AN ACCOUNT OF THE CONTINUUM ABSORPTION IN THE THREE–FREQUENCY METHOD OF TEMPERATURE SENSING

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Neglect of the finite absorption in the gap between the lines introduces the systematic error in lidar measurement of temperature by the three–frequency method of differential absorption. Working formulas for the estimate of the temperature from lidar returns that correct this error are derived and the order of magnitude of this correction is estimated.

Introduction. The principle of lidar sensing of the temperature profile proposed by Mason[1] and based on the temperature dependence of rotational levels of a chosen gas molecule was embodied in practical methods in a number of works, in particular, in Ref. 2.

These methods do much better (including maximum sensing range, accuracy, and spatial and temporal resolutions for prescribed energy parameters) in comparison with the Raman–scattering methods because of larger interaction cross sections, but they impose heavy demands on the spectral characteristics of a transmitter and receiver, necessary *a priori* information, signal processing, and interpetation of the results of sensing. Depending on a chosen strategy of overcoming these difficulties, stemming from the levels of technical and informational support, one uses: bifrequency method[3] which imposes less stringent requirements upon the transmitting system, three–frequency method[2] capable of decreasing the number of the *a priori* assumptions and assigned parameters, four–frequency method capable of selecting the lines with optimal relation between their absorptions, and other modifications.

The spatially resolved three–frequency method based on sensing at the wavelengths λ_1 and λ_2 lying at the centers of two absorption lines and at λ_0 lying in the gap between the lines, in fact is based on a comparison of the estimates for one and same gas concentration by the method of differential absorption at the pairs of the wavelengths λ_1 and λ_0 and λ_2 and λ_0. The classical working formula for the temperature estimate from lidar returns by the three–frequency method[2] was derived under the assumption that there is no absorption by a working gas at λ_0. This introduces the systematic error which is of primary significance when one or two weak lines are used. The present paper eliminates this drawback.

Equation for the temperature estimate. Let us introduce the following designations:

P_0 and T_0 are the pressure and temperature at which the parameters of the absorption lines of the gas in use were measured;

P and T are the real pressure and temperature in the examined layer of the atmosphere;

σ_{0i} is the absorption cross section of individual gas molecule at λ_i, P_0, and T_0 ($i = 0, 1, 2$); σ_i is the same but at P and T; $\Delta\sigma_i = \sigma_i - \sigma_0$ ($i = 1, 2$);

S_{0i} is the line intensity at T_0 ($i = 1, 2$); S_i is the same but at T;

E''_i is the energy of the lower state of the transition from λ_i ($i = 1, 2$); $\Delta E' = E''_1 - E''_2$;

γ_{0i} is the halfwidth of the line at P_0 and T_0 ($i = 1, 2$); γ_i is the same but at P and T;

$Q(T)$ is the incomplete function;

n_i is the coefficient of temperature dependence of the line halfwidth; $\Delta n = n_1 - n_2$;

$\tilde{M}_i$ and $\tilde{\tau}_i$ are the estimates of the working gas concentration and double optical depth of differential absorption in the strobe at λ_i and λ_0; the tilde denotes the estimate of the parameter from lidar returns;

k_B is Boltzmann's constant;

j is the serial number of spatial strobes ($j = 1, 2, 3, ...$);

α_{ij} and β_{ij} are the coefficients of elastic extinction and backscattering in the strobe ($i = 1, 2$);

n^s_{ij}, n^{bg}_{ij}, and n_{ij} are the numbers of single–electron pulses (SEP), dark–current background, and pulses in the temporal strobe $\Delta t = 2L/c$ of lidar return signal.

Having written down the estimates of the working gas concentration in the form

$$\tilde{M}_i = \tilde{\tau}_i / 2L\Delta\sigma_i \,, \ i = 1, 2 \,,$$

and having equated them, we obtain

$$\ln\Delta\sigma_1 - \ln\Delta\sigma_2 = \ln(\tilde{\tau}_1/\tilde{\tau}_2) \,. \tag{1}$$

Formulas for the estimates $\tilde{\tau}_i$ from lidar return signals are well known, in particular, in the photon counting regime of detection

$$\tilde{\tau}_{ij} = \ln \frac{\tilde{n}^s_{ij}\, \tilde{n}^s_{0,\,j+1}}{\tilde{n}^s_{0j}\, \tilde{n}^s_{i,\,j+1}} + B - C \,, \ i = 1, 2, j = 1, 2, 3, ...,$$

