INVESTIGATION OF THE EFFECT OF AEROSOL PARTICLES ORIENTATION ON THE AEROSOL LIGHT SCATTERING PARAMETERS

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Effect of a predominant orientation of cylindrical particles on the lidar return formation with an account for sounding radiation polarization is analyzed based on numerical simulation estimates. Model computational results for a polydisperse ice crystal ensemble with typical particle size $R_m = 1.0$ *and* 10.0 *μm and radiation wavelength* $\lambda = 1.06$ *μm as well as an algorithm for computing the conversion matrix for passing to the case of arbitrarily oriented cylinders with finite length are presented.*

The multiple phase transformations of the atmospheric moisture within the clouds of vertical evolution under conditions of their thermal nonhomogenity and intense exchange processes between their top and bottom parts give rise to a significant variability of the shape of aerosol particles in such clouds. It is also well known that in convective and cirrus clouds the ice crystals often have the shape of extended columns or needles. When analyzing the light–scattering parameters they may be approximated within certain accuracy limits by cylinders of finite lengths (CFL). Depending on the intensity of turbulent flows the factor of prevailing orientation of symmetry axes of such aerosol particles both relative to the sounding beam direction and with respect to the reference plane plays an important role in forming return signal from the scattering volume of crystalline formations.[1]

The problem on light–scattering by an infinite cylinder has been solved in the coordinate system which is related to the cylinder orientation. When the cylinder axis lies in the reference plane[2,3] the phase scattering matrix (PSM) for the CFL has a simple form. However, the model estimations of the scattering and extinction coefficients, which have been obtained in Ref. 2, and likewise for the scattering phase function of an ensemble of CFL, whose axes lie in the reference plane, have a limited applicability. An analysis of optical properties of an ensemble of spatially polyoriented particles assumes that numerical estimates obtained in different coordinate system are reduced to one systems in the reference plane.

In this paper we present an algorithm for computing the conversion matrix (CM) for backscttering of light by arbitrarily oriented CFL's. Some estimates of the variability of the Stokes vector–parameter components depending on the degree of prevailing orientation of the particles and on the polarization of incident radiation.

A plane electromagnetic wave when scattered on the CFL in the long–range zone is transformed into a spherical wave with the following components of the electric vector[2]:

$$\begin{bmatrix} E^s_{\theta'} \\ -E^s_{\varphi'} \end{bmatrix} = \frac{e^{-ikr'}}{ikr'} \begin{pmatrix} A_2 & A_3 \\ A_4 & A_1 \end{pmatrix} \begin{bmatrix} E^i_{l'} \\ E^i_{r'} \end{bmatrix}, \qquad (1)$$

where

$$A_i(\varphi', \theta', \beta, a, l) = \frac{kl}{\pi} U\left[\frac{kl}{2}(\cos\theta' - \cos\beta)\right] T_i(\varphi', \beta, a);$$

$T_i(\varphi', \beta, a)$ are the CM elements for an infinite cylinder, $\kappa = 2p/\lambda$ is the wave number, β is the angle between the direction of sounding radiation incidence and the cylinder axis, l is the cylinder length, a is the radius of the cylinder cross section, r', θ', and φ' are the spherical coordinates; the z' axis coincides with the cylinder axis; the vector **k** lies in the plane $x'z'$; $E^i_{l'}$ and $E^i_{r'}$ are the parallel and perpendicular components of the incident wave electric vector with respect to the $x'z'$ plane; and, $U(x) = \sin(x)/x$ is Kotel'nikov's function.

In order to proceed to the analysis of the scattering phase matrix of an ensemble of spatially polyoriented CFL, let us introduce the $x\ y\ z$ coordinate system (CS), whose z axis coincides with the direction of incident radiation, and assume that the plane xz is the reference plane. It is convenient to set the orientation of the cylinders in this CS by two Euler's angles α and β.[4]

The components E^s_h and E^s_z of the field scattered in the plane $\varphi = 0$, can be related to the components of incident radiation $E^i_l = E^i_x$ and $E^i_r = -E^i_y$ by making use of the conversion matrix S_i by the relation

$$\begin{bmatrix} E^s_{\theta} \\ -E^s_{\varphi} \end{bmatrix}_{\varphi=0} = \frac{e^{-ikr}}{ikr} \begin{pmatrix} S_2 & S_3 \\ S_4 & S_1 \end{pmatrix} \begin{bmatrix} E^i_x \\ -E^i_y \end{bmatrix}. \qquad (2)$$

