RECONSTRUCTION OF OVERCAST FRAGMENTS OF VIDEO IMAGES AS APPLIED TO STATISTICS OF SATELLITE OBSERVATIONS

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Received June 2, 1995

An algorithm for reconstructing overcast fragments of aerospace images of the Earth's surface has been proposed. It is based on a statistical approach to completing skipped components of the observation vector. Some examples of reconstruction of video images have been given with a sample of images of preceding statistically homogeneous observations.

Well known are the problems connected with the analysis of aerospace video images produced when some regions of the Earth's surface are shadowed by fog or cloud fragments of high optical density. In this case, standard methods of image restoration using, for example, linear model describing the process of image transfer through dense scattering media and having the form of functional of convolution of the sought–after image with the point spread function give no way to obtain results of any value.

At the same time, in mathematical statistics,[1,2] the apparatus has been developed for reconstructing skipped components and formulating decision rules of statistical inference in observations of incomplete vector data. Methods for reconstructing skipper values are based on the use of information coded in interconnections between the vector components of the data being observed, i.e., on the context of interrelations. Therefore, for a solution of the aforementioned problem, the methods based on estimating the unknown parameters of statistical models are widely used.[2]

Thus, for example, in order to reconstruct defect rows of raster images and to represent them efficiently, the filtration algorithms are used.[5] The linear models of data description in bases of linearly independent functions (in the simplest case it may be the Fourier trigonometric basis used for restoration of archive photographs[6]) offer reconstructing properties.

Although from the theoretical viewpoint the choice of a system of basis function is not crucial for a solution of the problem of reconstructing the skipped values, taking into account multidimensionality of video mages and instability of a solution of the system of linear equations whose dimensionality is determined by the number of functions of basis space, it becomes clear that progress in the problem solution is completely governed by the approximating properties of the basis chosen to describe the observed ensemble of images by relatively small number of basis functions.

In general let us consider a problem of reconstruction of fragments of vector fields modeling multicomponent aerospace images. We assume that the random vector field $\xi(\mathbf{u})=(\xi^1(\mathbf{u}), \ldots, \xi^s(\mathbf{u}))^{\mathrm{T}}$ of vector argument $\mathbf{u} = (u^1,\ldots,u^\nu)^{\mathrm{T}}$ (where s and ν are the dimensionality of the vector function $\xi(\cdot)$ and of vector argument respectively, and T denotes transposition), centered about mathematical expectation $\mu(\mathbf{u})$, is represented in its domain of definition

$$D = \{\mathbf{u}:\ u_a^i \le u^i \le u_b^i\ ,\quad i = 1, \ldots, \nu\}\ ,$$

where $\{u_a^i\}$ and $\{u_b^i\}$ are the boundaries of multidimensional square, by a set of N realizations $\xi_1(\mathbf{u}), \ldots, \xi_N(\mathbf{u})$.

With regard for restrictions imposed onto the field that were introduced in Refs. 3 and 4 and hold true for physically realizable fields, the orthonormal basis can be constructed in the space of process realizations, and the vector field can be represented as follows:

$$\xi(\mathbf{u}) = \lim_{k \to \$} \sum_{i=1}^{k} X^i\, \Phi_i(\mathbf{u})\ , \tag{1}$$

where the limit is understood as convergence in the rms sense, $\{\Phi_i(\mathbf{u})\}_1^k$ are the non–random basis vector functions of vector argument. The random coefficients $\{X^i\}_1^k$ are determined from conditions of minimum standard deviation

$$\varepsilon_k^2 = \mathrm{M} \left|\left| \xi(\mathbf{u}) - \sum_{i=1}^{k} X^i\, \Phi_i(\mathbf{u}) \right|\right|^2 , \tag{2}$$

where M denotes the mathematical expectation operator, é·é denotes the Euclidean norm in the space of observations, and k is the number of chosen basis functions. If the condition of orthonormality

$$(\Phi_i, \Phi_j) = \int_D \Phi_i^{\mathrm{T}}(\mathbf{u})\, \Phi_j(\mathbf{u})\, \mathrm{d}^{\nu}\mathbf{u} = \delta_{ij}\ , \tag{3}$$

is imposed onto the basis functions $\{\Phi_i(\mathbf{u})\}_1^k$, where $\delta_{ij}(i,\ j = 1, \ldots, k)$ is the Kronecker symbol, $\mathrm{d}^{\nu}\mathbf{u} = \mathrm{d}u^1\times \ldots \times\mathrm{d}u^{\nu}$, and (., .) denotes the scalar

