EFFECT OF TURBULENCE ON THE THERMAL SELF-ACTION OF A LASER BEAM IN THE ATMOSPHERE

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Received May 4, 1989

A method for calculating the distribution of the average intensity of a partially coherent laser beam, propagating in a turbulent atmosphere under conditions of thermal self-action, is developed based on an equation for the mutual coherence function of the wave field. Using the method developed it is shown that even small fluctuations of the wind velocity and refractive index strongly limit local self-focusing, which arises in the presence of nonlinear wind refraction. The computational data on the average beam intensity for the nonstationary state of thermal self-action, taking into account at the same time both the turbulent pulsations of the refractive index and fluctuations of the wind velocity, are presented for the first time.

The problem of taking into account the nonlinear interaction of radiation with a medium, when owing to the absorption of some of the energy of laser beams by gases and aerosol in the atmosphere the air near the beam is heated, arises in the analysis of the propagation of powerful laser radiation in the atmosphere.[1] In a real atmosphere the thermal self-action occurs against a background of turbulent fluctuations of the refractive index and the wind velocity, which further complicates the theoretical study of this process. The calculations of the characteristics of a beam under conditions of thermal self-action by the method of statistical tests[2–7] have been limited to taking into account either the fluctuations of the wind velocity[2,3,6,7] or random pulsations of the refractive index.[2,3,4]

In this paper we describe a method that permits calculating, based on the equation for the mutual coherence function of the wave field, the energy characteristics of laser beams propagating under conditions of thermal self-action, taking into account simultaneously the turbulent pulsations of the wind velocity and the refractive index. The distribution of the average intensity in the beam as a function of the conditions of propagation is analyzed. The results obtained are compared in particular cases with existing calculations performed by the method of statistical tests.

Let the laser beam propagate along the X-axis of a Cartesian coordinate system. Then the equation for the complex amplitude of the radiation field $U(x', \vec{\rho}, t)$ under the conditions of thermal self-action in the atmosphere has the form[1]

$$\left[2ik\frac{\partial}{\partial x'} + \Delta_{\vec{\rho}}\right] U(x',\vec{\rho},t) + k^2\left[\tilde{\varepsilon}_1(x',\vec{\rho},t) + \frac{\partial\varepsilon}{\partial T} \times T(x',\vec{\rho},t)\right] U(x',\vec{\rho},t) = 0, \quad (1)$$

where $U(x', \vec{\rho}, t)$ satisfies in the $x' = 0$ plane the boundary condition $u(0, \vec{\rho}, t) = U_0(\vec{\rho}, t)$;the vector $\vec{\rho} = \{z, y\}$ lies in a plane perpendicular to the direction of propagation; $k = 2\pi/\lambda$, where λ is the wavelength; $\Delta_{\vec{\rho}} = \partial^2/\partial z^2 + \partial^2/\partial y^2$, t is the running time; and, $\tilde{\varepsilon}_1(x', \vec{\rho}, t)$ and $\frac{\partial\varepsilon}{\partial T}T(x', \vec{\rho}, t)$ are, respectively, the change in the dielectric constant owing to turbulence and owing to heating of the air by the radiation.

The change in the temperature can be represented in the form[1]

$$T(x',\vec{\rho},t) = \frac{\alpha_{ab}}{\rho C_p}\int_0^t dt'\, I(x',\tilde{\vec{\rho}}(x',\vec{\rho},t',t),t'), \quad (2)$$

where $\tilde{\vec{\rho}}(x', \vec{\rho}, t', t) = \vec{\rho} - \int_{t'}^{t}\vec{V}_\perp(x', \vec{\rho}, \tau)d\tau$ is the trajectory of a "fluid" particle in the field of the transverse component of the Lagrangian wind velocity $\vec{V}_\perp = \{V_x, V_y\}$ (Ref. 8); α_a, ρ, and C_p are, respectively, the absorption coefficient, the density, and the heat capacity of air; and, $I(x', \vec{\rho}, t) = |U(x', \vec{\rho}, t)|^2$ is the intensity of the beam.

