Methods of retrieval of atmospheric optical parameters from polarization lidar sensing data. Part 2. Problems of *a priori* uncertainty in the scattering phase matrix

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We consider methods of inversion of polarization sensing data acquired in the presence of a significant multiple scattering (MS) in optical signals. The second component of Stokes vector is extensively studied and it is shown that the simple parameterization of MP contribution significantly reduces the bulk of *a priori* information required for signal interpretation. Numerical experiment has been conducted to analyze possible errors in the retrieved extinction coefficient and lidar ratio, caused by incorrect set of the scattering phase matrix in inverting the returns calculated by the Monte Carlo method.

Introduction

Inversion of polarization lidar sensing data, especially of those obtained in the presence of significant background of multiple scattering (MS), is an ill-posed problem because its solution requires *a priori* information on the scattering phase matrix (on lidar ratio in the single frequency sensing from the ground). Promising is the joint interpretation of the results of active and passive sensing,[1] including the use of different active sensing methods such as Raman, multifrequency, and polarization techniques.[2] Multifrequency sensing allows one to separate cloud and aerosol layers of isotropic scatterers,[3] while polarization sensing makes it possible to separate anisotropic particles[1]; therefore, within the simplest classification (aerosol – water cloud – ice cloud), the problem of identification of the scattering object can be considered to be solved. However, determination of concrete type of scatterers (within one class) is not unambiguous.[4]

In polarization sensing in the presence of significant MS level in return signal (such as in satellite sensing) it is possible to consider MS as informative component and retrieve the lidar ratio.[5,6] The joint estimate of depolarization and lidar ratios allows one to estimate, with some degree of confidence, the type and size of scattering particles. The applicability of the method is restricted to the assumption on microphysical homogeneity of the scattering medium and adequacy of mathematical model describing multiple scattering processes. The solution of the problem is further complicated by the absence of models of scattering phase matrices for both ice and mixed-phase polydisperse ensembles of cloud particles.

Description of analytical models for polarization components of the lidar returns and estimate of influence of the main parameters entering the models on the information content of MS contribution for aerosol and ice clouds are given in Section 1 of this paper. Section 2 considers retrieval algorithms for profiles of extinction coefficient and lidar ratio of a single component homogeneous medium. The methodical questions related to inversion of polarization components of the lidar returns are considered extensively in the first part of the paper by Samoilova.[7] We tested the algorithm for retrieval of the optical parameters from signals calculated by Monte Carlo method for three aerosol models and three ice cloud models. In Section 3 we show examples of retrieval of optical parameters under conditions of *a priori* uncertainty in the scattering phase matrix.

1. Model of lidar equation

In the case of large distances between lidar and the object sounded, the signal from the photodetector is described by the lidar equation, which, in the single-scattering approximation for linearly polarized radiation, has the following form

$$F_{\|,\perp}(z) \sim \frac{\beta_{\|,\perp}(z)}{z^2} T^2(z_0, z), \qquad (1)$$

where β is the backscattering coefficient;

$$T^2(z_0,z) = \exp\left\{-2\int_{z_0}^{z}\sigma(z')\mathrm{d}z'\right\} = \exp\{-2\tau(z_0,z)\}$$

is the integrated transmission; and σ is the extinction coefficient. Subscripts "$\|$" and "$\perp$" correspond to parallel and perpendicular polarization components of the lidar return; they are related to components of Stokes vector by the equations:

$$\mathbf{S}^{(1)}(z) = \begin{pmatrix} F_{\|} + F_{\perp} \\ F_{\|} - F_{\perp} \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} (\beta_{\|} + \beta_{\perp})T^2(z_0,z)/z^2 \\ (\beta_{\|} - \beta_{\perp})T^2(z_0,z)/z^2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \beta_1 T^2(z_0,z)/z^2 \\ \beta_2 T^2(z_0,z)/z^2 \\ 0 \\ 0 \end{pmatrix}. \qquad (2)$$

We assume that for the problem considered here the normalized scattering phase matrix has the form

$$A_{ij} = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{12} & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & -a_{43} \\ 0 & 0 & a_{43} & a_{44} \end{pmatrix}. \qquad (3)$$

This assumption is valid both for water and ice clouds containing symmetrical particles randomly oriented in space.[8,9] Lidar equation for sensing from a satellite, which takes into account the MS contribution for components of the Stokes vector, can be represented as[6]:

$$\mathbf{S}^{(\Sigma)} = \begin{pmatrix} S_1^{(1)} \times S_1^{(m)} \\ S_2^{(1)} \times S_2^{(m)} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} S_1^{(1)} \exp\{m_1\} \\ S_2^{(1)} \exp\{m_2\} \\ 0 \\ 0 \end{pmatrix}. \qquad (4)$$

