H M^2 / \left[ \frac{1}{3}(M+1)^3 - \frac{1}{2}(M+1)^2 + \frac{1}{6}M + \frac{1}{6} \right].$$

At  $M=100$   $P_{HM} = 0.0296P_H$  instead of  $0.01P_H$ , which is for uniform power distribution.

## Conclusion

The results of the studies allow making the following conclusions:

1. The use of photoresistors operating in the RH mode gives a multiple (by several orders of magnitude) decrease of the threshold power of received signals due to narrowing the difference (ranging) frequency output signal band, as well as a corresponding rise of energy characteristics of the infrared LFM lidars.

2. The use of optical fibers to conserve reference radiation phase in coherent LFM lidars provides for a multiple (up to five orders of magnitude) lengthening of the sensed path at conservation of mutual coherence between the reference and received radiations, thus allowing the use of semiconductor lasers in such systems with ranging up to 10 km. Uneven distribution of the reference radiation power among the conservation channels improves the metrological characteristics of the system at the sacrifice of considerable reduction  $((R_{max}/R_{min})^2$  times) of the dynamic range of photoreceiver's output signals.

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