We shall assume that the coherence time of the source is much shorter than the characteristic time of any thermal nonlinear interaction of the radiation with the medium. Then, since the medium has inertia, we shall assume that the temperature in Eq. (2) is the average temperature over fluctuations of the field of the source.[6]

The random pulsations of the induced temperature which are produced by fluctuations of the radiation intensity, which arise as the laser beam propagates in the turbulent atmosphere, likewise can be neglected when the average intensity is calculated.[4]

We shall neglect the random defocusing of the radiation by induced nonuniformities of the refractive index, which are formed by turbulent mixing of the heated air, compared with the defocusing by the average thermal lens in the region of the beam.[1,9,10]

Based on what was said above, we replace in Eq. (1) the induced temperature T by its average value $<T>$. Assuming that the distribution of the components of the vector of fluctuations of the wind velocity is Gaussian, the expression for the average temperature has the form[11]

$$<T(x',\vec{\rho},t)> = \frac{\alpha_{ab}}{2\pi\rho C_p\sigma_v^2}\int_0^t \frac{d\tau}{\tau^2}\times$$
$$\times\iint_{-\infty}^{+\infty} d^2\rho<I(x',\vec{\rho}\,'-\vec{\rho}\,',t-\tau)>\exp\left[-\frac{(\vec{\rho}\,'-<\vec{V}_\perp>\tau)^2}{2\sigma_v^2\tau^2}\right], \tag{3}$$

where σ_v^2 is the variance of the fluctuations of the wind velocity.

For the steady-state regime of self-action ($t > 3a_0/\sqrt{\sigma_y^2+\langle V_\perp\rangle^2}$ (a_0 is the effective radius of the beam in the $x' = 0$ plane)[12,13] assuming that the average intensity $\langle I\rangle$ does not depend on the time, and passing to the limit $t\to\infty$, Eq. (3) can be integrated over t', which gives

$$<T(x',\vec{\rho},t)> =$$
$$= \frac{\alpha_{ab}}{\rho C_p}\frac{1}{2\sqrt{2\pi}\sigma_v}\iint_{-\infty}^{+\infty} dz'dy'<I(x',z-z,y-y')>\times$$
$$\times(z'^2+y'^2)^{-1/2}\exp\left[-\frac{V^2}{2\sigma_v^2}\frac{y'^2}{z'^2+y'^2}\right]\times$$
$$\times\operatorname{erfc}\left[-\frac{V}{\sqrt{2}\sigma_v}\cdot\frac{\vec{z}\,'}{\sqrt{\vec{z}\,'^2+y'^2}}\right], \tag{4}$$

where the z-axis is oriented along the direction $\langle\vec{V}_\perp\rangle = \{V,\ 0\}$; $\operatorname{erfc}(\alpha) = 1 - \operatorname{erf}(\alpha)$; and, $\operatorname{erf}(\alpha)$ is the error function.

The following algorithm was employed to determine the average intensity $\langle I(x,\vec{\rho},t)\rangle$ in the general case. The path x is divided into N-layers. At the boundary of each layer the turbulent distortions of the wave field are taken into account in the phase-screen approximation

$$U_j(x_{j-1},\vec{\rho},t) = U_{j-1}(x_{j-1},\vec{\rho},t)\times$$
$$\times\exp\left[\frac{ik}{2}\int_{x_{j-1}}^{x_1} dx'\ \tilde{\varepsilon}_1(x',\vec{\rho},t)\right], \tag{5}$$

where $j = 1,\ 2,\ \ldots,\ N$. Then within the j-th layer the field $U_j(x',\ \vec{\rho},\ t)$ will be described by Eq. (1), where $\tilde{\varepsilon}_1\equiv 0$, T is replaced by $\langle T\rangle$ in accordance with Eq. (3), and $x'\in[x_{j-1},\ x_j]$.