When rotated by the angles a and b the $(x\ y\ z)$ coordinate system transforms into the $(x'\ y'\ z')$. In addition, the following transformations of the parallel and perpendicular components of the incident field occur:

$$\begin{bmatrix} E^i_{l'} \\ E^i_{r'} \end{bmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{bmatrix} E^i_x \\ -E^i_y \end{bmatrix}. \qquad (3)$$

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At the same time, the components of the scattered field in a system of spherical coordinate after analogous rotation undergo a transformation in accordance with the relation

$$\mathbf{E}^S(r, \theta, \varphi) = D\mathbf{E}^s(r', \theta', \varphi') \text{ , where } D = ABC. \tag{4}$$

$$A = \begin{pmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{pmatrix}, \; B = \begin{pmatrix} \cos\alpha\cos\beta & -\sin\alpha & \cos\alpha\sin\beta \\ \sin\alpha\cos\beta & \cos\alpha & \sin\alpha\cos\beta \\ -\sin\beta & 0 & \cos\beta \end{pmatrix},$$

$$C = \begin{pmatrix} \sin\theta'\cos\varphi' & \cos\theta'\cos\varphi' & -\sin\varphi' \\ \sin\theta'\sin\varphi' & \cos\theta'\sin\varphi' & \cos\varphi' \\ \cos\theta' & -\sin\theta' & 0 \end{pmatrix},$$

where α is in fact the angle between the scattering plane and the zz' plane.

One can show that the matrix D has the form

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & D_{22} & D_{23} \\ 0 & D_{32} & D_{33} \end{pmatrix}.$$

Taking into account that for spherical waves $E^s_{r'} = E^s_r = 0$ one can write relation (4) in the form

$$\begin{bmatrix} E^s_\theta \\ -E^s_\varphi \end{bmatrix}_{\varphi=0} = R \begin{bmatrix} E^s_{\Theta'} \\ -E^s_{\varphi'} \end{bmatrix}, \tag{5}$$

where $R_{11} = D_{22}$, $R_{12} = -D_{23}$, $R_{21} = -D_{32}$, $R_{22} = D_{33}$.

By substituting Eq. (3) into Eq. (1) and then the resulting expression into Eq. (5) accounting for the fact that $r' = r$, one obtains

$$\begin{bmatrix} E^s_\theta \\ -E^s_\varphi \end{bmatrix}_{\varphi=0} = \frac{e^{-ikr}}{ikr} R \begin{pmatrix} A_2 & A_3 \\ A_4 & A_1 \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{bmatrix} E^i_x \\ -E^i_y \end{bmatrix}. \tag{6}$$

By comparing formulas (2) and (6) we obtain the following relationship between the conversion matrices in the $(x\ y\ z)$ and $(x'\ y'\ z')$ coordinate systems:

$$\begin{pmatrix} S_2 & S_3 \\ S_4 & S_1 \end{pmatrix}_{\varphi=0} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}_{\varphi=0} \begin{pmatrix} A_2 & A_3 \\ A_4 & A_1 \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}. \tag{7}$$

When making final calculations of the elements of the conversion matrix S_i for each specific $\theta, \alpha,$ and β the relationships between the spherical coordinates are taken into account

$$\cos\theta' = \cos\theta\cos\beta + \sin\theta\sin\beta\cos\alpha \text{ ,}$$

$$\cot\varphi' = \cot(\varphi - \alpha)\cos\beta - \cot\theta\sin\beta/\sin(\varphi - \alpha) \text{ .}$$

Thus, the scattering phase matrix can be computed for each specific pair of α and β values. The total SPM of an ensemble of polyoriented CFL is a sum of SPM's of individual particles, provided that single scattering approximation is valid, and no correlation between the phases of radiation scattered by individual CFL occurs.

In the case of backscattering ($\theta = \pi$) the matrix R degenerates to

$$R = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix},$$

therefore for an arbitrarily oriented CFL we can write the elements of the conversion matrix in the form

$$S_2 = A_2 \cdot \cos^2\alpha - A_1 \cdot \sin^2\alpha \text{ , } S_1 = A_1 \cdot \cos^2\alpha - A_2 \cdot \sin^2\alpha \text{ ,}$$

$$S_3 = -(A_1 + A_2) \cdot \sin\alpha\ \cos\alpha \text{ , } S_4 = -S_3 \text{ .}$$

We analyze the variability of the normalized values of the Stokes vector–parameter components for the scattering angle $\theta = \pi$ as a function of the angle α of the cylinder orientation both for the case of linear (1, 1, 0, 0) and circular (1, 0, 0, 1) polarizations of incident radiation.