The factor $S_i^{(m)}$, accounting for multiple scattering contribution, depends on the extinction coefficient, lidar ratio, and on the function representing a combination of elements of the scattering phase matrix:

$$m_i(z) = m_i[\sigma S_i, G_i] =$$

$$= 2\pi \int_{z_1}^{z_2} \sigma(x) S_i(x) \int_0^{\varphi(z,x)} G_i(\varphi)\,\mathrm{d}\varphi\,\mathrm{d}x, \quad i = 1,\ 2, \qquad (5)$$

where $S_1 = \sigma/\beta_1 = 1/a_{11}(\pi)$ is the lidar ratio,

$$S_2 = \sigma/\beta_2 = 1/a_{22}(\pi);$$

$$G_1(\varphi) = \sin(\varphi)\left[a_{11}(\varphi)a_{11}(\pi - \varphi) + a_{12}(\varphi)a_{12}(\pi - \varphi)\right],$$

$$G_2(\varphi) = \frac{1}{2}\sin(\varphi)\left[a_{12}(\varphi)a_{12}(\pi - \varphi) + a_{22}(\varphi)a_{22}(\pi - \varphi) - \right.$$

$$\left. -a_{33}(\varphi)a_{33}(\pi - \varphi) + a_{34}(\varphi)a_{34}(\pi - \varphi)\right];$$

$$\varphi(z,x) = \arccos\frac{2ac - (a^2 + c^2)\cos(\varphi_0)}{2ac\cos(\varphi_0) - (a^2 + c^2)},$$

φ_0 is the receiver's field-of-view (FOV) angle; and parameters $a = z - x/2$ and $c = \sqrt{a^2 - b^2} = x/2$, where a and b are, respectively, major and minor semiaxes of the ellipse defining the scattering geometry in the theory.[10]

One of the factors reducing the applicability of the model (4) and (5) is the multiplicative representation of the MS contribution to the total signal. Definition of the signal in the form of the product of singly and multiply scattered components (in the same way as, e.g., in small-angle approximation) leads to omission of regions beyond the cloud boundary, which is at the distance where $S_i^{(1)}(z) \equiv 0$. In addition, for satellite-based viewing geometry, when ellipse totally falls within the receiver's field of view, $m_i(z) \to \tau(z)$.[6] Thus, model (4) and (5) can be used for description of signals coming from optically thin media, namely aerosols or cirrus clouds. For the first component of the Stokes vector it is an extension of well known model,[11,12] which takes into account the MS contribution, and which is used to interpret the ground-based measurements:

$$S_1^{(\Sigma)}(z) = S_1^{(1)}(z)\exp\left\{2\int_{z_0}^{z}\left[\sigma(z')(1 - \eta_1(z'))\right]\mathrm{d}z'\right\}, \qquad (6)$$

where $\eta_1(z) \in \,]0,\,1[$. If it is assumed that $\eta_1(z) \cong$ $\cong \mathrm{const}$, then the following expression is valid:

$$\eta_1(z) = 1 - \frac{\ln\left\{S_1^{(\Sigma)}(z)/S_1^{(1)}(z)\right\}}{2\tau(z)} = 1 - \frac{m_1(z)}{2\tau(z)},$$

where $m_1(z)$ is defined by formula (5). Winker[5] has shown that model (6) can be used to interpret satellite-based measurements. In that paper, Monte Carlo calculations of the parameter η_1 for different scattering phase functions demonstrated that $\eta_1(z) \cong$ $\cong \mathrm{const}$, and that this constant is totally determined by the type of the scattering particles. Let us write the model formula analogous to Eq. (6), now for the second component of Stokes vector:

$$S_2^{(\Sigma)}(z) = S_2^{(1)}(z)\exp\left\{2\int_{z_0}^{z}\left[\sigma(z')(1 - \eta_2(z'))\right]\mathrm{d}z'\right\}, \qquad (7)$$

where

$$\eta_2(z) = 1 - \frac{\ln\left\{S_2^{(\Sigma)}(z)/S_2^{(1)}(z)\right\}}{2\tau(z)} = 1 - \frac{m_2(z)}{2\tau(z)},$$

and $m_2(z)$ is defined by formula (5).

Figure 1 presents parameters $\eta_1(z)$ and $\eta_2(z)$ calculated by Monte Carlo method (asterisks) and from formulas (6) and (7) (solid lines).

In constructing the Monte Carlo algorithm we used known local estimate of the particle flux.[13] The main principles and details of construction of such estimates are given in Ref. 14. Calculations were performed for lidar located at a distance $z_0 = 690$ km from the upper boundary of the scattering layer (wavelength $\lambda_0 = 532$ nm, receiver's FOV $\varphi_0 =$

= 130 mrad, and divergence of sounding radiation $\psi_0 = 100$ mrad). In calculations of the curves in Figs. 1*a*–*b* we assumed a homogeneous aerosol layer with the extinction coefficient 0.5 km^{-1} and geometrical thickness of 2 km. The results presented in the lower part of the figure are obtained for a homogeneous cloud composed of different types of crystals randomly oriented in space ($\sigma = 0.5$ km^{-1}, $\Delta z = 2$ km). We assumed three aerosol models, namely "background" (Fig. 1*a*), "dust" (Fig. 1*b*), and "maritime" aerosols (Fig. 1*c*).[5] In addition, we assumed three models of cloud composed of ice crystals in the form of "columns" with $L/a = 50$ μm/10 μm (Fig. 1*e*), "plates" with $L/a = 8$ μm/10 μm (Fig. 1*f*), and a "mixture of plates and columns" (Fig. 1*d*).[15,16] The "mixture of plates and columns" is a model matrix for a polydisperse ice cloud; it is a combination of scattering phase matrices for hexagonal fixed-size crystals. The mixture consists of "plates" with sizes $L/a = 8/10$, 9/15, 32/40, 24/50, and 42 μm/100 μm and columns with sizes $L/a = 50/10$, 100/20, 100/40, 100/50, and 300 μm/60 μm; all crystal sizes are assumed to have identical contributions.