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product, the representation coefficients $\{X^i\}_1^k$ that minimize Eq. (2) have the form

$$X^i = (\xi, \Phi_i) = \int_D \xi^{\mathrm{T}}(\mathbf{u})\ \Phi_i(\mathbf{u})\ \mathrm{d}^{\nu}\mathbf{u}\ , \quad i = 1, \ldots, k\ . \qquad (4)$$

Let us now assume that the next realization being observed has skipped values in some connected subsets of the domain D. These skipped values are due to shadowing effect of broken cloudiness. Let us denote a set of all fragments of the domain D with indefinite realizations $\xi(\mathbf{u})$ by E, where $E \subset D$.

It should be noted that the rms error ε_k^2 (see Eq. (2)) of representation of this incomplete realization(using statistical terms of Ref. 1 and 2) is uncertain, since $\xi(\mathbf{u})$ has unknown values in the domain $E \subset D$, whereas the basis functions $\{\Phi_i(\mathbf{u})\}_1^k$ are defined everywhere over the domain D.

The idea of reconstructing the skipped values is to complement realization $\xi(\mathbf{u})$ in E in any way and to find the coefficients $\{X^i\}_1^k$ that minimize ε_k^2. In connection with the aforesaid, let us complement the uncertain realizations $\xi(\mathbf{u})$ in the domain E by a linear combination of some (thus far unknown) coefficients and basis functions $\{\Phi_i(\mathbf{u})\}_1^k$. It is natural to choose the coefficient of model (1) as unknown coefficients of these linear combinations. In other words, we introduce a modified realization of the following form:

$$\zeta(\mathbf{u}) = \begin{cases} \xi(\mathbf{u}), & \mathbf{u} = D \setminus E, \\ \sum\limits_{j=1}^{k} X^j\ \Phi_j(\mathbf{u}), & \mathbf{u} \in E, \end{cases} \qquad (5)$$

and the complement $\sum\limits_{j=1}^{k} X^j\ \Phi_j(\mathbf{u})$ of realization $\xi(\mathbf{u})$ in a shadowed region does not increase the error ε_k^2, because it enters into both terms of the difference in Eq. (2).

Let us now write down expression (4) for the representation coefficients dividing the domain D of integration into non–intersecting subdomain $D \setminus E$, in which the modified observation $\zeta(\mathbf{u})$ has the form of true observation $\xi(\mathbf{u})$, and subdomain E, where $\zeta(\mathbf{u})$ has the form of approximation $\sum\limits_{j=1}^{k} X^j\ \Phi_j(\mathbf{u})$ of the field to be reconstructed in E. Then expression X^i for coefficients (4) comprises the following components:

$$X^i = \int_{D \setminus E} \xi^{\mathrm{T}}(\mathbf{u})\ \Phi_i(\mathbf{u})\mathrm{d}^{\nu}\mathbf{u} + \int_E \sum_{j=1}^{k} X^j\ \Phi_j^{\mathrm{T}}(\mathbf{u})\ \Phi_i(\mathbf{u})\mathrm{d}^{\nu}\mathbf{u}. \qquad (6)$$

Now we introduce the designations

$$A = (a_{ji})\ , \quad a_{ji} = \int_E \Phi_j^{\mathrm{T}}(\mathbf{u})\Phi_i(\mathbf{u})\mathrm{d}^{\nu}\mathbf{u}\ , \quad (j, i = 1, \ldots, k)\ ,$$

$$\mathbf{b} = (b^1 \ldots b^k)^{\mathrm{T}}\ , \ b_i = \int_{D \setminus E} \xi^{\mathrm{T}}(\mathbf{u})\ \Phi_i(\mathbf{u})\mathrm{d}^{\nu}\mathbf{u}\ , \ (i=1, \ldots, k)\ ,$$

$\mathbf{x} = (X^1 \ldots X^k)^{\mathrm{T}}$, and $I = \mathrm{diag}(1, \ldots, 1)$ (the unit $k \times k$ matrix).

