EFFECT OF TIME-DEPENDENT PULSATIONS OF THE WIND VELOCITY ON THE TRANSFORMATION OF A LASER BEAM IN THE ATMOSPHERE

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The role of time-dependent pulsations of the wind velocity in the self-action of a laser beam in the atmosphere is studied analytically by the nonlinear phase channel method. Formulas are derived for the average statistical spatial-temporal scales of the light field. It is shown that the temporal fluctuations of the wind velocity degrade both spatial and the temporal structure of the laser radiation. In addition, the correlation radius of the beam in the plane of the wind varies nonmonotonically along the propagation path.

Introduction

The statistics of the nonlinear transformation of laser beams in the atmosphere is a rapidly developing field, which encompasses both purely physical problems and practical numerical applications. Progress in experiments on the thermal self-action of laser radiation in the atmosphere, on the one hand, became possible owing to the extensive use of the methods of nonlinear optics, but on the other hand it confronted the theory with a number of problems associated with the fact that the combined influence of nonlinear effects and fluctuations of the parameters of the medium must be taken into account.[1,2] The transformation of the characteristics of a light beam in a randomly nonuniform atmosphere have been studied in detail, both experimentally[3–5] and theoretically[2,6] using different approximate analytical methods. Numerical modeling has been used to study the change in the spatial structure of a laser beam on a path with variable velocity[5–7] and taking into account fluctuations of the refractive index of the atmosphere.[1]

In nonlinear media, however, the spatial and temporal fluctuations of the light wave become coupled.[8] This greatly enlarges the range of statistical phenomena compared with traditional phenomena, which have been studied quite well in the works cited.

This paper is devoted to the theoretical analysis of one of these new phenomena – the role of time-dependent pulsations of the wind velocity in the nonlinear transformation of the spatial-temporal structure of an initially coherent laser beam in the atmosphere.

Formulation of the problem and method of solution

In the geometric-optics approximation the thermal self-action of laser beams in a turbulent atmosphere, where large-scale pulsations of the wind velocity are significant, are described by the nonlinear equation of quasioptics for the complex amplitude of the light field A together with a material equation describing heat conduction:

$$\left(\frac{\partial}{\partial \vec{z}} + \frac{i}{2k} \Delta_\perp \right) A = - \frac{ik}{2n_0} n_T T A; \tag{1}$$

$$\frac{\partial T}{\partial \tau} - \chi \Delta_\perp T + \upsilon \frac{\partial T}{\partial \vec{x}} = \frac{\alpha}{\rho \vec{C}_p} |A|^2. \tag{2}$$

In the system of equations (1) and (2) k is the wave number of the radiation propagating along the z-axis, which is oriented along the surface of the earth; $\Delta_\perp$ is the transverse Laplacian operator in the XOY plane; $n_T T = \dfrac{dn}{dT}$, $T < 0$ is the perturbation of the refractive index from the equilibrium value n_0 owing to heating of the atmosphere by intense radiation; τ is the time in the commoving coordinate system; α, X, and pC_p are the absorption coefficient, the thermal diffusivity, and the heat capacity per unit volume at constant pressure; and, υ is the velocity of the medium along the X-axis (along the earth's surface).

To study the effect of pulsations of the wind velocity on the nonlinear transformation of the light radiation we shall study a coherent laser beam which at the inlet to the medium has a Gaussian envelope $A_0(\bar{r}) = \sqrt{I_0} \exp(-r^2 / a_0^2)$, where I_0 is the characteristic value of the peak intensity and a_0 is the width of the beam.

