POWER OF A LIDAR RETURN FROM A SURFACE WITH COMPLICATED SCATTERING PHASE FUNCTION SOUNDED THROUGH THE ATMOSPHERIC LAYER

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This paper deals with study of optical power of a lidar return from a surface in the atmosphere with a complicated scattering phase function. Expressions that describe the received power from a surface with a scattering phase function having the diffuse and quasispecular components are derived for the case in which sensing is conducted through an optically dense aerosol atmosphere. It is shown that the received power essentially depends on the ratio of the diffuse and quasispecular components of the surface scattering phase function.

Energy characteristics of lidar returns recorded through the atmosphere, from a surface having either specular or a Lambertian scattering phase function have been studied in a number of papers (see, e.g., Refs. 1–4). Below we consider the power of a lidar return received through the atmospheric layer from a surface with a complicated scattering phase function.

Let the sensed surface possess the scattering phase function having the diffuse and the quasispecular component with a narrow reflection phase function, the maximum of which coincides with the direction of specular reflection. The brightness $I(\mathbf{R}, \mathbf{m})$ of radiation reflected from such a surface is

$$I(\mathbf{R}, \mathbf{m}) = I(\mathbf{R})\left[\alpha \cos^n\theta + \beta \exp\left\{-\frac{(\mathbf{m}_\perp - \mathbf{m}_{0\perp})^2}{\Delta^2}\right\}\right], \qquad (1)$$

where $\mathbf{R}$ is the spatial coordinate; $\mathbf{m}$ is the unit vector describing the direction of observations; $I(\mathbf{R})$ is some function of coordinates; α and β are coefficients determining the fractions of the diffuse and quasispecular reflections; $\mathbf{m}_\perp$ and $\mathbf{m}_{0\perp}$ are the vectors characterizing the direction of observations and that of the maximum of the reflected radiation (for the quasispecular component of the reflectivity); $\mathbf{m}_\perp = \{m_x, m_y\}$; $\mathbf{m}_{0\perp} = \{m_{x0}, m_{y0}\}$; $m_x = \sin\theta\cos\varphi$; $m_y = \sin\theta\sin\varphi$; $m_{y0} = \sin\theta_0\sin\varphi_0$; $m_{x0} = \sin\theta_0\cos\varphi_0$; (θ, θ_0) and (φ, φ_0) are the zenith angles and the azimuths of an observation and of the maximum quasispecular reflection. Angles θ_0 and φ_0 are interrelated according to the laws of geometric optics to corresponding angles θ_s and φ_s, which describe the direction of sounding radiation incidence; Δ is the parameter characterizing the angular width of the quasispecular component of the scattering phase function; and, n is the parameter characterizing the angular width of the diffuse component of the scattering phase function. For $\Delta \ll 1$ formula (1) takes the form

$$I(\mathbf{R}, \mathbf{m}) \simeq I(\mathbf{R})\left[\alpha \cos^n\theta + \beta \exp\left\{-\frac{(\varphi - \varphi_0)^2\cos^2\theta_0 + (\varphi - \varphi_0)^2\sin^2\theta_0}{\Delta^2}\right\}\right]. \qquad (2)$$

Let us normalize Eq. (2). The integral of $I(\mathbf{R}, \mathbf{m})\cos\theta$ over the half–space (i.e., over the solid angle of 2π into which the radiation is reflected) should be equal to the irradiance of the "secondary" source E at the surface[2] ($E(\mathbf{R}) = AE_s(\mathbf{R})$); $E_s(\mathbf{R})$ is the irradiance of the surface produced by sounding radiation incident upon it; A is the reflectance).

For $E(\mathbf{R})$ we obtain

$$E(\mathbf{R}) = \int_{2\pi} I(\mathbf{R}, \mathbf{m}) \cos\theta \, d\Omega = I(\mathbf{R})\left[\alpha \frac{2\pi}{n+2} + \beta\pi\Delta^2\right]. \qquad (3)$$

Thus, with normalization taken into account radiation brightness (2) is

$$I(\mathbf{R}, \mathbf{m}) = \frac{E(\mathbf{R})}{\alpha \dfrac{2\pi}{n+2} + \beta\pi\Delta^2}\left[\alpha \cos^n\theta + \beta \exp\left\{-\frac{(\theta - \theta_0)^2\cos^2\theta_0 + (\varphi - \varphi_0)^2\sin^2\theta_0}{\Delta^2}\right\}\right]. \qquad (4)$$