Then from Eq. (6) we obtain the equation for unknown components of vector $\mathbf{x}$, namely,

$$(I - A)\mathbf{x} = \mathbf{b}\ . \qquad (7)$$

Solving it by any method, we find the desired coefficients $\{X^i\}_1^k$. These coefficients can be used for reconstruction of the realization $\xi(\mathbf{u})$ being observed using either model (1) or modified realization of Eq. (5) only for shadowed regions.

It should be noted that the basis $\{\Phi_i(\mathbf{u})\}_1^k$ of orthonormal functions was taken arbitrarily, but the quality of approximation and the accuracy of reconstruction will be much higher for the Karhunen–Loeve (KL) basis best suited for description a random process in the sense of the rms error, which ensures the minimum approximation error[3,4] ε_k^2 as compared with other bases for fixed number k of basis functions.

An algorithm for constructing such a basis and other adaptive bases from a sample of complete realizations $\xi_1(\mathbf{u}), \ldots, \xi_N(\mathbf{u})$ was described in detail in Ref. 4. Let us consider the problems of construction of such a basis from incomplete realizations $\xi_1(\mathbf{u}), \ldots, \xi_N(\mathbf{u})$ in subdomains $E_1, \ldots, E_N$, respectively, of the domain D.

At the first step, we choose a certain basis $\{\Psi_i(\mathbf{u})\}_1^k$ and, after complement of realizations as described above, obtain a corrected set of images $\{\zeta_i^0(\mathbf{u})\}_1^N$, in which the skipped realization $\xi_1(\mathbf{u}), \ldots, \xi_N(\mathbf{u})$ of overcast fragments of video images $E_1, \ldots, E_N$ have been complemented by modifications (5). Having the reconstructed data of zero approximation $\{\zeta_i^0(\mathbf{u})\}_1^N$ at hand, we can construct the KL basis of the first approximation $\{\Phi_i^1(\mathbf{u})\}_1^k$, and on this basis solve the problem of reconstruction of realizations of the basis sample for model (5).

To eliminate degeneracy of matrix A in Eq. (7), which results in singular solutions, a moving regime of correction and construction of basis should be used, that is, when reconstructing the ith realization $\xi_i(\mathbf{u})$, its modified analog should be excluded from the sample $\{\zeta_i^0(\mathbf{u})\}_1^N$ and the KL basis of the first approximation $\{\Phi_i^1(\mathbf{u})\}_1^k$ should be constructed from the sample $\zeta_1^0(\mathbf{u}), \ldots, \zeta_{i-1}^0(\mathbf{u}),\ \zeta_{i+1}^0(\mathbf{u}), \ldots, \zeta_N^0(\mathbf{u})$ of size $N - 1$, where $i = 1, \ldots, N$.

Having thus reconstructed each of $\{\xi_i(\mathbf{u})\}_1^N$ images and obtained their modifications $\{\zeta_i^1(\mathbf{u})\}$, we construct the basis of the second approximation $\{\Phi_i^2(\mathbf{u})\}_1^k$ from these data.

Continuing this iterative process of construction of the next basis and reconstruction of realizations, we finally obtain at the mth step a set of basis functions

$\{\Phi_i^m(\mathbf{u})\}_1^k$, that can be used to correct ultimately the images $\{\xi_i(\mathbf{u})\}_1^N$, converting them in a set $\{\zeta_i^m(\mathbf{u})\}_1^N$ with the rms error of approximation

$$\varepsilon_k^2(m) = \frac{1}{N}\sum_{j=1}^{N}\left|\left|\zeta_j^m(\mathbf{u}) - \sum X_j^i\,\Phi_i^m(\mathbf{u})\right|\right|^2.$$

An increment to the error at the next step of iteration serves as a criterion for termination of this process, and when $\varepsilon_k(m-1) - \varepsilon_k(m) \le \varepsilon$, where ε is a preset parameter, the process of basis correction is terminated, and the obtained basis $\{\Phi_i^m(\mathbf{u})\}_1^k$ is used for reconstruction of newly recorded video images.