The model estimates made for the incident radiation polarized parallel with the reference plane (1, 1, 0, 0) may be quite simply extended to the case of incident radiation polarized perpendicularly (1, –1, 0, 0) to the reference plane

$$I^s_\perp(\alpha) = I^s_{||}\left(\frac{\pi}{2} - \alpha\right); \quad Q^s_\perp(\alpha) = -Q^s_{||}\left(\frac{\pi}{2} - \alpha\right);$$

$$U^s_\perp(\alpha) = -U^s_{||}\left(\frac{\pi}{2} - \alpha\right); \quad V^s_\perp(\alpha) = V^s_{||}\left(\frac{\pi}{2} - \alpha\right), \tag{8}$$

where $0 \le \alpha \le \pi/2$.

Accounting for a need to average the backscattering parameters over the polydisperse ensemble of particles having spectra of cross size and lengths as well as the spectrum of orientations of cylinders in a system of spherical coordinates (relative to angles α and β) the sought values were calculated based on a statistical ensemble of all possible dimensions and orientations of the CFL, encompassing 30,000 realizations. The problem on the convergence of the estimation method used has been analyzed in Ref. 3 using numerical experiments.

Based on the algorithm under consideration we have analyzed the variation of the Stokes vector–parameter values in the problem of monostatic lidar sensing in which the ensemble of CFL has a lognormal distribution of the cross section radii with parameters $r_m = 1.0$ and 10.0 μm, and $\sigma_r = 0.5$. The ratio of a cylinder length to its radius is assumed to be uniformly distributed over the interval [3, 5]. The distribution of particle orientations over the angle β was assumed to be normal with $\beta_m = 90°$ and 45°, σ_b being 30°. The same was accepted for the angle α with $\sigma_a = 30°$ and $\alpha_m \in [0; 180°]$. The calculations were done for the wavelength $\lambda = 1.06$ 1m and the refractive index $m = 1.296–0.0001\ i$, i.e., for the ice CFL's.

Variation of the values of the J component of the Stokes vector–parameter as a function of the angle α is presented in Fig. 1. The results of calculations show that the value of Stokes vector–parameter J component, normalized by the coefficient of total scattering, decreases as the radius of particles grows. For a linearly polarized incident radiation the value of the J^s_l component of a scattered field monotonically increases as the angle α_m of predominant orientation of the CFL's grows and reaches its maximum at $\alpha_m = 90°$.

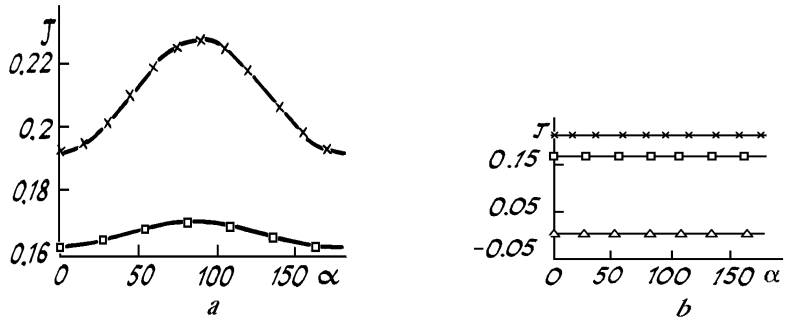

FIG. 1. Variations of the J component of the Stokes vector–parameter of radiation scattered at the angle Θ = 180°, *as a function of the angle* α: *for linear (a) and circular* (b) *polarizations. The notation at the curves: * stands for* β_m = 90° *and* r_m = 1.0 μm, — β_m = 90° *and* r_m = 10.0 μm, *and* Δ – β = 45° *and* r_m = 10.0 μm.

Thus, the linearly polarized radiation is most efficiently scattered by the CFL's, whose axes lie predominantly in the plane orthogonal to the polarization plane. For the incident radiation with circular polarization the J_C^s component of the Stokes vector–parameter is independent of the angle α_m, what is quite obvious.

At a dominantly slant incidence of radiation with respect to the cylinder axes, β_m = 45°, for both states of polarization the $J_{l,C}^s$ values sharply decrease compared to the normal incidence, β_m = 90°, almost by two orders of magnitude.

As follows from Figs. 2, 3, and 4, the variation of the lidar return polarization state when radiation is scattered on the cylinders with r_m = 10.0 μm at the normal incidence of sounding radiation is qualitatively analogous to scattering on spheres, and in particular, Q_l^s is close to unity and V_C^s is close to minus unity, while the rest components U_l^s, V_l^s, Q_C^s, and U_C^s of the Stokes vector–parameter are close to zero.