Since the error of Monte Carlo calculations of the of Stokes vector components depends on the errors in calculation of all the four components, in calculating the matrix of a polydisperse ensemble one fails to attain quite smooth dependences $\eta_1(z)$ and $\eta_2(z)$ (see Fig. 1*d*). This is due to high degree of asymmetry of scattering phase function and strong variations of the components of the matrix for the directions of scattering close to 180°.

Analysis of the results obtained allows us to identify the following interesting interrelations. First, the conclusions formulated in Ref. 5 concerning the parameter $\eta_2(z)$ are also valid: model (7) is applicable to description of the second component of the Stokes vector, and $\eta_2(z) \cong \text{const}$ and is fully determined by the type of scatterers. Second, the ratio $\eta_1(z)/\eta_2(z) \cong$ $\cong \text{const}$ and, therefore, the polarization characteristics of lidar returns in the case of long-distance sensing depend, in accordance with expression (5), on the lidar ratio (for the first component of the Stokes vector this same conclusion was drawn in Ref. 5) and on the integral of the functions, which depend on the combination of elements of the scattering phase matrix. The angular differences between elements of the matrix, found to be highly significant for different types of crystals, are not that critical in sensing from space. This conclusion is very important for signal interpretation because it substantially reduces the bulk of *a priori* information required for signal inversion.

2. Methods of inverting the polarization lidar returns

Among the merits of the model (4) and (5) there is the simplicity of the polarization ratio:

$$p(z) = \frac{S_2^{(\Sigma)}(z)}{S_1^{(\Sigma)}(z)} = p^{(1)}(z)\exp\{m_2(z) - m_1(z)\} = p^{(1)}(z)\times$$
$$\times\exp\left\{2\pi\int\limits_{z_1}^{z_2}\sigma(x)\left[S_2(x)\int\limits_0^{\varphi(z,x)}G_2(\varphi)\mathrm{d}\varphi - S_1(x)\int\limits_0^{\varphi(z,x)}G_1(\varphi)\mathrm{d}\varphi\right]\mathrm{d}x\right\}, \tag{8}$$

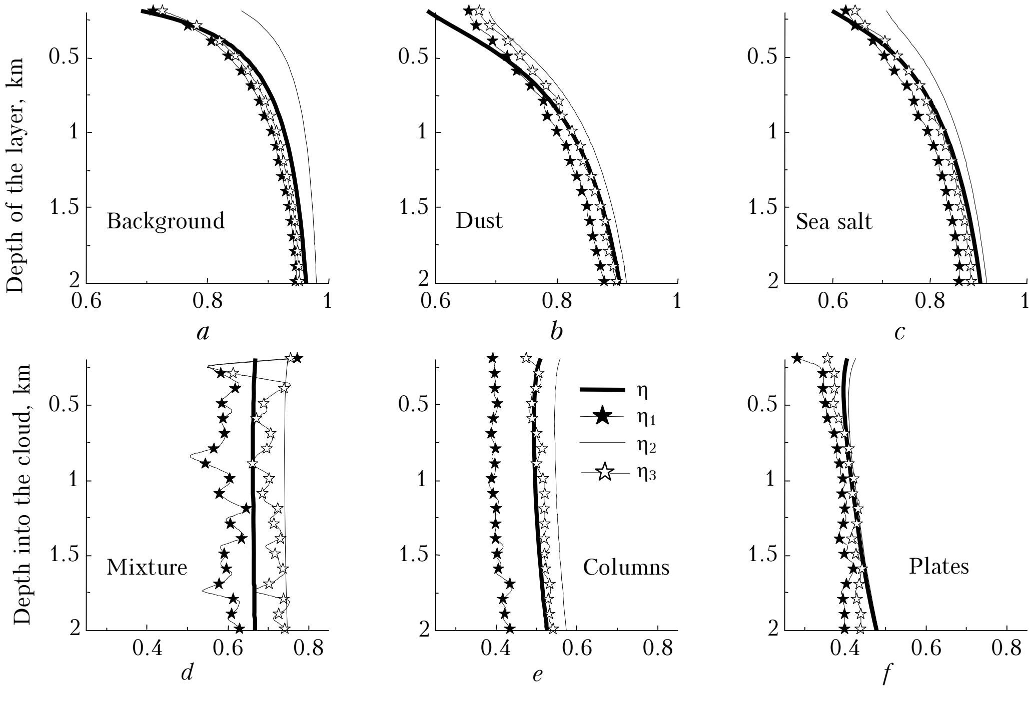

Fig. 1. Comparison of methods of calculation of functions characterizing MS contribution to polarization components of the lidar return: Monte Carlo method (asterisks); analytical model by formulas (6) and (7) (solid lines).

where

$$p^{(1)}(z) = \frac{\beta_{\parallel}(z) - \beta_{\perp}(z)}{\beta_{\parallel}(z) + \beta_{\perp}(z)}$$

is the polarization ratio caused by single scattering. For the logarithmic derivative Eq. (8) the following relations are valid[6]:

$$\frac{\partial}{\partial z}\ln\left[p(z)\right] \approx \frac{\partial p^{(1)}/\partial z}{p^{(1)}(z)} +$$

$$+2\pi\sigma(z)S_1(z)\left[\frac{S_2(z)}{S_1(z)}\int\limits_0^{\pi}G_2(\varphi)\mathrm{d}\varphi - \int\limits_0^{\pi}G_1(\varphi)\mathrm{d}\varphi\right] =$$

$$= \frac{\partial p^{(1)}/\partial z}{p^{(1)}(z)} + 2\pi\sigma(z)S_2(z)\left[\int\limits_0^{\pi}G_2(\varphi)\mathrm{d}\varphi - \frac{S_1(z)}{S_2(z)}\int\limits_0^{\pi}G_1(\varphi)\mathrm{d}\varphi\right]. \qquad (9)$$