According to Ref. 3 the reflection phase function of the surface, corresponding to brightness (4) is equal to

$$\rho(\theta, \varphi) = \frac{I_{\mathrm{H}}(\mathbf{R}, \mathbf{m})}{I_0(\mathbf{R}, \mathbf{m})} = \frac{1}{\alpha \dfrac{2}{n+2} + \beta\Delta^2}\left[\alpha\cos^n\theta + \beta \exp\left\{-\frac{(\theta - \theta_0)^2\cos^2\theta_0 + (\varphi - \varphi_0)^2\sin^2\theta_0}{\Delta^2}\right\}\right], \qquad (5)$$

where $I_0(\mathbf{R}, \mathbf{m})$ is the brightness of the Lambertian surface.

When $\beta = 0$ and $n = 0$ formulas (1)–(5) are transformed into corresponding expressions for a Lambertian surface. At $\alpha = 0$ and as $\Delta \to 0$, Eqs (1)–(5) are transformed into the formula for a specularly reflecting surface.

We assume that the scattering surface is characterized by brightness (4). From the distribution of brightness over the scattering surface $I_n(\mathbf{R}, \mathbf{m})$ one can find the brightness

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of the radiation $I(\tilde{\mathbf{R}},\tilde{\mathbf{m}})$ incident upon the receiver,[5] and then, applying the reciprocity theorem to the scattering medium[5] and using the results from Ref. 3, construct an integral expression for the power of radiation collected by a receiver (we assume mutual shadowing of the surface elements to be negligible)

$$P = \int_S d\mathbf{R} \int d\Omega \cos\theta_d I_H(\mathbf{R}, \mathbf{m}) I_d(\mathbf{R}, \mathbf{m}) , \qquad (6)$$

where $I_d(\mathbf{R}, \mathbf{m})$ is the brightness at a point $\mathbf{R}$ of the surface S, produced by radiation incident upon it through the atmosphere from a "fictious" source with parameters of the receiver; θ_d is the angle between the normal to the surface and the direction to the receiver.

Further calculations are conducted for a homogeneously scattering atmosphere with a strongly–forward–peaked scattering phase function. We assume that the angle at which the receiving aperture is seen from the points at the scattering surface is much less than the angular width of the phase function for radiation reflected from that surface and the receiver field of view. This condition significantly simplifies calculating the integral in Eq. (6). It means physically that we neglect variations of the reflection phase function for those points of the surface, from which the radiation enters the receiver. Within the approximation of small angles for the polar diagrams of both the transmitter and receiver, assuming that the optical axes of the receiver and transmitter are in the XOZ plane, and using the results from Refs. 3, 6, and 7, one obtains the equation for power of radiation collected by the receiver in the following form:

$$P = \frac{A}{\pi} \frac{1}{\alpha \frac{2}{n+2} + \beta\Delta^2} \left[\alpha \cos^n\theta_s \int_S d\mathbf{R}\, E_s(\mathbf{R}') E_d(\mathbf{R}'') + \right.$$

$$+ \beta \int_S d\mathbf{R}\, E_s(\mathbf{R}') E_d(\mathbf{R}'') \times$$

$$\left. \times \exp\left\{ -\frac{1}{\Delta^2} \left[((\sin\theta_0 - \sin\theta_d) + R_x t)^2 + R_y^2 s^2 \right] \right\} \right] , \qquad (7)$$

where

$$S = \frac{A_s}{B_s} + \frac{A_d}{B_d} ; \quad t = \frac{A_s \cos^2\theta_s}{B_s} + \frac{A_d \cos^2\theta_d}{B_d} ;$$

$$A_{s,d} = \frac{1}{2} \sqrt{\alpha_{s,d}^2 + \sigma L_{s,d} \langle\gamma^2\rangle} ;$$

$$B_{s,d} = L_{s,d} \frac{\left[\frac{\alpha_{s,d}^2}{2} + \frac{\sigma L_{s,d} \langle\gamma^2\rangle}{4} \right]}{\sqrt{\alpha_{s,d}^2 + \sigma L_{s,d} \langle\gamma^2\rangle}} ;$$

$$\mathbf{R}' = \{R_x \cos\theta_s, R_y\} ; \ \mathbf{R}'' = \{R_x \cos\theta_d, R_y\} ;$$