$\tilde{n}^s_{ij} = n_{ij} - \tilde{n}^{bg}_{ij}$, where $\tilde{n}^{bg}_{ij}$ is the estimate of the number of the dark–current SEP in the corresponding strobe obtained by *a priori* calculations or by counting of the SEP in the time intervals between the lidar returns,

$$B = \ln(\beta_{i,\,j+1}\beta_{0j}/\beta_{ij}\beta_{0,\,j+1}) \,, \ C = 2L\,(\alpha_{ij} - \alpha_{0j}) \,.$$

Let us write down the left side of Eq. (1) in the form $\ln(\sigma_1/\sigma_2) + A$, where

$$A = \ln\left(1 - \frac{\sigma_0}{\sigma_1}\right) - \ln\left(1 - \frac{\sigma_0}{\sigma_2}\right) = \sum_{k=1}^{\infty} \frac{\sigma_0^{\kappa}}{\kappa}\left(\frac{1}{\sigma_2^{\kappa}} - \frac{1}{\sigma_1^{\kappa}}\right). \qquad (2)$$

As is well known[2],

$$\sigma_i = \frac{S_i}{\pi\gamma_i}, \quad \gamma_i = \gamma_{0i}\frac{P}{P_0}\left(\frac{T_0}{T}\right)^{n_i},$$

$$S_i = S_{0i}\frac{Q(T_0)}{Q(T)}\exp\left[\frac{E'_i}{\kappa_B}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]$$

from which it follows that

$$\sigma_i = \sigma_{0i}\frac{Q(T_0)P_0}{Q(T)P}\left(\frac{T}{T_0}\right)^{n_i}\exp\left[\frac{E''_i}{\kappa_B}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right], \qquad (3)$$

where $\sigma_{0i} = S_{0i}/\pi\gamma_{0i}$. Let us approximate Eqs. (2) and (3) making use of the fact that

$$|\Delta T| = |T - T_0| \ll T_0 .$$

By designating

$$\mu = \Delta n + \frac{E''}{\kappa_B T_0}, \quad \nu = \mu T_0\left(\frac{1}{T_0} - \frac{1}{T}\right), \qquad (4)$$

we find

$$\frac{\sigma_1}{\sigma_2} \simeq \frac{\sigma_{01}}{\sigma_{02}}(1 + \nu),$$

$$\frac{1}{\sigma_2^{\kappa}} - \frac{1}{\sigma_1^{\kappa}} = \frac{1}{2}\left[\frac{1}{\sigma_2^{\kappa}}\left(1 - \frac{\sigma_2^{\kappa}}{\sigma_1^{\kappa}}\right) - \frac{1}{\sigma_1^{\kappa}}\left(1 - \frac{\sigma_1^{\kappa}}{\sigma_2^{\kappa}}\right)\right]|$$

$$\simeq \frac{1}{2}\left[\frac{1}{\sigma_{02}^{\kappa}} - \frac{1}{\sigma_{01}^{k}} - \frac{1}{\sigma_{01}^{\kappa}(1 + \nu)^{\kappa}} + \frac{1}{\sigma_{02}^{\kappa}}(1 + \nu)^{\kappa}\right]. \qquad (5)$$

By taking $\sigma_0 = \sigma_{00}$ and designating $\rho_i = \sigma_{00}/\sigma_{0i}$ ($i = 1, 2$) and $\eta = 1 + \nu$, we obtain

$$A = \frac{1}{2}\left[\ln(1 - \rho_1) - \ln(1 - \rho_2) + \ln(1 - \rho_1/\eta) - \ln(1 - \rho_2\eta)\right].$$

Then Eq. (1) for the estimate takes the form

$$\ln\eta + \frac{1}{2}\ln\frac{(1 - \rho_1)(1 - \rho_1/\eta)}{(1 - \rho_2)(1 - \rho_2\eta)} = \ln\xi ,$$

where

$$\xi = \frac{\tilde{\tau}_1 \sigma_{02}}{\tilde{\tau}_2 \sigma_{01}} ,$$

or after taking the antilogarithms,

$$(1 - \rho_1)\eta^2 - [(1 - \rho_1)\rho_1 - (1 - \rho_2)\rho_2 \xi^2]\eta - (1 - \rho_2)\xi^2 = 0. \qquad (6)$$