For a homogeneous medium, $\partial p^{(1)}/\partial z = 0$, so the first term in Eq. (9) vanishes, the function in square brackets (which depends on scattering phase matrix) is constant, and, hence, the logarithmic derivative is proportional to the product of the extinction coefficient and the lidar ratio:

$$\zeta_1(z) = \sigma(z)S_1(z) \approx \frac{1}{2\pi W_1}\frac{\partial}{\partial z}\left[\ln p(z)\right], \qquad (10)$$

where

$$W_1 = \int\limits_0^{\pi}\left[\frac{S_2(z)}{S_1(z)}G_2(\varphi) - G_1(\varphi)\right]\mathrm{d}\varphi.$$

Also, the following formula is valid

$$\zeta_2(z) = \sigma(z)S_2(z) \approx \frac{1}{2\pi W_2}\frac{\partial}{\partial z}\left[\ln p(z)\right], \qquad (11)$$

where

$$W_2 = \int\limits_0^{\pi}\left[G_2(\varphi) - \frac{S_1(z)}{S_2(z)}G_1(\varphi)\right]\mathrm{d}\varphi.$$

The algorithm of signal inversion is constructed as follows: at the first stage, it is necessary to correctly differentiate the logarithm of the polarization ratio. Below we use the quadratic spline approximation of the profile of logarithm of the polarization ratio averaged over 200 m range interval. It is noteworthy, that this method is recommended in reconstructing the profile of the lidar ratio from data of Raman lidar sensing.[17] The function $\zeta_1(z)$ obtained in accordance with the expression (10) makes it possible to estimate MS contribution (see Eq. (5)) and, accordingly, the signal caused by single scattering. Using the method of local calibration with known $\sigma(z_*)$ value at the far end of sounding path (see Ref. 7 for details), the first equation in Eqs. (2) is inverted with respect to $\sigma(z)$, and profile of the lidar ratio is estimated from formula

$$S_1(z) = \zeta_1(z)/\sigma(z).$$

In the upper part of Fig. 2 we present results of retrieval of optical parameters of three clouds composed of "plates/columns mixture" (Figs. 2*a* and *d*), "columns" (Figs. 2*b* and *e*), and "plates" (Figs. 2*c* and *f*).

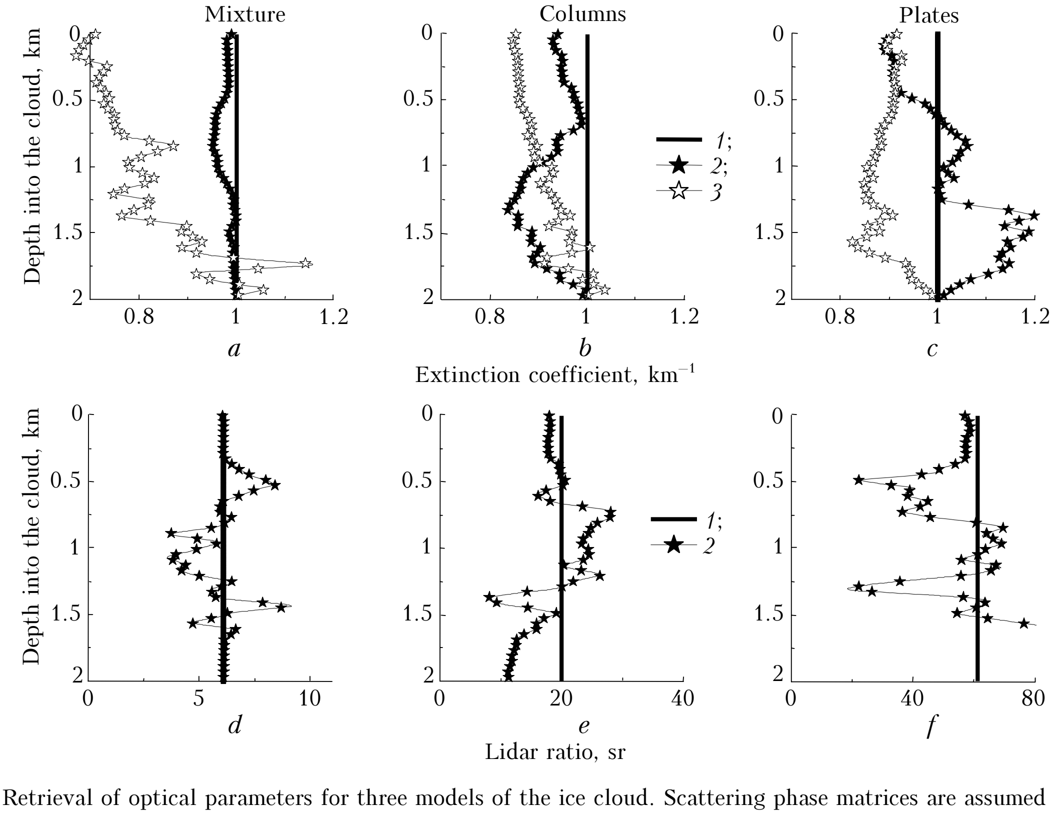

Fig. 2. Retrieval of optical parameters for three models of the ice cloud. Scattering phase matrices are assumed to be known.