In the rms sense convergence of the above–described procedure of reconstruction of basis and realizations follows from convergence of monotonically decreasing finite sequences.

It should be noted that to solve the problem of reconstruction of overcasted fragments of video images, for construction of adaptive bases[4] it is natural to use the *a priori* information in the form of:

– landscape maps of ground reflectivity that can be used to form model images of underlying surface of a chosen region,

– statistics of microwave satellite images recorded with superhigh resolution, from which the optical images scaled to the images being observed can be synthesized,

– samples of satellite images recorded at stations of routine survey at stages of preliminary observations with regard to seasonal stationarity of situation.

Just the last case is presented as an example of operation of an algorithm. We consider the simplified geometry of satellite observations without regard for cloud shadow, adjacency effect, sun elevation, and other interfering effects believing that their influence during the period of data accumulation does not disturb statistical homogeneity of observations.

To illustrate the operation of the algorithm for reconstructing overcast fragments of video images, linearly independent realizations of images were formed. As a basis for collecting image statistics, three satellite photographs taken from onboard the Resurs satellite were chosen in the following wavelength ranges: 0.5–0.6 μm (channel I), 0.6–0.7 μm (channel II), and 0.8–0.9 μm (channel III). They were digitized by 256 brightness levels.

Fragments of images of 256×256 pixels, denoted by $\xi_1(x, y)$, $\xi_2(x, y)$, and $\xi_3(x, y)$ for each wavelength range, served as initial information to collect statistics of images. The next model image $\xi(x, y)$ of the sampling ensemble was formed using a linear combination of three initial images, namely,

$$\xi(x, y) = \alpha\xi_1(x, y) + \beta\xi_2(x, y) + \gamma\xi_3(x, y)\ , \qquad (9)$$

where α, β, and γ were set by a random-number generator. They obeyed uniform distribution and normalization condition $\alpha + \beta + \gamma = 1$.

To collect statistics of linearly independent images, we used the nonlinear transformation of equalization in Eq. (9). The sample of 25 images was thus obtained, which modeled the results of satellite observations at a station of routine survey of the same region of the Earth's surface.

As model clouds, plane fragments bounded by ellipses with random center positions, orientation, and lengths of axes were taken. On average, 10.5% of total area was shadowed by these clouds. As a criterion of image reconstruction quality, the following quadratic integral criterion

$$\varepsilon = \sqrt{\int_D [\xi(x, y) - \hat{\xi}(x, y)]^2 \mathrm{d}x\mathrm{d}y \Big/ \int_D [\xi(x, y)]^2 \mathrm{d}x\mathrm{d}y}\ 100\% \qquad (10)$$

was chosen. Here, $\xi(x, y)$ is the initial sharp (complete) image, $\hat{\xi}(x, y)$ is the image reconstructed from $\xi_0(x, y)$, and $\xi_0(x, y)$ is the overcast image $\xi(x, y)$.

The first run of experiments used the KL basis constructed from a sample of 12 complete images. Approximating properties of the KL basis so obtained are characterized by the spectrum of the eigenvalues shown in Fig. 1*a* (curve *1*), where λ determine the contribution from each basis function to the generalized variance. Figure 1*b* (curve *1*) shows the error of approximation of an ensemble of realization by a set of basis functions.

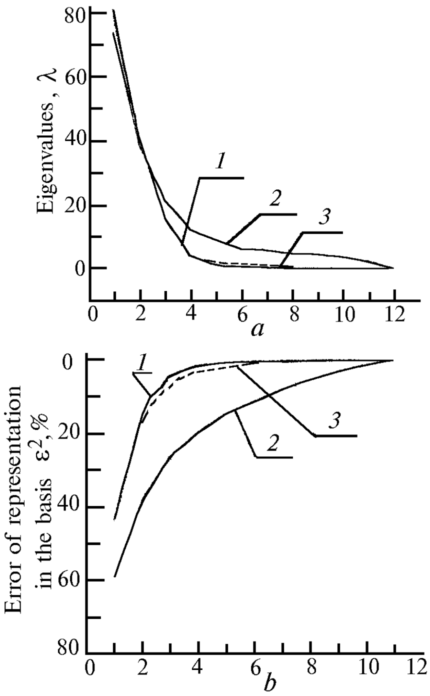

FIG. 1. Spectra of eigenvalues (a) and error of approximation in the KL basis (b).