Assuming that δ-correlation holds along the propagation path and that the turbulent fluctuations of the dielectric constant have a Kolmogorov spectrum,[14] we obtain for the mutual coherence function

$$\Gamma(x',\vec{R},\vec{\rho},t) = <U\left[x',\vec{R}+\frac{1}{2}\vec{\rho},t\right]U^*\left[x',\vec{R}-\frac{1}{2}\vec{\rho},t\right]>$$

in the "j-th" layer, in accordance with Eqs. (1) and (5) and the approximations made above, the equations

$$\left[ik\frac{\partial}{\partial x'}+\nabla_{\vec{R}}\nabla_{\vec{\rho}}\right]\Gamma_j(x',\vec{R},\vec{\rho},t)+\frac{k^2}{2}\frac{\partial\varepsilon}{\partial T}\sum_{n=0}^{1}(-1)^n\times$$
$$\times<T\left[x_{j-1},\vec{R}+\left[\frac{1}{2}-n\right]\vec{\rho},t\right]>\Gamma_j(x',\vec{R},\vec{\rho},t) = 0; \tag{6}$$

$$\Gamma_j(x_{j-1},\vec{R},\vec{\rho},t) = \Gamma_{j-1}(x_{j-1},\vec{R},\vec{\rho},t)\exp(-g\vec{\rho}^{\,2}), \tag{7}$$

where $q = (1.46k^2\Delta xC_n^2)^{6/5}$; $\Delta x = x_j - x_{j-1}$; C_n^2 is the structure constant of the refractive-index fluctuations; and $x'\in[x_{j-1},\ x_j]$, $j = 1,\ 2,\ \ldots,\ N$, $x_0 = 0$, $x_N = x$. The quadratic approximation was employed in the exponential in Eq. (7).[15]

In the case of a partially coherent, collimated, Gaussian beam we shall represent the coherence function Γ_0 in the plane of the source in the form[16]

$$\Gamma_0(0,\vec{R},\vec{\rho},t) = I_0\exp\left[-\frac{\vec{R}^2}{a_0^2}-\left[1+\frac{a_0^2}{a_c^2}\right]\frac{\vec{\rho}^{\,2}}{4a_0^2}\right], \tag{8}$$

where I_0 is the maximum value of the average intensity and a_c is the coherence radius.

Retaining in the Taylor series expansion of the temperature difference in Eq. (6) in $\vec{\rho}$ the first nonvanishing term in the series and Fourier transforming Eq. (6)

$$J(x', \vec{R}, \vec{\kappa}, t) = \iint_{-\infty}^{+\infty} d^2\rho\ \Gamma(x', \vec{R}, \vec{\rho}, t)\ e^{-i\vec{\kappa}\vec{\rho}}, \tag{9}$$

we arrive at the radiation transfer equation.[17]

We shall now transform to dimensionless variables:

$$\vec{R} \to \vec{R}/a_0,\ \vec{\rho} \to \vec{\rho}(1 + a_0^2/a_c^2)^{1/2},\ \vec{\kappa} \to \vec{\kappa}R_{nl}/(a_0 k),$$

$$\langle T\rangle \to \frac{\partial\varepsilon}{\partial T}\langle T\rangle R_{nl}^2/a_0^2,\ x' \to x'/R_{nl},\ \langle I\rangle \to \langle I\rangle/I_0,$$

$$t \to Vt/a_0,$$

where

$$R_{nl} = \left[\frac{1}{2}\sqrt{\pi}\ \left|\frac{\partial\varepsilon}{\partial T}\right|\ \alpha_{ab} I_0/(\rho C_p V a_0)\right]^{-1/2} \tag{10}$$

is the effective thermal self-action length in the case of uniform wind.

Solving the radiation transfer equation by the method of characteristics we can represent the coherence function in the x_j plane in the form

$$\Gamma_j(x_j, \vec{R}, \vec{\rho}, t) = \frac{P^2}{\pi} \iint_{-\infty}^{+\infty} d^2\kappa\ \exp\left[- S_j\left(x_{j-1}, \tilde{\vec{R}}(x_{j-1}), \frac{d}{dx'}\tilde{\vec{R}}(x_{j-1})\right)\right] e^{iP\vec{\kappa}\vec{\rho}}, \tag{11}$$

where

$$S\left[x_{j-1}, \tilde{\vec{R}}(x_{j-1}), \frac{d}{dx'}\tilde{\vec{R}}(x_{j-1})\right] = S_j(x_j, \vec{R}, \vec{\kappa}) = -\ln J_j(x_j, \vec{R}, \vec{\kappa}),$$

