# SEPARATION OF LIGHT SCATTERING COMPONENTS IN MULTIFREQUENCY LASER SENSING OF THE UPPER ATMOSPHERE

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*An algorithm for separating the aerosol and molecular scattering components in multifrequency laser sensing of the upper atmosphere is considered. It is shown that the proposed algorithm is stable with respect to errors in the input data.*

Separation of the scattering components is an important and complicated problem in laser sounding of the atmosphere. This problem becomes particularly important in sounding of the upper layers of the atmosphere. The following two factors should be noted here. First, scattering on aerosol particles makes a contribution to the optical characteristics of the atmosphere which may be as large as the contribution of the molecular component.[1] Therefore, the aerosol component of scattering should be taken into account. Second, the values of the optical characteristics of the upper atmosphere are very small. They are of the order of $10^{-2}$ or $10^{-3}$ km$^{-1}$. It immediately follows that stringent requirements must be imposed on the accuracy of algorithms for separating the scattering components.

As a rule, such algorithms are based on an *a priori* assignment of aerosol models.[2] In this case the problem of separation is reduced to determining one or several (depending on the bulk of experimental information) normalizing factors. It stands to reason that the efficiency of such an approach is largely determined by the selection of the appropriate models. This approach is well justified for single–frequency sounding. However, if lidars with three or more operating wavelengths are used, then it is possible to develop more efficient algorithms for separating the aerosol and molecular components of scattering. Such algorithms call for a lesser amount of an *a priori* information.

Recall that in sounding of the upper layers of the atmosphere the recorded signals are equal to

$$N(\lambda_i, h_l) = b(\lambda_i)\, h_l^{-2}\, T_{\mathrm{a}}^2(\lambda_i, h_0) \left[\beta_\pi^{\mathrm{m}}(\lambda_i, h_l) + \beta_\pi^{\mathrm{a}}(\lambda_i, h_l)\right] \times$$

$$\times\, T_{\mathrm{a}}^2(\lambda_i, h_0 - h_l)\, T_{\mathrm{m}}^2(\lambda_i, h_l)\, \Delta h_l\ , \ i = 1, 2, \ldots, n, \qquad (1)$$

where $N(\lambda_i, h_l)$ is the number of photoimpacts from the distance $h_l$; $\lambda_i$ is the wavelength; $b(\lambda_i)$ is the scaling factor; $T_{\mathrm{a}}^2(\lambda_i, h_0)$ is the square atmospheric aerosol transmission; $\beta_\pi^{\mathrm{m}}(\lambda_i, h_l)$ and $\beta_\pi^{\mathrm{a}}(\lambda_i, h_l)$ are the molecular and aerosol backscattering coefficients, respectively; $T_{\mathrm{a}}^2(\lambda_i, h_0 - h_l)$ is the square aerosol transmission of the layer lying between $h_0$ and $h_l$; $T_{\mathrm{m}}^2(\lambda_i, h_l)$ is the square molecular transmission of the layer below $h_l$; and, $\Delta h_l$ is the distance between two adjacent counts (strobe length).

In general, system (1) consists of $n$ equations and contains $2n$ unknowns. However, if we make use of an analytic expression defining the spectral behaviour of the molecular scattering coefficients, the number of unknowns may be reduced in this system. Really, the molecular backscattering coefficients can be represented in the form

$$\beta_\pi^{\mathrm{m}}(\lambda_i, h_l) = p(\lambda_i)\, \beta_\pi^{\mathrm{m}}(\lambda_1, h_l)\ , \qquad (2)$$

where $p(\lambda_i)$ is the fractional–rational function of the known form determined by the theory of molecular scattering. In general, it can be written down as

$$p(\lambda_i) = \left(\frac{\lambda_1}{\lambda_i}\right)^4 \left(\frac{m_{0i} - 1}{m_{01} - 1}\right)^2 . \qquad (3)$$

In Eq. (3) the refractive index of air is determined by the Edlen formula[3]

$$(m_{0i} - 1) \cdot 10^6 = 64.328 + \frac{29498.1}{146 - \dfrac{1}{\lambda_i^2}} + \frac{255.4}{41 - \dfrac{1}{\lambda_i^2}} ,$$

which holds at $T_0 = 288$ K and $P_0 = 1013$ nbar. As a result, the number of unknowns in system (1) is reduced to $n + 1$. It is possible to make the problem well defined at the expense of the aerosol component.[4] This operation is correct since the signals form a set of interrelated values.