As a rule, the wind velocity in the layer of the atmosphere near the ground is oriented virtually horizontally, and only the component of the wind velocity transverse to the direction of propagation (to the z-axis), which can be written in the form $\upsilon = \upsilon_0 + \tilde{\upsilon}$, where $\upsilon_0 = \langle \upsilon \rangle$ is the average component of the crosswind and $\tilde{\upsilon}$ is the fluctuational correction, affects the transformation of the laser beam.[6] The pulsations of the wind velocity are related with the large-scale fluctuations of the atmosphere, whose characteristic scale $I_\upsilon \gg a_0$ and the characteristic pulsation time is of the order of $\tau_\upsilon \sim h/\upsilon_0$ (h is the height above the earth's surface).[7,9] Therefore in the transverse cross section of the beam the velocity of the crosswind can be assumed to be constant and over the transit time of the medium $a_0\upsilon_0 \ll \tau_\upsilon$ there is virtually no time for the wind velocity to change. Even under conditions of not very strong wind $\upsilon_0 \sim 1$ m/sec the effect of the heat condition on the formation of the temperature channel is vanishingly small, since the characteristic heating time $a_0^2 / X \gg a_0\upsilon_0$. Disregarding the spatial fluctuations of the velocity of the crosswind along the propagation path and the small-scale turbulence of the atmosphere for quasicontinuous beams, Eg. (2) for the temperature channel can be written in the form

$$\frac{\partial T}{\partial \vec{x}} = \frac{\alpha}{\rho C_p \upsilon_0}\left(1 - \frac{\tilde{\upsilon}}{\upsilon_0} + \frac{\tilde{\upsilon}^2}{\upsilon_0^2}\right)|A|^2, \tag{3}$$

where $\tilde{\upsilon}(\tau)$ is a random function of time with zero mean value, $\langle\tilde{\upsilon}(\tau)\rangle = 0$, the correlation function $\langle\tilde{\upsilon}(\tau_1)\tilde{\upsilon}(\tau_2)\rangle = \sigma_\upsilon^2 \exp(-(\tau_1 - \tau_2)^2 / \tau_\upsilon^2$ and a small variance $\sigma_\upsilon < \upsilon_0$.

If diffraction in the atmosphere is neglected, only phase self-modulation of the amplitude of the light field occurs, without any change in the average intensity over a distance equal to the characteristic nonlinear phase modulation length,

$$L_{nl} = \left[\frac{n_0 \rho C_p \upsilon_0 a_0}{|n_T|\alpha I_0\sqrt{\pi/2}}\right]^{1/2} \tag{4}$$

For light beams with intensity $I_0 \sim 1$ kW/cm^2 and width $a_0 \sim 3$ cm with an average wind velocity $\upsilon_0 \sim 1$ m/sec the characteristic nonlinear phase modulation length is of the order of $L_{nl} \sim 100$ m. For radiation with wavelength $\lambda \sim 1$ μm and beamwidth $a_0 \sim 3$ cm the characteristic length of the diffraction broadening of the beam is equal to $L_d = 1 / 2ka_0^2 \sim 3$ km. Thus for intense light beams in the atmosphere $L_d/L_{nl} \gg 1$. This ratio of the nonlinear and diffraction lengths makes it possible to solve the system of equations (1) and (3) by the nonlinear phase channel method: in this method the problem is solved in two stages: at the first stage only the nonlinear self-action, resulting in random phase modulation of the amplitude of the light field along the path z owing to thermal self-action and existing fluctuations of the wind velocity, while the intensity of the radiation remains constant, is studied; at the second stage the linear parabolic equation (1) without the nonlinear term, but with initial conditions obtained at the first stage, is solved. This approach, naturally, limits the length of the part of self-action, which must not exceed the characteristic self-action length $z < L_{sv} = (L_{nl}\ L_d)^{1/2}$.

Thus as result of the first stage the amplitude of the light field assumes the form

$$A_1(\vec{r}, \tau, z) = A_0(r)\exp\left[-\frac{ikn_T z}{2n_0}T(\vec{r}, \tau)\right], \tag{5}$$

where the random (as a function of time) disturbance of the temperature $T(\bar{r}, \tau)$ is the same along the path z and is given by Eq. (3), in which the intensity of the radiation is equal to its input value $|A_0(\bar{r})|^2$.