The solution of the equation for the estimate. The solution of Eq. (6) $\tilde{\eta}$ gives the estimate $\tilde{\nu} = \tilde{\eta} - 1$ and by virtue of Eq. (4), the temperature estimate

$$\tilde{T} = T_0(1 - \tilde{\nu}/\mu)^{-1} . \qquad (7)$$

In the zeroth approximation in σ_{00} (i.e., when ρ_1 and $\rho_2 = 0$) we obtain from Eq. (6)

$$\tilde{\nu}^{(0)} = \xi - 1, \quad \tilde{T}^{(0)} = T_0\left(1 - \frac{\xi - 1}{\mu}\right)^{-1} ,$$

that for $|\nu| \ll 1$, as could be expected, gives the relation

$$\tilde{T}^{(0)} = T_0\left[1 - \frac{\ln(\tilde{\tau}_1 \sigma_{02}/\tilde{\tau}_2 \sigma_{01})}{\Delta n + \Delta E''/\kappa_B T_0}\right]^{-1} , \quad \nu \ll 1 , \quad \sigma_{00} = 0 , \qquad (8)$$

which agrees with formula (5) of Ref. 2. On the other hand, for $\nu \ll 1$ formula (5) gives the estimate

$$\tilde{\nu} \simeq \frac{(1 - \rho_2)^2\xi^2 - (1 - \rho_1)^2}{(1 - \rho_1)(2 - \rho_1) - (1 - \rho_2)\rho_2\xi^2} , \quad \nu \ll 1$$

that for ρ_1 and $\rho_2 = 0$ gives Eq. (8) once again. In general the sought—for root of Eq. (6) is determined by the correct asymptote over ρ_1 and ρ_2 and is equal to

$$\tilde{\eta} = \frac{\rho_1 - \rho_2 g}{2} + \sqrt{\frac{(\rho_1 - \rho_2 g)^2}{4} + g} , \qquad (9)$$

where

$$g = \frac{1 - \rho_2}{1 - \rho_1}\xi^2 .$$

In practice ρ_1 and ρ_2 are rather small and instead of Eq. (9) we may use the estimate in the first approximation in σ_{00}

$$\tilde{\eta}^{(1)} = \xi^2 + \rho_1(1 + \xi^2)/2 - \rho_2\xi^2 . \qquad (10)$$

Relative correction. To detemine the contribution of the continuum absorption, we consider the relative correction for the temperature estimate

$$\delta_T = \frac{\tilde{T} - \tilde{T}^{(0)}}{\tilde{T}^{(0)}}\bigg|_{\xi=1+\nu} \simeq \frac{\tilde{T} - \tilde{T}^{(0)}}{T_0}\bigg|_{\xi=1+\nu} = \frac{\tilde{\nu} - \tilde{\nu}^{(0)}}{\mu}\bigg|_{\xi=1+\nu} ,$$

taking into account that in accordance with Eq. (4)

$$\nu \simeq \frac{\Delta T}{T_0}\mu .$$

In the general case from Eq. (9) we obtain

$$\delta_T = \frac{1}{\mu}\left[\frac{\rho_1 - \rho_2 h}{2} + \sqrt{\frac{(\rho_1 - \rho_2 h)^2}{4} + h} - 1 - \nu\right], \qquad (11)$$

where

$$h = \frac{1 - \rho_2}{1 - \rho_1}(1 + \nu)^2 .$$

For clarity let us write down Eq. (11) in the first approximation σ_{00}

$$\delta_T^{(1)} = \frac{\Delta T}{T_0}\left[\varphi + \frac{\rho_1}{2}(1 + \varphi^2) - \rho_2\varphi^2\right],$$

where

$$\varphi = 1 + \nu \ .$$

For definitness we take $\rho_2 \leq \rho_1$. It is obvious that for given ρ_1 the quantity $\delta_T^{(1)}$ is linearly dependent on $\theta = \rho_2/\rho_1$ and takes the extremum values

$$\delta_{T_{\max}^{\min}}^{(1)} = \frac{\Delta T}{T_0} \left[\varphi + \frac{\rho_1}{2} \left(1 \mp \varphi^2 \right) \right]$$

for $\theta = 1$ and 0, respectively, and the average value

$$\delta_{T_{\mathrm{av}}}^{(1)} = \frac{\Delta T}{T_0} \left(\varphi + \frac{\rho_1}{2} \right)$$

for $\theta = 1/2$.

Conclusion. The derived simple working formulas (9) and (10) with the finite absorption in the gap between the lines taken into account correct the standard temperature estimate by the three–frequency method of differential absorption. The relative correction has the order of relative deviation of the real temperature from the reference one. Such a consideration, falling outside the scope of this paper, is necessary for the other methods of temperature measurement by differential absorption.

REFERENCES

1. J. Mason, Appl. Opt. **14**, No. 1, 76–78 (1975).
2. M. Endemann and R.L. Byer, Appl. Opt. **28**, No. 18, 3211–3217 (1981).
3. C.L. Korb, G.K. Schwemmer, M. Dombrowski, and C.Y. Weng, in: *Optics and Laser Remote Sensing* (Springer, Berlin, 1983), pp. 120–127.
4. G. Korn and T. Korn, *Mathematical Handbook for Scientists and Engineers* (McGraw–Hill, New York, 1961).