the characteristic $\tilde{R}(x')$ satisfies the equation

$$\frac{d\tilde{\vec{R}}(x')}{dx'} = \frac{1}{2}\nabla_{\tilde{\vec{R}}} \langle T(x_{j-1}, \tilde{\vec{R}}(x'), t)\rangle \tag{12}$$

with the boundary conditions $\tilde{\vec{R}}(x_j) = R$, $\frac{d}{dx'}\tilde{\vec{R}}(x_j) = \vec{\kappa}$, $x' \in [x_{j-1}, x_j]$, and $P = L_D/R_{NL}$, $L_D = ka_0^2/(1 + a_0^2/a_k^2)^{1/2}$ is the effective diffraction length.

It follows from Eqs. (8) and (9) that the function S_j in the emission plane can be represented as

$$S_0(0, \vec{R}, \vec{\kappa}) = \vec{R}^2 + P^2\vec{\kappa}^2. \tag{13}$$

We shall calculate the integral over $\vec{\kappa}$ in Eq. (11) approximately. For this we expand the function S_j in a Taylor series in a neighborhood of the point $\vec{\kappa}^*$ of the maximum of the integrand in the expression (11) and we shall retain terms which are no higher than second order in $\vec{\kappa}$:

$$S_j(x_j, \vec{R}, \vec{\kappa}) \approx S_j(x_j, \vec{R}, \vec{\kappa}^*) + \frac{1}{2}\left[(\vec{\kappa}-\vec{\kappa}^*)\nabla_{\vec{\kappa}}\right]^2 S_j(x_j, \vec{R}, \vec{\kappa}^*) \tag{14}$$

The values of $\vec{\kappa}$ are determined from the system of equations

$$\nabla_{\vec{\kappa}} S_j(x_j, \vec{R}, \vec{\kappa}^*) = 0. \tag{15}$$

The formula (14) for performing analytic calculations in Eq. (11) can be employed if the nonlinearity is weak $P^2 < 1$, and also for large parameters $P^2 \gg 1$, (Ref. 17), if S_j has one extreme point and

$$\det\left[\frac{\partial^2 S_j(x_j, \vec{R}, \vec{\kappa}^*)}{\partial x_m \partial x_n}\right] \neq 0, \tag{16}$$

where $m, n = 1$ and 2, and κ_1 and κ_2 are components of the vector $\vec{\kappa}$.

In accordance with Eq. (14) we shall represent S_j in the form

$$S_j(x_j, \vec{R}, \vec{\kappa}) = A^{(j)}(x_j, \vec{R}) + \frac{1}{2}\sum_{m,n=1}^{2} B_{mn}^{(j)}(x_j, \vec{R}) \times \left[\vec{\kappa}_m - D_m(x_j, \vec{R})\right]\left[\vec{\kappa}_n - D_n(x_j, \vec{R})\right]. \tag{17}$$

It follows from Eq. (13) that

$$A^{(0)}(0, \vec{R}) = \vec{R}^2;\ B_{11}^{(0)}(0, \vec{R}) = B_{22}^{(0)}(0, \vec{R}) = 2P^2;$$

$$B_{12}^{(0)}(0, \vec{R}) \equiv B_{21}^{(0)}(0, \vec{R}) = 0;\ D_1(0, \vec{R}) = 0;$$

$$D_2(0, \vec{R}) = 0.$$

Based on the formulas (17), (11), (7), and (9) it is easy to establish a relation between the functions $S_j(x_j, \vec{R}, \vec{\kappa})$ and $S_{j+1}(x_j, \vec{R}, \vec{\kappa})$ with whose help the turbulent distortions of the average intensity can be taken into account. This relation can be expressed in terms of the functions A, B_{11}, B_{22}, and B_{12}:

$$A^{(j+1)}(x_j, \vec{R}) = A^{(j)}(x_j, \vec{R}) + \frac{1}{2}\ln\mu,$$
$$B_{mn}^{(j+1)}(x_j, \vec{R}) = \left[B_{mn}^{(j+1)}(x_j, \vec{R}) + \delta\tilde{g}\nu\right]/\mu, \tag{18}$$