At the second stage of the solution with the initial condition (5) the amplitude of the light field assumes the form

$$A(\vec{r}, \tau, z) = \int d\vec{r}\,' A_1(\vec{r}\,', \tau, z)\, G(\vec{r}, \vec{r}\,'), \tag{6}$$

where

$$G(\vec{r}, \vec{r}\,') = -\frac{ik}{2\pi\vec{z}}\exp\left[-\frac{ik}{2\vec{z}}(\vec{r} - \vec{r}\,')^2\right] \tag{7}$$

is the Green's function for the regular linear atmosphere.

To determine the spatial-temporal average statistical characteristics of the laser radiation we shall analyze the second-order spatial-temporal correlation function (STCF) for the complex amplitude (6)

$$\Gamma(\vec{r}_1, \vec{r}_2, \tau_1, \tau_2, z) = \langle A(\vec{r}_1, \tau_1, z)A^*(\vec{r}_2, \tau_2, z)\rangle =$$
$$= \iint d\vec{r}\,'_1 d\vec{r}\,'_2 A_0(\vec{r}\,'_1)A_0(\vec{r}\,'_2)G(\vec{r}_1, \vec{r}\,'_1)G^*(\vec{r}_2, \vec{r}\,'_2)\langle e^{-i\Delta\Phi}\rangle, \tag{8}$$

where the difference of the nonlinearly modulated phases is given by

$$\Delta\Phi \frac{\sqrt{2}zL_d}{\sqrt{\pi}\,L_{nl}^2 a_0} - \left[\left[1 - \frac{\tilde{\upsilon}(\tau_1)}{\upsilon_0} + \frac{\tilde{\upsilon}^2(\tau_1)}{\upsilon_0^2}\right]\times\right.$$
$$\times\int_{-\infty}^{x'_1} dx'' e^{-2\vec{r}\,''^2_1/a_0^2} - \left[1 - \frac{\tilde{\upsilon}(\tau_2)}{\upsilon_0} + \frac{\tilde{\upsilon}^2(\tau_2)}{\tau}\right]\times$$
$$\left.\times\int_{-\infty}^{x'_2} dx''_2\, e^{-2\vec{r}\,''^2_2/a_0^2}\right];$$
$$\vec{r}\,'_{1,2} = (x'_{1,2}, y'_{1,2}),\ \vec{r}\,''_{1,2} = (x''_{1,2}, y''_{1,2}). \tag{9}$$

To determine the correlation function on the right side of (8) we shall calculate the average value $\langle\Delta\Phi\rangle$ and the variance of the fluctuations of the phase difference $\langle\Delta\tilde{\Phi}^2\rangle = \langle(\Delta\Phi - \langle\Delta\Phi\rangle)^2\rangle$. For the times $(\tau_1 - \tau_2) < \tau_\upsilon$ and in the region of the beam near the axis $|\tilde{r}'_{12}| < a_0$ we obtain

$$\langle\Delta\Phi\rangle = \frac{zL_d\sqrt{2}}{\sqrt{\pi}L_{nl}^2}\left[1 + \frac{\sigma_v^2}{\upsilon_0^2}\right] \times \left[\frac{x'_1 - x'_2}{a_0} + \sqrt{\frac{\pi}{2}}\,\frac{y_1'^2 - y_2'^2}{a_0^2}\right]^2; \quad (10)$$

$$\langle\Delta\tilde{\Phi}^2\rangle = \left[\frac{zL_d\sqrt{2}}{\sqrt{\pi}L_{nl}^2}\right]^2 \frac{\sigma_v^2}{V_0^2}\left[\frac{(x'_1 - x'_2)^2}{a_0^2} + \frac{\pi}{8}\frac{(\tau_1 - \tau_2)^2}{\tau_v^2}\right] \quad (11)$$

For small values of the relative variance of the fluctuations of the wind velocity $(\sigma_\upsilon < \upsilon_0)$ the condition $(\langle\Delta\tilde{\Phi}^2\rangle)^{1/2} < \langle\Delta\Phi\rangle$, holds, and therefore

$$\langle e^{-i\Delta\Phi}\rangle = \exp\left\{-i\langle\Delta\Phi\rangle - \frac{1}{2}\langle\Delta\tilde{\Phi}^2\rangle\right\}. \quad (12)$$