Parameters of lidar, used for Monte Carlo calculations of the signal, are given in description of Fig. 1, while functions determining the MS contribution are presented in the lower part of Fig. 1. Curves *1* correspond to model profiles of the sought parameters. Curves *2* are obtained from formula (4) by the method of logarithmic derivative in inverting of equation for the first component of the Stokes vector. The estimate of MS contribution has been performed in accordance with Eqs. (5) and (10). Additionally the method makes it possible to estimate the lidar ratio (lower part of the figure). Curves *3* (in the upper part of the figure, for extinction coefficient) are also obtained from Eq. (4), but corrected for the MS background by use of iteration method. Retrieval errors are on the order of 20% for extinction coefficient and approximately coincide for both methods. For lidar ratio, the errors do not exceed 50%, so it is possible to estimate the average lidar ratio.

Figure 3 presents the results on optical parameters retrieved for three aerosol models. The lidar signals were calculated by Monte Carlo method, while the functions accounting for the MS contribution are presented in the upper part of Fig. 1. Figures at the curves are the same as in Fig. 2. For "background" aerosol (Figs. 3*a* and *d*) the MS contribution is very small (see Fig. 1*a*), and so the function $\zeta_1(z)$ is estimated with large errors. This leads to an increase of errors in processing by the method of logarithmic derivative (as compared with the iteration method) during $\sigma(z)$ retrieval and to unstable retrieval of the lidar ratio. For "dust" (Figs. 3*b* and *e*) and "maritime" (Figs. 3*c* and *f*) aerosols the errors are on the order of 10% for extinction coefficient and 30% for the lidar ratio.

Note that the problems of *a priori* uncertainty in setting W_i and $G_i(\varphi)$ were not considered here; and the type of scatterers and scattering phase matrices were assumed known. Oscillations of the profiles of extinction coefficient, retrieved by iteration method (see curves *3*), coincide with the oscillations of signals calculated by Monte Carlo method (we deliberately did not smoothed them out since random errors are always present in real signals). Spline approximation was used to smooth the profiles $\ln p(z)$, for which the mean relative errors ranged from 30% for "plates" to 120% for the "background" aerosol. Methodical aspects of stable numerical differentiation of logarithm of experimentally measured functions (ratio of two measured functions, for our method) have been extensively discussed by Pappalardo et al.,[17] and are not addressed here.

3. Retrieval of optical parameters for unknown scattering phase matrix

The problem of *a priori* uncertainty concerning the type of scatterers is seemingly one of the most complex problems in interpretation of lidar sensing data. The simplest illustration is the need for *a priori* setting the lidar ratio in inverting the equations (1) for two-component medium, and a complete solution

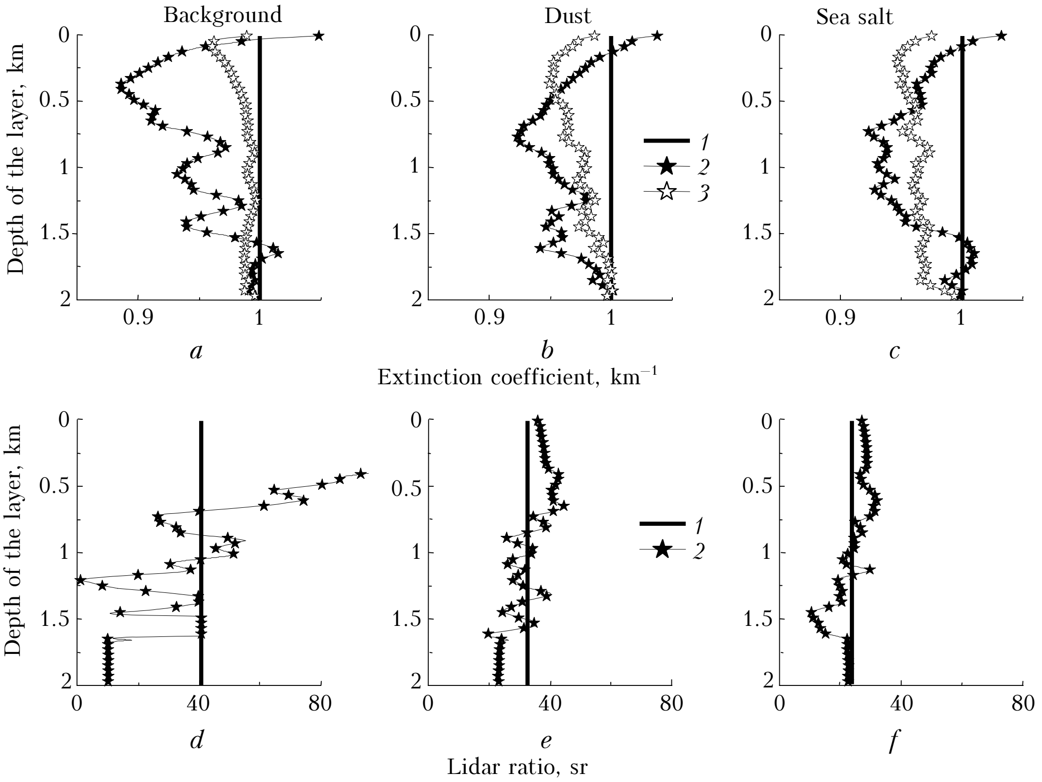

Fig. 3. Retrieval of optical parameters for three aerosol models. Scattering matrices are assumed to be known.