where

$$\nu = \det\left[B_{mn}^{(j)}(x_j, \vec{R})\right];$$

$$\mu = 1 + \tilde{g}\sum_{n=1}^{2} B_{nn}^{(j)}(x_j, \vec{R}) + \nu\tilde{g}^2;\ \tilde{g} = g_T(x/N)^{6/5}/P;$$

$$q_T = 2.43\beta_N^{12/5}/(1 + a_0^2/a_c^2)^{1/2};$$

$$\beta_N^2 = 1.24C_n^2 k^{7/6} R_{NL}^{11/6}$$

is a complex parameter characterizing the turbulent conditions of propagation over a distance $\vec{R}_{nl}$ (Ref. 16); $\delta = 1$ if $m = n$, and $\delta = 0$ if $m \neq n$.

The vectors $\vec{R}$ and $\frac{d}{dx}\vec{R}$ in the x_j plane are determined by solving numerically Eq. (12) with the boundary conditions in the x_{j+1} plane by the Runge-Kutta method for fixed value of $\vec{R}$, which is a node of a uniform coordinate grid, and different values of $\vec{\kappa}$. Once the values of the function $S_{j+1}(x_{j+1}, \vec{R}, \vec{\kappa})$ at the nodes of a uniform grid $\{z_i, y_k\}$ are known, the function $A^{(j+1)}$, $B_{mn}^{(j+1)}$, D_n, which, according to Eq. (11), is equal to

$$S_{j+1}(x_{j+1}, \vec{R}, \vec{\kappa}) = S_{j+1}\left[x_j, \tilde{\vec{R}}(x_j), \frac{d}{dx'}\tilde{\vec{R}}(x_j)\right]$$

in the plane x_j can be found by constructing a spline or by interpolating according to the formula (17). Next its derivatives $\frac{\partial S_{j+1}}{\partial \kappa_n}$, $\frac{\partial^2 S_{j+1}}{\partial \kappa_n \partial \kappa_m}$ are calculated by numerical differentiation.

To present $S_{j+1}(x_{j+1}, \vec{R}, \vec{\kappa})$ in the form (14) and then perform the integration in Eq. (11) over $\vec{\kappa}$ it is necessary to find $\vec{\kappa}^*$ from the system (15). This can be done using the following iteration scheme:

$$\kappa_n^{(k+1)} = \kappa_n^{(k)} + \left[\frac{\partial \bar{S}}{\partial \kappa_m}\frac{\partial^2 \bar{S}}{\partial \kappa_m \partial \kappa_n} - \frac{\partial \bar{S}}{\partial \kappa_n}\frac{\partial^2 \bar{S}}{\partial \kappa_m^2}\right]/M, \qquad (19)$$

where

$$\bar{S} = S_{j+1}(x_{j+1}, \vec{R}, \vec{\kappa}^{(k)}),\ m,\ n = 1,\ 2,\ m \neq n,$$

$$M = \det[\partial^2 \bar{S}/\partial \kappa_m \partial \kappa_n],\ k = 0, 1, 2, \ldots$$

The vector $\frac{d}{dx'}\tilde{\vec{R}}(x_{j+1})$ obtained by solving Eq. (11) with the boundary conditions

$$\tilde{\vec{R}}(x_j) = \vec{R},\ \frac{d}{dx'}\tilde{\vec{R}}(x_j) = \left\{D_1(x_j, \vec{R}), D_2(x_j, \vec{R})\right\}$$

is employed as zero approximation $\vec{\kappa}^{(0)}$.

After $\vec{\kappa}^* = \vec{\kappa}^{(k+1)}$ is determined, by comparing Eqs. (14) and (17) an array of values of the functions $A^{(j+1)}$, $B_{mn}^{(j+1)}$, D_n at the nodes of the grid $\{z_i, y_k\}$ in the x_{j+1} plane is constructed:

$$D_n(x_{j+1}, \vec{R}) = \kappa_n^{(k+1)},\ B_{mn}^{(j+1)}(x_{j+1}, \vec{R}) = \frac{\partial^2 \bar{S}}{\partial \kappa_m \partial \kappa_n},$$

$$A^{(j+1)}(x_{j+1}, \vec{R}) = \bar{S},$$

where m, n = 1, 2.