Results and discussion

Substituting Eqs. (7), (10), and (11) into Eq. (8) we obtain the following expression for the modulus of the STCF of the field:

$$|\Gamma(\vec{r}_1, \vec{r}_2, \tau_1, \tau_2, z)| = I_0\frac{a_0^2}{a_x a_y}\exp\left[-\frac{(x_1 - x_0)^2}{a_x^2} - \frac{(x_2 - x_0)^2}{a_x^2} - \frac{y_1^2 + y_2^2}{a_y^2} - \frac{(x_1 - x_2)^2}{r_{kx}^2} - \frac{(\tau_1 - \tau_2)^2}{\tau_k^2}\right], \quad (13)$$

where the average values of the beamwidth along the X-and Y-axes are

$$a_x(z) = a_0\left[1 + \frac{z^2}{L_d^2} + \pi\frac{\sigma_\upsilon^2}{\upsilon_0^2}(z/L_{nl})^4\right]^{1/2}; \quad (14)$$

$$a_y(z) = a_0\left[1 + \frac{z^2}{L_d^2} + \frac{z^2}{L_{nl}^2}\left[2 + \frac{z^2}{L_{nl}^2}\right] + 2\frac{\sigma_v^2 z^2}{\upsilon_0^2 L_{nl}^2}\left[1 + \frac{z^2}{L_{nl}^2}\right]\right] \quad (15)$$

the average windward displacement of the energy axis of the beam is

$$x_0(z) = -\frac{a_0}{\sqrt{2\pi}}\left[1 + \frac{\sigma_\upsilon^2}{\upsilon_0^2}\right]\frac{z^2}{L_{nl}^2}, \quad (16)$$

the correlation radius along the X-axis is

$$r_{kx}(z) = a_0\frac{\upsilon_0}{\sigma_\upsilon}\frac{\sqrt{\pi}}{4}\frac{L_{nl}^2}{zL_d^2} \times \left[1 + \frac{z^2}{L_d^2}\left[1 + \pi\frac{\sigma_\upsilon^2}{\upsilon_0^2}\frac{z^2L_d^2}{L_{nl}^4}\right]\right]^{1/2}, \quad (17)$$

and the coherence time of the beam is

$$\tau_k(z) = 2\tau_\upsilon\frac{\upsilon_0}{\sigma_\upsilon}\frac{L_{nl}^2}{zL_d}. \quad (18)$$

It is obvious from the expressions for the beam width along the X- and Y-axes that owing to the induced inhomogeneous temperature channel asymmetric defocusing of the beam occurs. In the case of regular wind $(\sigma_\upsilon = 0)$, owing to the attenuation of the thermal self-action downwind the beam is broadened less along the X-axis $(a_y > a_x)$[10]. The pulsations of the wind velocity give rise to additional defocusing of the beam, which is caused by the induced fluctuations of the thermal channel. In the process the downwind broadening of the beam is also lessened. The average displacement of the energy axis of the beam increases as the relative variance $\sigma_\upsilon/\upsilon_0$ of the wind velocity fluctuations increases (Fig. 1).

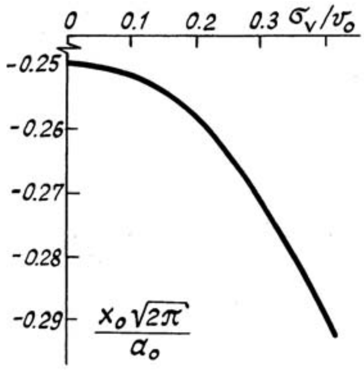

FIG. 1. The position of the energy center of the beam as a function of the standard deviation of the wind velocity fluctuations at a distance $z = 0.5L_{nl}$.

The transformation of the spatial-temporal correlation properties of an initially coherent laser beam is of special interest. In a real situation it is sometimes necessary to analyze either the spatial or the temporal coherence of a random laser field. However it is obvious that the spatial and temporal fluctuations are closely interrelated.[8] These fluctuations can be separated and the transformation of one form into another and vice versa can be studied only by analyzing the STCF of the field.