of this problem is possible only in the case of Raman sensing. In this paper, we did not pose the problem of identification of the type of scattering object according to lidar measurements: within the classification "aerosol—water cloud—ice cloud" the problem is considered to be solved. Our purpose is to show, by use of numerical simulation, how large errors may result from incorrectly set scattering phase matrix in retrieval of optical parameters from polarization measurements, and, possibly, offer recommendations concerning the use of a particular algorithm. Note that for water clouds the choice of the scattering phase matrix in processing is not critical because the variability range of the lidar ratio is not wide, matrices are close, and retrieval of the parameter in the presence of MS background is stable.[18]

3.1. Ice cloud

Figure 4 presents the results of retrieval of optical parameters of polydisperse cloud composed of the "plates/columns mixture." Conditions of numerical experiment are analogous to those in Fig. 1*a*; the polarization characteristics describing MS contribution are presented in Fig. 1*d*. Retrieval of extinction coefficient (the upper part of the figure) and lidar ratio (the lower part) was performed for known scattering phase matrix (Fig. 4*a*, *d*), as well as for "error" matrices for "columns" (Fig. 4*b* and *e*) and "plates" (Fig. 4*c* and *f*). Curves *1* correspond to model profiles of the parameters sought. Curves *2* are obtained by the method of logarithmic derivative and curves *3* (for the extinction coefficient) by iteration method. Analysis of results shows that (1) incorrect setting of the scattering phase matrix practically does not influence the accuracy of the lidar ratio retrieval, with only small overestimation by no more than retrieval error for the case of known scattering phase matrix; and (2) both the method of logarithmic derivative and iteration method produce approximately the same error in retrievals of the extinction coefficient for known scattering phase matrix, both for aerosol and clouds (see Figs. 2 and 3). An exception is just the result for mixed-phase cloud (see Figs. 2*a* and 4*a*) in which case the iteration method gives large error of retrieval. Therefore, best retrieval of $\sigma(z)$ under conditions of *a priori* uncertainty by iteration method (Figs. 4*b* and *c*) must seemingly be interpreted as an overestimation due to the use of scattering phase functions with large lidar ratio. On the contrary, for the method of logarithmic derivative the retrieved $\sigma(z)$ decreases with the increase of the lidar ratio used in the matrices; however, overall the errors of determination of the parameter do not exceed retrieval error for the known scattering phase matrix, being on the order of 20%.

Figure 5 shows the results of determination of optical parameters of monodisperse cloud composed of "columns"; functions accounting for the MS contribution to lidar returns are presented in Fig. 1*e*.

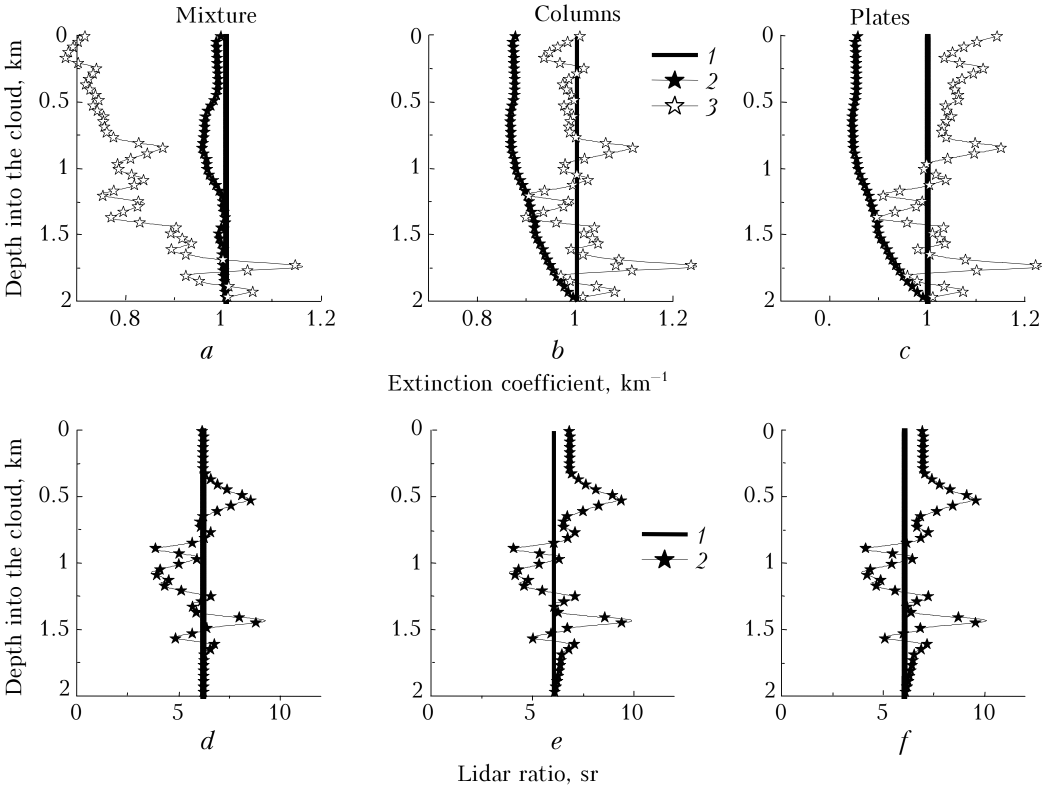

Fig. 4. Retrieval of optical parameters of polydisperse ice cloud for the known scattering phase matrix (*a* and *d*), as well as for "erroneous" matrices for "columns" (*b* and *e*) and "plates" (*c* and *f*).