Let us consider the algorithm for solving this problem. This algorithm is iterative. The shortest wavelength is separated out for determining the molecular component and the remaining $n - 1$ equations are considered. We write down these equations in the form

$$\frac{N(\lambda_i, h_l)\, h_l^2}{b(\lambda_i)\, T_{\mathrm{a}}^2(\lambda_i, h_0)\, T_{\mathrm{m}}^2(\lambda_i, h_{l+1})\, T_{\mathrm{a}}^2(\lambda_i, h_0 - h_{l+1})\, \Delta h_l} - \beta_\pi^{\mathrm{m}}(\lambda_i, h_{l+1}) =$$

$$= \beta_{\pi,0}^{\mathrm{a}}(\lambda_i, h_l)\ , \ i = 2, \ldots, n, \qquad (4)$$

where $h_0$ is the altitude at which the lidar has been calibrated, $h_{l+1}$ is the altitude of the preceding strobe at which all optical characteristics have been determined. Recall that the interpretation is made from above, i.e., $h_0 > h_{l+1} > h_l$. Solving then the inverse problem of aerosol light scattering in the form

$$\alpha s(r) + \frac{1}{(\Lambda_2 - \Lambda_1)(R_2 - R_1)^3} \int_{R_1}^{R_2} \int_{R_1}^{R_2} \int_{\Lambda_1}^{\Lambda_2} G(r, \eta)\, K_{\mathrm{p}}(\lambda_i, m_i, \xi) \times$$

$$\times\, K_\pi(\lambda_i, m_i, \eta)\, s(\xi)\, \mathrm{d}\lambda\, \mathrm{d}\xi\, \mathrm{d}\eta = \frac{1}{(\Lambda_2 - \Lambda_1)(R_2 - R_1)^3} \times$$

$$\times \int\limits_{R_1}^{R_2}\int\limits_{\Lambda_1}^{\Lambda_2} G(r,\eta)\,K_\pi(\lambda_i, m_i, \eta)\,\beta^{a}_{\pi,0}(\lambda_i, h_l)\,d\lambda\,d\eta , \qquad (5)$$

we find the particle size distribution function $s_a(r)$. In Eq. (5), $\alpha$ is the regularization parameter, $K_\pi(\lambda_i, m_i, r)$ are the backscattering efficiencies, and $G(r, \eta)$ is the Green's function for the integrodifferential Euler equation. Since the structure of such algorithms has been studied extensively in Ref. 5, we do not dwell on these questions here.

Having determined the particle size distribution function we next calculate any characteristic of aerosol light scattering. In particular, with a knowledge of the size distribution function we can calculate the aerosol extinction coefficient

$$\beta^{a}_{ex,1}(\lambda_i, h_l) = \int\limits_{R_1}^{R_2} K_{ex}(\lambda_i, m_i, r)\,s_\alpha(r)\,dr \qquad (6)$$

and the square aerosol transmission

$$T^2_{a,1}(\lambda_i, h_0 - h_l) = T^2_{a}(\lambda_i, h_0 - h_{l+1})\,\exp\{[\beta^{a}_{ex}(\lambda_i, h_{l+1}) +$$

$$+ \beta^{a}_{ex,1}(\lambda_i, h_l)]\,\Delta h_l\} . \qquad (7)$$

It is evident that the calculated values are the first approximation to the real values. The process is repeated starting with formula (4) until the condition

$$|\beta^{a}_{\pi,j}(\lambda_i, h_l) - \beta^{a}_{\pi,j-1}(\lambda_i, h_l)| \le \delta_1 \qquad (8)$$

is satisfied, where $j$ is the serial number of iteration.

It should be noted that the process converges sufficiently rapidly. This is due to the fact that the exponential in Eq. (7) is close to unity.

Knowing the particle size distribution function, it is possible to predict the aerosol optical characteristics for the particular wavelength. Let $\beta^{a}_{\pi,q}(\lambda_i, h_l)$ be the backscattering coefficient calculated by formula (5). It is assumed that $T^2_{m,0}(\lambda_1, h_{l+1} - h_l) = 1$. Then the equation

$$\frac{N(\lambda_i, h_l)\,h_l^2}{Z_1\,T^2_{m,0}(\lambda_1, h_{l+1} - h_l)} - \beta^{a}_{\pi,q}(\lambda_i, h_l) = \beta^{m}_{\pi,0}(\lambda_i, h_l) ,$$

where

$$Z_1 = b(\lambda_1)T^2_a(\lambda_1, h_0)\,T^2_m(\lambda_1, h_{l+1})\,T^2_a(\lambda_1, h_0 - h_{l+1})\Delta h_l , \qquad (9)$$

determines the zeroth approximation of the molecular component. The molecular scattering coefficient is found based on the well–known relation

$$\beta^{m}_{sc,0}(\lambda_i, h_l) = \frac{2}{3}\,\beta^{m}_{sc,0}(\lambda_1, h_l) . \qquad (10)$$

Then we calculate the first approximation of the molecular transmission

$$T^2_{m,1}(\lambda_1, h_{l+1} - h_l) = \exp\{[\beta^{m}_{sc}(\lambda_1, h_{l+1}) + \beta^{m}_{sc,0}(\lambda_1, h_l)]\,\Delta h_l\}, \qquad (11)$$

and substitute it into Eq. (9). If the condition

$$|\beta^{a}_{\pi,\kappa}(\lambda_i, h_l) - \beta^{m}_{\pi,\kappa-1}(\lambda_i, h_l)| \le \delta_2 \qquad (12)$$

is not satisfied, the process [starting with formula (9)] is repeated ($\kappa$ is the serial number of iteration). It should be noted that this process converges rapidly as well, since the square transmission, determined from formula (11), is close to unity.