Finally, substituting Eq. (17) into Eq. (11), setting $\vec{\rho} = 0$, and integrating over $\vec{\kappa}$, we obtain the following expression for the average intensity in the x_{j+1} plane:

$$\langle I(x_{j+1}, \vec{R}, t)\rangle = \frac{2P^2 \exp\left[-A^{(j+1)}(x_{j+1}, \vec{R})\right]}{\sqrt{\det\left[B_{mn}^{(j+1)}(x_{j+1}, \vec{R})\right]}}. \qquad (20)$$

In this work we calculated the average intensity of a partially coherent Gaussian beam using Eqs. (17)–(20) for different conditions of propagation. We studied the nonstationary regime of thermal self-action.

The calculations show that for the case of a uniform wind[1,17] and large nonlinearity parameters ($P^2 > 10$), at certain distances x the maximum value of the intensity $\langle I\rangle_m$ increases owing to the formation of a region in the beam where the radiation-induced nonuniformity of the refractive index forms an extended focusing lens. The fluctuations of the wind velocity spread out the average profile of' the induced temperature and, therefore, reduce the focusing action of the induced nonuniformity. For this reason the maximum intensity under conditions of fluctuations of the wind velocity will increase at distances greater than in the case of a uniform wind. This is illustrated in Fig. 1a. Moreover; growth of $\langle I\rangle_m$ owing to local wind-induced self-focusing does not occur at all, if one of the $\sigma_v^2 / V^2 \geq 1$ $g_T = 0$ or $g_T \geq 3$ $\sigma_v^2 / V^2 = 0$ is satisfied (at least in the region $P^2 \leq 100$, for which the calculations were performed). When fluctuations of the wind velocity and refractive index are taken into account at the same time $\langle I\rangle_m$ is not observed to increase with the distance x for values less than $\sigma_v^2 / V^2 = 1$ and $q_T = 3$ (curve 2′). Figure 1a also presents results which show how the position of the coordinate of the maximum intensity R_m and the energy "center of gravity" R_c changes:

$$R_c = \frac{1}{P}\iint_{-\infty}^{+\infty} d^2\rho\ z\langle I(x, \vec{\rho})\rangle \quad \left[P = \iint_{-\infty}^{+\infty} d^2\rho\langle I(x, \vec{\rho})\rangle\right]$$

as the path length increases. (The displacements are given in units determined by normalizing to the initial radius of the beam a_0).

Figure 1b presents results for the effective radius of the beam normalized to a_0;

$$g_{z,y} = \left[\frac{2}{P}\iint_{-\infty}^{+\infty} d^2\rho \left\{\begin{matrix}(z-R_c)^2\\ y^2\end{matrix}\right\}\langle I(x, \vec{\rho})\rangle\right]^{-1/2}$$

along the z and, y axes, respectively. One can see from the figure that if the fluctuations of the wind velocity cause the asymmetry of the beam to decrease (broken curves), then taking turbulent fluctuations of the refractive index along the path into account merely gives an additional uniform broadening of the beam along the axes; the same degree of asymmetry remains as in the case $q_T = 0$ (dot-dashed curves).