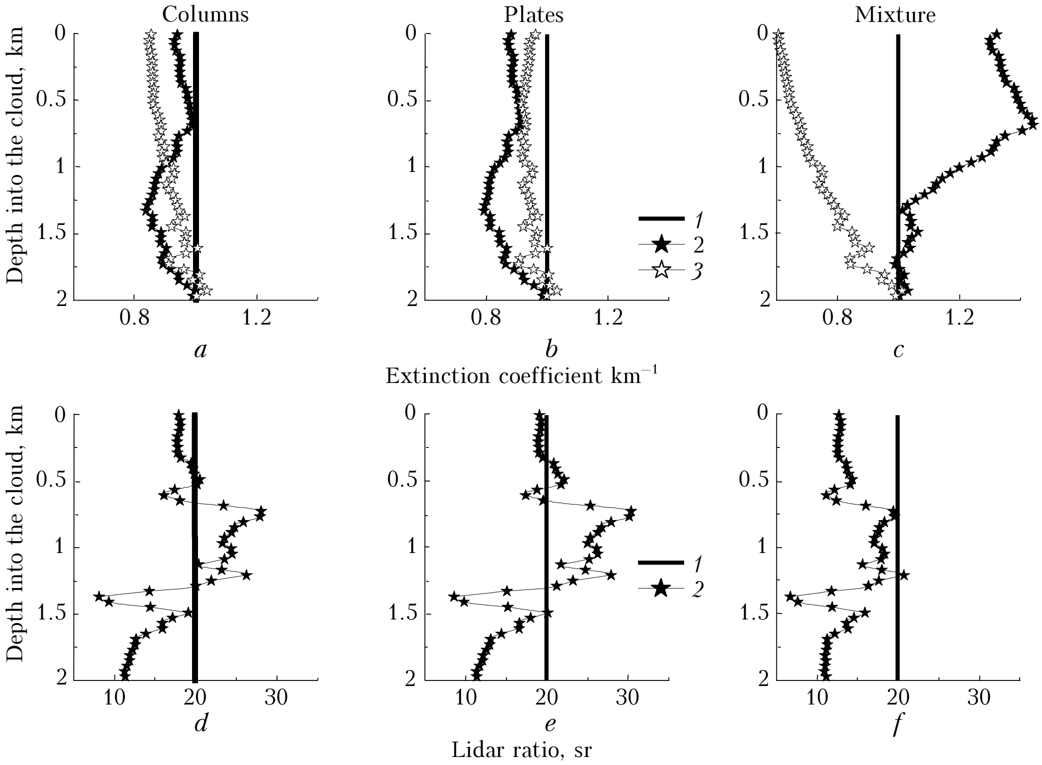

Fig. 5. Retrieval of optical parameters of monodisperse ("columns") ice cloud for known scattering phase matrix (*a* and *d*), as well as for "erroneous" matrices for "plates" (*b* and *e*) and polydisperse cloud (*c* and *f*).

Retrieval was performed for the known scattering phase matrix, as well as for "error" matrices for "plates" and "plates/columns mixture." Figures at the curves are as in Fig. 4.

Retrieval accuracies of both extinction coefficient and the lidar ratio practically coincide if the matrix for "plates" rather than "columns" is used in processing. The retrieved profiles are markedly distorted if the "error" matrices for polydisperse cloud are used in signal inversion, with about 40% underestimation of the lidar ratio, and with errors increased up to 50% for the extinction coefficient. Moreover, as was already noted in analysis of Fig. 4, *a priori* uncertainty concerning the scattering phase matrix leads to opposite distortions in $\sigma(z)$ retrieved by different methods (see Fig. 5*c*).

Here we do not present the results of retrieval of optical parameters for cloud composed of "plates" because they nearly replicate the results shown in Fig. 5. Summarizing, we can note that (1) the profile of lidar ratio is more stable in retrieval using "error" scattering phase matrix; and (2) large discrepancy in the retrieval of extinction coefficient by different methods clearly indicates that the matrix used for processing is incorrect.

3.2. Aerosol

Figure 6 presents the results of determination of optical parameters of "maritime" aerosol. The results of signal processing with "erroneous" matrices are presented in Figs. 6*b* and *e* ("dust" aerosol) and Figs. 6*c* and *f* ("background" aerosol). The structure of Fig. 6 is analogous to that of Figs. 4 and 5. Analysis of the results shows that the aerosol parameters have smaller retrieval errors than cloud parameters under conditions of *a priori* uncertainty, with both parameters being determined accurate to the retrieval accuracy for known scattering phase matrix. Here we do not present the results of signal processing for "dust" aerosol because they practically replicate the results shown in Fig. 6. For the "background" aerosol, because of insignificant level of MS contribution to the return signal, the $\zeta_1(z)$ estimation according to Eq. (10) leads to considerable errors and instability of optical parameter retrievals; these results are omitted here either.