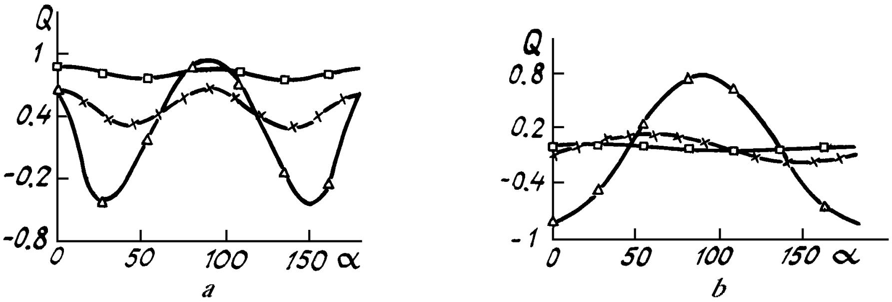

FIG. 2. The variation of the Q component of the Stokes vector–parameter of radiation scattered at the angle θ = 180°, *as a function of the angle* α: *for the linear (a) and circular (b) polarizations.*

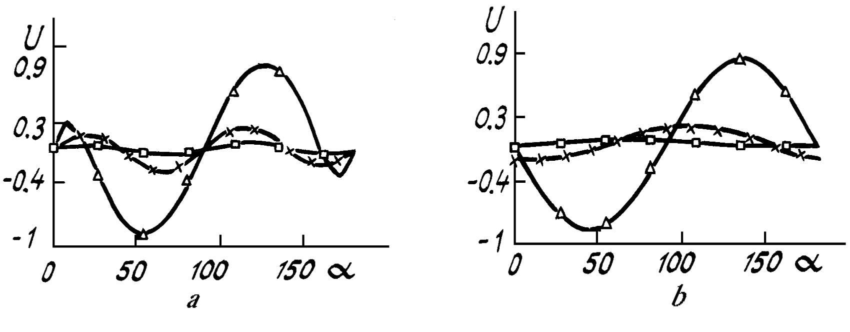

FIG. 3. The variation of the U–component values of the Stokes vector–parameter, of radiation scattered at the angle θ = 180° *as a function of the angle* α: *for the linear (a) and circular (b) polarization.*

The scattering on the CFL's with the cross sections lying in the resonance region has a pronounced dependence on the angle of predominant orientation even at normal incidence of sounding radiation. In addition, such particles, when irradiated by linearly polarized light produce the backscattered radiation of circular polarization of the highest degree of polarization, e.g., at the angle α_m = 45° (Fig. 4*a*), and at the same time the circularly polarized incident radiation becomes totally depolarized (Fig. 4*b*).

The components $Q_{l,C}^{s}$ and $U_{l,C}^{s}$ are most sensitive to variations of the angle α_m at $\beta_m = 45°$. It is in this case the circularly polarized radiation, when it is backscattered and $\alpha_m = 0$ and 90°, acquires the properties close to those of linearly polarized radiation, in particular, as is seen from Fig. 2*b*, $Q_C^s(0°) = -0.8$ and $Q_C^s(90°) = 0.8$, and from Figs. 3*b* and 4*b* one can see that $U_C^s(0°) = 0$, $V_C^s(0°) = 0.18$, $U_C^s(90°) = 0$, and $V_C^s(90°) = 0.18$.

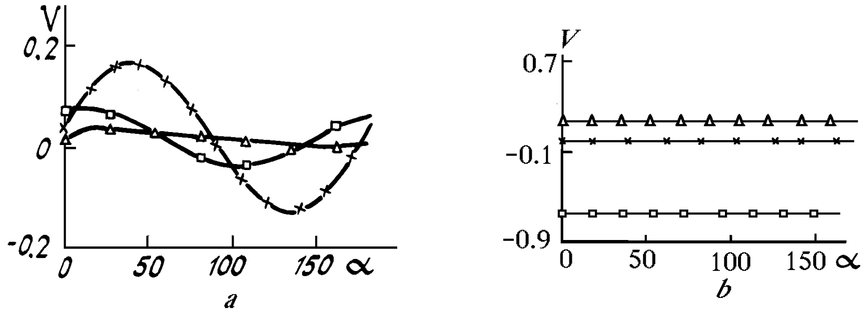

FIG. 4. The variation of the V–component values of the Stokes vector–parameter of radiation scattered at the angle θ = 180° *as a function of the angle* α*: for the linear (a) and circular (b) polarizations.*

In conclusion, we should like to emphasize that the obtained results promise good prospects in application to retrieval of the information about the orientation of aerosol particles from lidar sensing data, especially using the polarization methods. It is obvious that the direction and degree of predominant orientation of the particles bear information about the direction and intensity of air flows. And this may be used to develop the methods for optical sensing the atmospheric state at these quite remote altitudes.

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