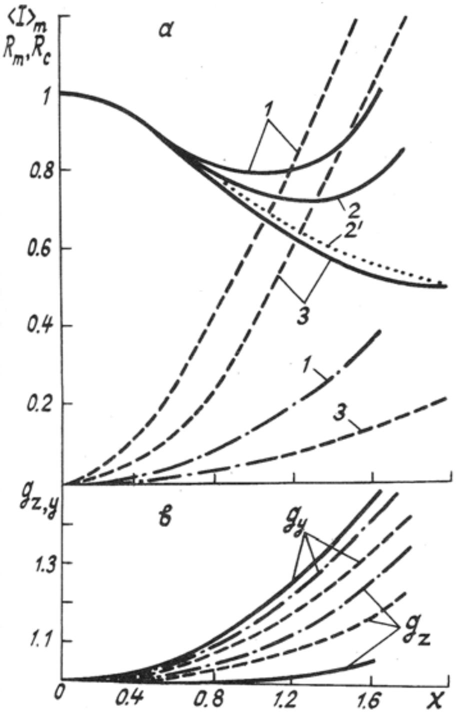

FIG. 1. The characteristics of the laser beam as a function of the path length for $P = 10$: (a) $\langle I\rangle_m$ – solid lines and dots; R_m – dashed lines; R_c – dot-dashed lines: $q_T = 0$(1, 2, 3); $q_T = 1$ (2′); $\sigma_v^2 / V^2 = 0$ (1), 0.3 (2, 2′), and 1 (3); (b) $\sigma_v^2 / V^2 = q_T = 0$ (solid lines), $\sigma_v^2 / V^2 = 0.3$, $q_T = 0$ (dashed lines), and $\sigma_v^2 / V^2 = 0.3$, $q_T = 1$ (dot-dashed lines).

On the whole the calculations for the steady-state regime of thermal self-action show that the fluctuations of the wind velocity greatly change the aberrational picture of the distribution of the average intensity. The presence of turbulent nonuniformities of the refractive index on the propagation path leads primarily to broadening of the beam, whose effect on the distribution of the average intensity in the observation plane is much smaller than that of fluctuations of the wind velocity.

Figure 2 shows the effect of fluctuations of the wind velocity on the characteristics of a beam propagating in the atmosphere under conditions of thermal self-action. It is obvious from the figure that the maximum intensity starts to decrease when the fluctuations of the wind velocity increase (see Fig. 2); then, starting with the values $\sigma_v^2 / V^2 \simeq 1$ the maximum intensity increases owing to intensified turbulent diffusion of the heated air out of the region of the beam to a level corresponding to a linear medium.

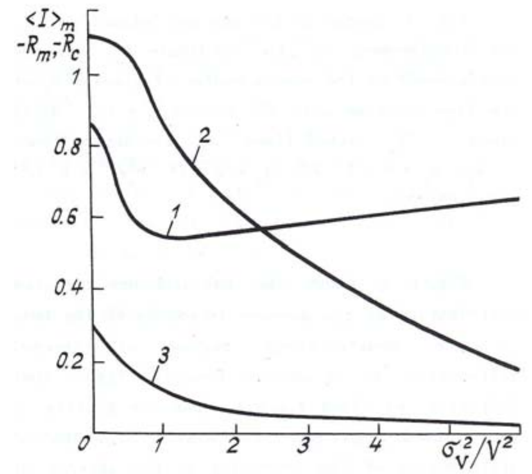

FIG. 2. The effect of fluctuations of the wind velocity on the characteristics of the laser beam $\langle I\rangle_m$, R_m, and R_c with $P = 10$, $x = 1.5$, and $q_T = 0$: 1 – $\langle I\rangle_m$; 2 – R_m; 3 – R_c.

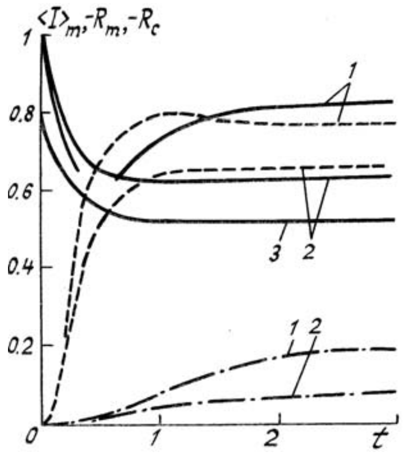

FIG. 3. Change in the maximum intensity $\langle I\rangle_m$, the displacement of its coordinate R_m, and the displacement of the energy center of gravity R_c of the beam in time with $P = 10$ and $x = 1.2$: solid lines – $\langle I\rangle_m$; dashed lines – R_m; dot-dashed lines – R_c; $q_T = 0$ (1, 2), $q_T = 2$ (3), $\sigma_v^2 / V^2 = 0$ (1) and 1 (2, 3).