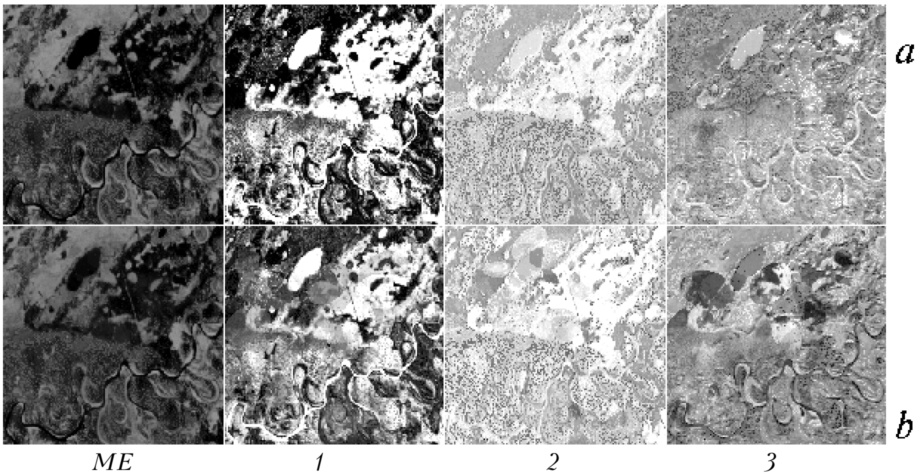

FIG. 2. Mathematical expectation (ME) and the first three functions of the KL basis (1, 2, and 3).

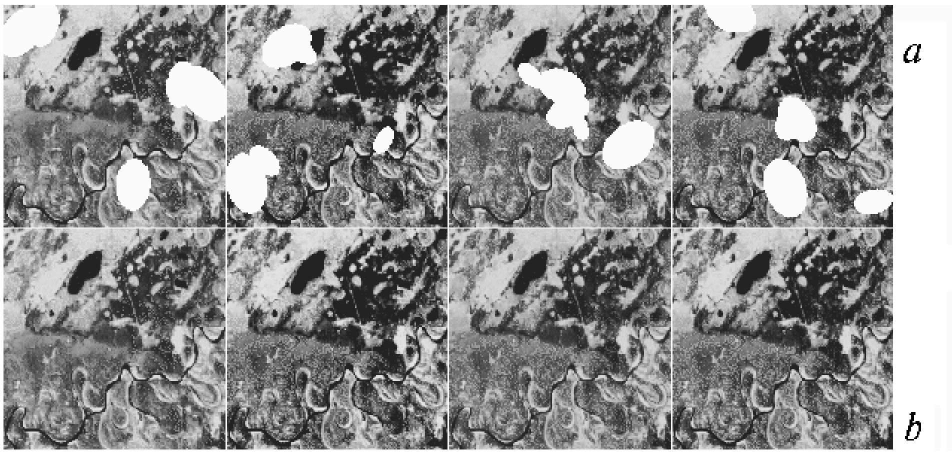

FIG. 3.