Conclusion

We considered the method of interpretation of polarization sensing data obtained in the presence of considerable MS contribution to the return signal. Data of the numerical Monte Carlo simulation, performed for satellite lidar, are used to examine information content of the MS contribution. For a single-component homogeneous medium the MS background is shown to depend significantly on the value of the backscattering phase function; also it is confirmed that the multiple scattering is informative with respect to the type of scatterers.

The algorithm of simultaneous retrieval of the profiles of extinction coefficient and lidar ratio, the so-called method of logarithmic derivative of polarization coefficient, was tested in assessing three

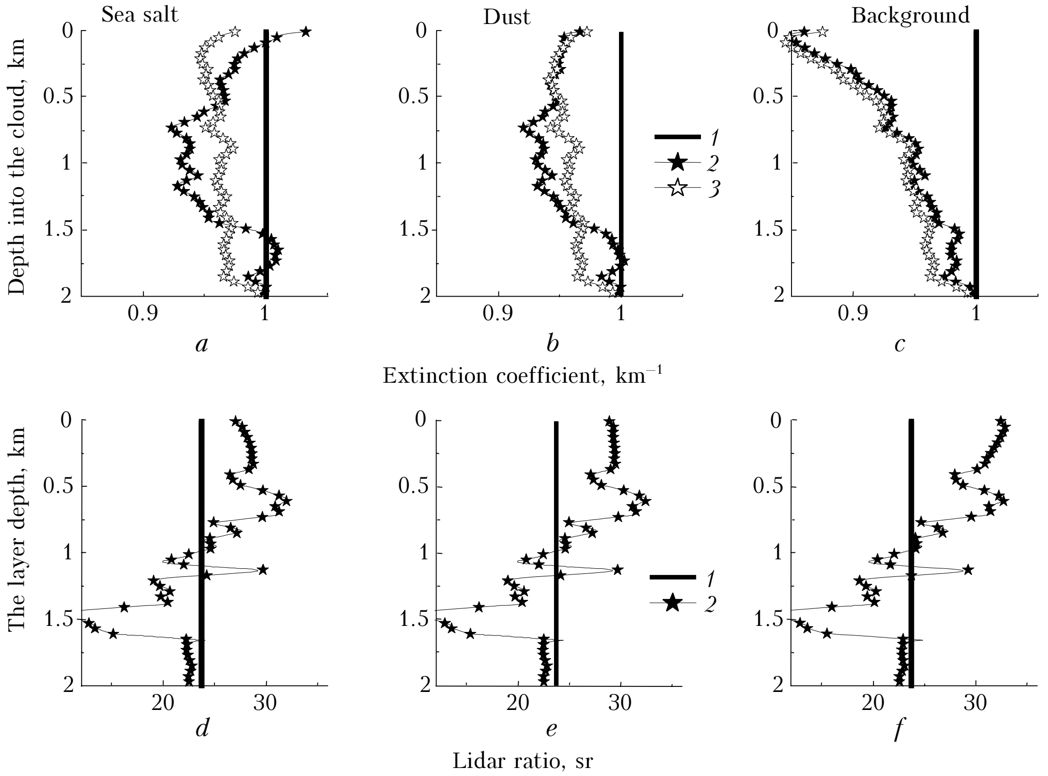

Fig. 6. Retrieval of optical parameters of “maritime” aerosol for known scattering phase matrix (*a* and *d*), as well as “erroneous” matrices for “dust” (*b* and *e*) and “background” (*c* and *f*) aerosols.

aerosol models and three ice cloud models. The main methodical difficulty of the algorithm implementation is the need in correct numerical differentiation of experimental data. Determination of the profiles for “background” aerosol, having minimum MS contribution to return signal, is demonstrated to be unstable. For other models the accuracy of retrieval of the optical parameters is satisfactory and comparable with the accuracy of traditional methods.

In studying possible errors arising under conditions of *a priori* uncertainty concerning determination of the type of scatterers, it is shown that (1) the profile of lidar ratio is more stable if incorrect scattering phase matrix is used in retrieval, and (2) *a priori* uncertainty leads to opposite distortions in retrieval of extinction coefficient by the method of logarithmic derivative and by the iteration method. It is reasonable to perform estimation of $\sigma(z)$ by both methods, because the discrepancies are large, clearly indicating that the matrix used in processing is incorrect.

Of course, the three models of scattering phase matrix considered here are insufficient to account for all crystal habits in ice clouds. Systematic analysis (if possible at all) undertaken by specialists in order to reveal specific features in behavior of the components of scattering matrices of crystals of similar (different) types and different (similar) sizes is of great interest. We also realize that the performed studies are rather qualitative because of the neglect of inconstancy of optical parameters along the sounding path. Moreover, applicability of Eq. (10) heavily relies upon the condition of homogeneity of the polarization ratio, which for the real ice clouds is almost always not the case. To correctly use the method for estimation of parameters, the total polarization profile should be preprocessed to separate out the component caused by multiple scattering. For this, it is first necessary to estimate the polarization ratio caused by single scattering (feasibility of this estimate, even without the account of MS effect, was demonstrated earlier[7]), and then to divide the total polarization profile by the obtained estimate. An implementation of the algorithm was discussed by Samoilova et al.,[19] who presented for satellite-based lidar the calculations of polarization components of lidar signal for the case of two-component and inhomogeneous (along the sounding path) medium.

Acknowledgments

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