Figure 3 shows the calculations for the distribution of the average intensity of the beam in the nonstationary regime of thermal self-action. It is obvious from the figure that initially, at times $t < 0.1$, when the profile of the induced temperature is close to the spatial distribution of the intensity in the absence of self-action, the maximum value of the intensity decreases owing to defocusing. As heat is carried out of the region of the beam, however, defocusing on the leeward side decreases, which causes $\langle I\rangle_m$ to increase and results in its subsequent saturation at some level at $t \approx 3$ (Refs. 12 and 13). Other characteristics of the beam also behave nonmonotonically as a function of time: R_m and $g_{z,y}$. This effect is most clearly seen when there are no fluctuations of the velocity. Turbulent mixing of the air impedes heat transfer in the direction of the wind, so that in this case the emergence into a steady-state regime occurs more smoothly. For $\sigma_v^2 / V^2 = 1$ $\langle I \rangle_m$ and R_m (curves 2) are monotonic functions of the time and reach a steady level somewhat earlier. The presence of turbulent nonuniformities $\tilde{\varepsilon}_1$ in the medium does not qualitatively change the time dependence of the average intensity (curve 3).

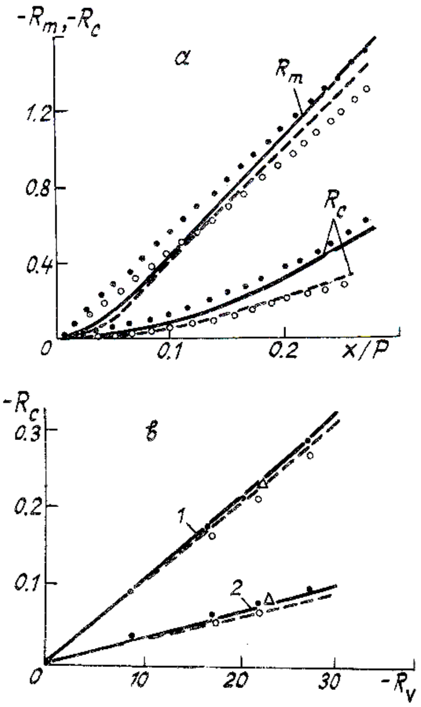

FIG. 4. Comparison of the calculations of the displacements of the coordinate of the maximum intensity and the energy center of gravity of the beam (solid and dashed curves) with calculations by the method of statistical tests (triangles, dark and light circles), performed in Ref. 3 (a) and Ref. 4 (b): (a) solid lines and dark circles — $\sigma_v^2 / V^2 = 0$; dashed lines and light circles — $\sigma_v^2 / V^2 = 0.4$; (b) solid lines — $B_N^2 x^{11/16} = 0.55$; dark and light circles show the results for the calculation of R_c with a given temperature field; the triangles show the results obtained by solving the self-consistent problem; 1 — $x/P = 0.3$; 2 — $x/P = 0.16$; $\vec{R}_v = -2P^2 / \sqrt{\pi}$.

The calculations performed with the help of the algorithm described above and the calculations formed by the method of statistical tests are compared in Fig. 4.[3,4] Figure 4b shows a comparison with the data of Ref. 4, where the fluctuations of the wind velocity were ignored ($\sigma_v^2 = 0$); Fig. 4a shows a comparison with the results of Ref. 3, where the propagation of a laser beam under conditions of thermal self-action was analyzed neglecting the turbulent pulsations $\tilde{\varepsilon}_1 = 0$. One can see that the results obtained by the different methods are in satisfactory agreement with one another.

Thus in this work a method for calculating the average intensity of a partially coherent laser beam, propagating in the atmosphere under conditions of thermal self-action, was developed based on the equation for the mutual coherence function of the field. The method permits taking into account simultaneously the effect of turbulent pulsations of the refractive index and fluctuations of the wind velocity. In this approach, in contradistinction to the method of statistical tests, the random processes occurring in the medium and at the laser output need not be modeled; this substantially reduces the computing time without lowering the accuracy.

We have shown that even weak fluctuations of the wind velocity and the refractive index greatly limit the local focusing of the beam, which arises under conditions of wind refraction.

The method developed permits tracing in time the evolution of the distribution of the average intensity of the beam under conditions of thermal self-action for real atmospheric conditions of propagation. This is important for analyzing beams with arbitrary pulse duration.

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