In what follows formula (2) can be used to calculate the backscattering coefficient. Formulas (3) enable one to find the molecular scattering coefficients. Then, following the relation

$$T^2_{m,1}(\lambda_i, h_{l+1} - h_l) = \exp\{[\beta^{m}_{sc}(\lambda_i, h_{l+1}) - \beta^{m}_{sc,1}(\lambda_i, h_l)]\Delta h_l\} \qquad (13)$$

we calculate the first approximation of the square molecular transmission. The difference

$$\frac{N(\lambda_i, h_l)\,h_l^2}{Z_i\,T^2_m(\lambda_i, h_{l+1} - h_l)} - \beta^{m}_{\pi,\kappa}(\lambda_i, h_l) = \beta^{a}_{\pi,1}(\lambda_i, h_l),\ i = 2, \ldots, n, \qquad (14)$$

where $Z_i = b(\lambda_i)\,T^2_a(\lambda_i, h_0)\,T^2_m(\lambda_i, h_{l+1})T^2_a(\lambda_i, h_0 - h_l)\,\Delta h_l$ ,

is formed, which represents the first approximation of the backscattering coefficient. If the condition

$$|\beta^{m}_{\pi,s}(\lambda_i, h_l) - \beta^{m}_{\pi,s-1}(\lambda_i, h_l)| \le \delta_3 \qquad (15)$$

holds, the process is terminated, otherwise it is repeated starting with formula (5). As the numerical calculations show, from 5 to 7 iterations must be carried out for the given iterative process to converge.

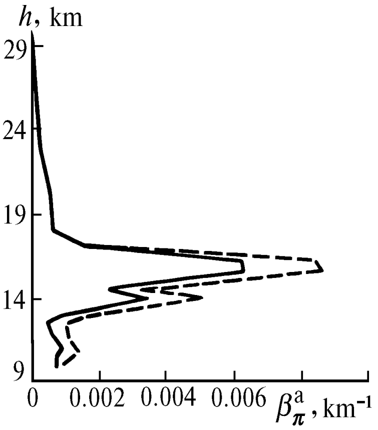


*FIG. 1. Vertical profile of the backscattering coefficient at a wavelength of* 0.339 μm. *Dashed curve is for the exact distribution, solid curve is for the reconstructed one.*

To check the stability of the above–described algorithm, the numerical experiment was conducted. The microstructure model of the aerosol in the upper atmosphere was for Deirmendjian's $H$ haze model.[6] At altitudes of from 12 to 18 km "perturbation" was introduced. Its microstructure obeyed the lognormal law.[7] The refractive index of aerosol particles was taken to be 1.43 – 0 which corresponded to 75 % sulfuric acid.[8] The molecular component was calculated for the model borrowed from Ref. 9. Results of numerical experiment are shown in Figs. 1 and 2. The results refer to the lidar operating at four wavelengths ($\lambda_i$ = 0.339, 0.353, 0.532, and 0.683 μm). Depicted in Fig. 1 is the altitude behaviour of the aerosol backscattering coefficient at a wavelength of 0.339 μm. It illustrates the systematic error. Peculiar feature of the algorithm is that the largest deviation of the exact coefficient from the reconstructed one is observed at the above–indicated wavelength.

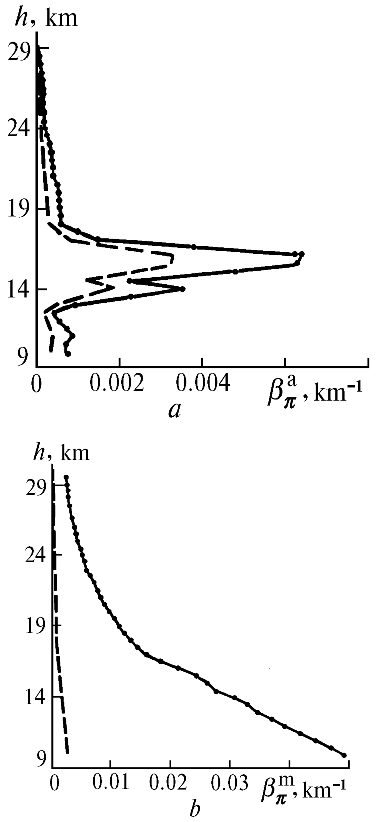


*FIG. 2. Vertical profiles of the backscattering coefficients.*

As can be seen from Fig. 1, the deviation of the exact solution (dashed curve) from the reconstructed one (solid curve) increases with altitude decrease. It reaches maximum in the layer of aerosol perturbation. Figure 2 illustrates the algorithmic stability. The altitude behaviour of aerosol (Fig. 2*a*) and molecular (Fig. 2*b*) backscattering coefficients at wavelengths of 0.339 μm (solid curve) and 0.683 μm (dashed curve) are shown in this figure. Plotted on the graph with dots are the backscattering coefficients derived with the 5 % error in the lidar returns. It follows from the figure that the additionally introduced error only slightly distorts the backscattering coefficients.

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