To analyze theoretically the correlation properties of the laser beam we write down the expression for the modulus of the degree of SCTF of the field

$$|\gamma(\vec{r}_1, \vec{r}_2, \tau_1, \tau_2, z)| = \frac{|\Gamma(r_1, r_2, \tau_1, \tau_2, z)|}{\sqrt{\langle I(r_1, \tau_1, z)\rangle \langle I(r_2, \tau_2, z)\rangle}} =$$

$$= \exp\left[-\frac{(x_1 - x_2)^2}{r_{kx}^2} - \frac{(\tau_1 - \tau_2)^2}{\tau_k^2}\right] =$$

$$= |\gamma_r(\vec{r}_1, \vec{r}_2, z)|\ |\gamma_\tau(\tau_1, \tau_2, z)|,$$

where γ_r and γ_τ are the degrees of spatial and temporal coherence, respectively.

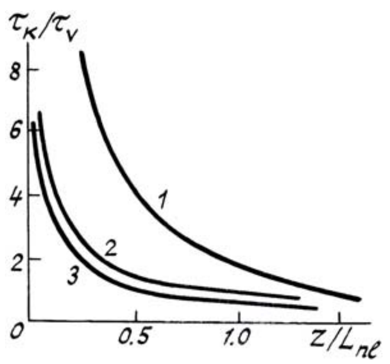

FIG. 2. Degratation of the temporal coherence of a laser beam along the propagation path with $L = 10L_{nl}$ and $\sigma_\upsilon/\upsilon_0 = 0.1$ (1), 0.3 (2), and 0.5 (3).

It is obvious that the time-dependent pulsations of the wind velocity along the X-axis give rise to the appearance of both spatial and temporal fluctuations into spatial fluctuations and vice versa is characteristic for a nonlinear medium.[8] The coherence time of the laser beam decreases hyperbolically as the beam propagates through the atmosphere and depends mainly on the relative variance of the pulsations (Fig. 2). For characteristic problems of the propagation of beams in the surface layer of the atmosphere $\tau_\upsilon \sim 1$ sec, $L_d/L_{nl} \sim 10$, and already with $\sigma_\upsilon/\upsilon_0 \sim 0.1$ over short distances $X \sim L_{nl}$ the interval of temporal coherence of the field becomes comparable with the characteristic pulsation time of the wind velocity $\tau_c \sim \tau_\upsilon$. Our model contains a preferred direction of time-dependent pulsations of υ (X-axis), so that they degrade the spatial coherence only along the X-axis. Points with the same x coordinate remain completely coherent in space. Competition between the nonlinear and diffraction effects causes the correlation radius to vary nonmonotonically along the X-axis. On the initial section of the self-action path $z < L_{nl}\sqrt{\upsilon_0 / \sigma_\upsilon}$ the correlation radius $\bar{r}_{cx}$ decreases; as the beam broadens and the nonlinear effects become weaker diffraction starts to play a dominant role and $\bar{r}_{cx}$ increases (Fig. 3).

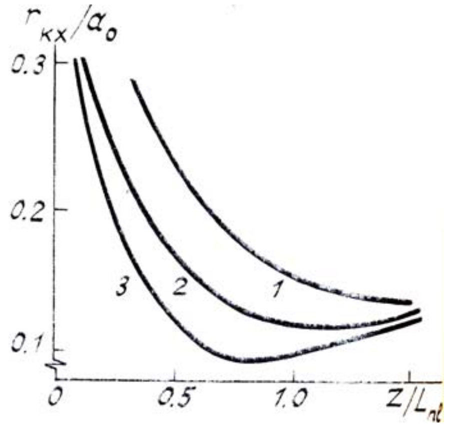

FIG. 3. The transformation of the coherence radius of the beam $r_{cx}(z)$ along a propagation path with $L_d = 10L_{nl}$ and $\sigma_\upsilon/\upsilon_0 = 0.1$ (1), 0.3 (2), and 0.5 (3).

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