$E_s(\mathbf{R})$, $E_d(\mathbf{R})$ are the irradiances of the surface produced by the actual and the "fictious" sources, respectively (the latter having the parameters of the receiver) (see Refs. 3 and 6) ; L_s and L_d are the distances from source and the receiver to the surface, respectively; $2\alpha_s$, $2\alpha_d$ are the divergence angles of the source and the receiver field of view, respectively; σ is the scattering coefficient of the atmosphere; $\langle\gamma^2\rangle$ is the variance of the angle of incident radiation deflection during an elementary act of scattering.

When $\beta = 0$, $n = 0$, Eq. (7) is transformed into that for the power received from the Lambertian surface, and at $\alpha = 0$ as $\Delta \to 0$ it transforms into the expression for a power received from a specularly reflecting surface.

By calculating the integrals entering into Eq. (7) we obtained

$$P = \frac{1}{\alpha \frac{2}{n+2} + \beta\Delta^2} \left\{ \alpha \cos^n\theta_d\, p^{-1/2} q^{-1/2} + \beta \left[p + \frac{s^2}{\Delta^2} \right]^{-1/2} \times \right.$$

$$\left. \times \left[q + \frac{t^2}{\Delta^2} \right]^{-1/2} \exp\left\{ - \frac{(\sin\theta_0 - \sin\theta_d)^2}{\Delta^2} \frac{q}{q + \frac{t^2}{\Delta^2}} \right\} \right\} \times$$

$$\times \frac{A P_0 \cos\theta_s \cos\theta_d\, r_d^2\, \alpha_d^2 \exp\{-(\varepsilon - \sigma)(L_s + L_d)\}}{16 B_s^2 B_d^2} , \qquad (8)$$

where

$$p = \frac{1}{4B_s^2} + \frac{1}{4B_d^2} ; \quad q = \frac{\cos^2\theta_s}{4B_s^2} + \frac{\cos^2\theta_d}{4B_d^2} ;$$

P_0 is the power emitted by the source; r_d is the effective size of the receiving aperture; ε is the atmospheric extinction coefficient.

At $\beta = 0$, $n = 0$, and $\sigma = 0$ Eq. (8) is transformed into the expressions for power received from a flat Lambertian surface through a transparent aerosol atmosphere,[3] and at $\alpha = 0$ as $\Delta \to 0$ it transforms into the expression for power received from a flat specularly reflecting surface.

Figure 1 shows the dependence of N (which is the ratio of the power P to power $P(\beta = 0, n = 0)$, calculated for a Lambertian surface), on the parameter β/α. The calculations were made using Eq. (8) and the following values of the parameters: $n = 0$ (Lambertian surface);

$$\theta_0 = \theta_d ; \Delta = 10^{-3} \cdot (0.057°); L_s = L_d = 10^3 \text{ m}; \alpha_d = \frac{1}{2} \cdot 10^{-3};$$

$$\alpha_s \ll \alpha_d ; \sigma\langle\gamma^2\rangle = \frac{1}{2} \cdot 10^{-10} \frac{1}{\text{m}} \text{ (curve } \mathit{1}); \sigma\langle\gamma^2\rangle = \frac{1}{2} \cdot 10^{-8} \frac{1}{\text{m}} \text{ (curve } \mathit{2}).$$

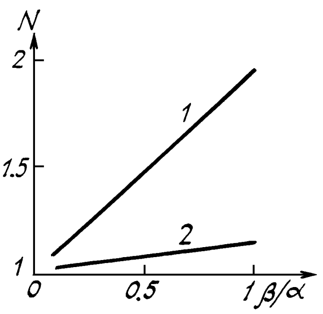

FIG. 1. Received power vs. the ratio of diffuse to the quasispecular component of the surface scattering phase function.

It can be seen from the figure that the received power strongly depends on the ratio of the quasispecular component of the scattering phase function to the diffuse. The more transparent is the atmosphere, the stronger is this dependence.

The equations derived in this paper can be useful for development of lidar systems and for analysis of their operation.

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