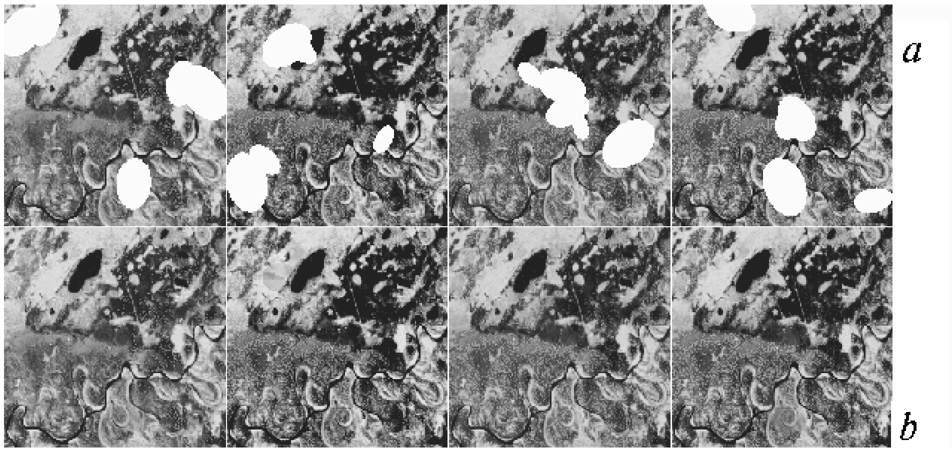

FIG. 4.

The mathematical expectation estimated from the sample and the first three basis functions (in the form of images) of the KL basis obtained in this experiment are illustrated by Fig. 2*a*.

Then the problem of reconstruction of ten images with cloud fragments was solved with the use of the basis so constructed. Figure 3*a* illustrates four of these images with clouds, whereas Fig. 3*b* displays their reconstructed copies.

In this case, the quality of image reconstruction to the initial sharp images, estimated by criterion (10), was equal to $\mu[\varepsilon] \pm \sigma[\varepsilon] = (0.53 \pm 0.13)\%$, where $\mu[\ ]$ is the mathematical expectation, and $\sigma[\ ]$ is the standard deviation of the reconstruction error ε estimated from ten images.

When criterion (10) is integrated only over the region occupied by clouds, the accuracy is equal to $\mu[\varepsilon] \pm \sigma[\varepsilon] = (1.6 \pm 0.45)\%$. At the same time, the difference between the complete image and the image in the form of mathematical expectation displayed in Fig. 2*a* was $\mu[\varepsilon] \pm \sigma[\varepsilon] = (11.8 \pm 2.5)\%$. If only overcast fragments of video images were replaced by their average values, then $\mu[\varepsilon]\pm\sigma[\varepsilon] = (12.1 \pm 1.5)\%$.

The second run of experiments corresponds to the case in which the KL basis was reconstructed from images with clouds. At the first step of iteration process of basis construction, the overcast fragments of video images were replaced by the values of brightness averaged over unshadowed fragments of corresponding images.

The spectrum of eigenvalues of the basis of the first approximation is shown in Fig. 1*a* (curve *2*), whereas Fig. 1*b* (curve *2*) shows the quality of approximation of observation ensemble in this basis. The quality of reconstruction of ten images in this case was $\mu[\varepsilon] \pm \sigma[\varepsilon] = (2.1\pm0.6)\%$, and upon integrating Eq. (10) over the cloud area, it was $\mu[\varepsilon] \pm \sigma[\varepsilon] = (6.5 \pm 2.0)\%$.

Figure 2*b* shows the estimated mathematical expectation and the first three basis functions corresponding to this case. Figures 4*a* and *b* show four images with clouds and their reconstructed copies, respectively. Then in the moving regime of reconstruction, when the next basis image is reconstructed in the first approximation basis, all basis images were reconstructed.

Once the realizations had been reconstructed, the KL basis of the second approximation was constructed. The spectrum of eigenvalues of this basis is shown in Fig. 1*a* (curve *3*), whereas the approximation error in this basis is shown in Fig. 1*b* (curve *3*). The behavior of the curves is indicative of the increasing role of the first basis functions and eigenvalues. The quality of ten images being reconstructed in this basis has increased, and $\mu[\varepsilon]\pm\sigma[\varepsilon] = (0.96 \pm 0.36)\%$. In this case, the modified quality criterion mentioned above was $\mu[\varepsilon] \pm \sigma[\varepsilon] = (2.97 \pm 1.1)\%$. Preliminary experiments on reconstruction of video images demonstrate high efficiency of the proposed method.

ACKNOWLEDGMENT

I would like to express my deep gratitude to Dr. V.V. Belov for his support of this work and to V.I. Khamarin, Senior Scientific Worker of the Institute of Ecology of Natural Complexes, for satellite photographs kindly placed at my disposal.

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