PHENOMENOLOGICAL STUDY OF THE PROBABILITY DENSITY OF THE INTENSITY FLUCTUATIONS IN TURBULENT ATMOSPHERE

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Phenomenological model is investigated which enables separation of two contributions of the total received field, one from direct radiation, and another one from multiple scattering. Using the Beckman distribution, based on this model, we have calculated the higher moments and derived the relationships between the distribution parameters and experimentally measured moments. It has been found that this distribution, due to limitations on the value of its third moment, cannot be used for describing intensity fluctuations along some paths, particularly under strong turbulence. Results are compared with the experiments, in which the Beckman distribution is applicable.

Contemporary theories of optical waves fail to describe comprehensively the probability density of intensity fluctuations at different atmospheric paths. The law of the intensity distribution is shown to be controlled by a single dimensionless parameter β_0 (Ref. 1)

$$b_0^2 = 1.23\ C_n^2\ k^{7/6}\ L^{11/6}\ , \tag{1}$$

where C_n^2 is the intensity of the refractive index fluctuations, L is the path length, and $k = 2\pi/\lambda$ is the wave number. In the weak fluctuation limit, $\beta_0 < 1$ and the theory and experiment both exhibit lognormal distribution of intensity fluctuations, while at $\beta_0 \gg 1$ (strong turbulence, long paths), the theory predicts exponential distribution of the intensity. However, these models do not provide a satisfactorily good description of the intensity fluctuations in the intermediate, though quite wide, region most interesting in practice.

In recent years the phenomenological approach[2,3] to the description of the probability density of fluctuations in a wide range of turbulent conditions is being widely used. The present paper validates experimentally one such a fluctuation model, proposed by Beckman and then adapted to optical range in Ref. 5.

According to the model chosen, the total received field consists of two components

$$A\ e^{i\phi} = A_1\ e^{i\phi_1} + A_2\ e^{i\phi_2}\ . \tag{2}$$

The first, $A_1\ e^{i\phi_1}$, is due to forward scattered radiation on the inhomogeneities located at the transmitter-receiver axis, and has a lognormal distribution of the amplitude and normal distribution of the phase. The second, $A_2\ e^{i\phi_2}$, is due to multiple scattering on the off-axis inhomogeneities and has Rayleigh distribution of the amplitude and uniform distribution of the phase. In fact, the same model is used in Ref. 3 except that it assumes that amplitudes A_1 and A_2 of the first and the second waves are distributed according to the Nakajimi m–distribution. Also essentially close to it is the model of optical wave intensity fluctuations in the atmosphere,[6] with precipitation which considers the probability density as a mixture of two contributions, one from the atmospheric turbulence, and the other due to scattering on hydrometeors.

A universal distribution from Ref. 3 was explored in Ref. 4 and it was shown that in a weak turbulence regime, it does not obey lognormal law and deviates considerably from the experimental data obtained under deep fading regime.

Based on the model (2) Ref. 5 introduces the integral representation of the Beckman probability density in the form

$$P(I) = \left(\sqrt{2\pi}\ \sigma_{I_1}\ m_{I_2}\right)^{-1} \times$$

$$\times \int\limits_0^\infty \frac{1}{I_1} \exp\left[-\frac{(\ln I_1 - m_{I_1})^2}{2\ \sigma_{I_1}^2} - \frac{I + I_1}{m_{I_2}}\right] \times$$

$$\times\ \mathbf{I}_0\left(\frac{2\sqrt{I\ I_1}}{m_{I_2}}\right) \mathrm{d}I_1\ , \tag{3}$$

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where σ_{I_1} and m_{I_1} are the variance and mean value of the logarithm of the lognormal component intensity, respectively, m_{I_2} is the mean intensity of the Rayleigh component of the field, and $\mathbf{I}_0()$ is the modified Bessel function of zero order. Using Eq. (3) we can write the expression for the high normalized intensity moments immediately as

$$m_n = \frac{<I^n>}{<I>^n} = \frac{1}{(1+R)^n} \sum_{k=0}^{n} (C_n^k)^2 \, m_r^{n-k} \, m_l^k \, R^k , \qquad (4)$$

where $m_l^k = <A_1^{2k}>/<A_1^2>^k$ is the kth normalized moment of the lognormal component, $m_r^k = <A_2^{2k}>/<A_2^2>^k$ is kth normalized moment of the Rayleigh component, and $R = <A_1^2>/<A_2^2>$ is the parameter controlling the relation between the lognormal and Rayleigh components of the total field.

Then, in order to relate the parameters of Eq. (3) to the moments measured experimentally, we solved the system composed of the equations for the first three moments, m_n, and for the parameters of the distribution (3). As a result, we have

$$m_{I_2} = (1 + R)^{-1};$$

$$R = \left\{ \sqrt{\frac{e^{\sigma_{I1}^2} - 2}{m_2 - 2}} - 1 \right\}^{-1} ; \qquad (5)$$

$$\frac{e^{\sigma_{I1}^2} - 2}{\left(e^{3\sigma_{I1}^2} - 9\, e^{\sigma_{I1}^2} + 12\right)^{2/3}} = \frac{m_2 - 2}{\left(m_3 - 9\, m_2 + 12\right)^{2/3}} .$$

It is important to note that the function

$$f(\sigma_{I_1}) = \frac{e^{\sigma_{I1}^2} - 2}{\left(e^{3\sigma_{I1}^2} - 9\, e^{\sigma_{I1}^2} + 12\right)^{2/3}}$$

is limited, $-0.4 < f(\sigma_{I_1}) < 0.2$, and therefore, the ratio $\dfrac{m_2 - 2}{(m_3 - 9\, m_2 + 12)^{2/3}}$ must take values from this interval in order for σ_{I_1} to be determined. This condition, together with the condition $m_{I_2} > 0$, limits possible range of the third moment, specifically, to those m_3 values falling within the shaded zone in Fig. 1. This, therefore, casts some doubts on the conclusion drawn empirically in Ref. 5 about the universality of the Beckman distribution. We note that the relation (4), for normalized moments, coincides exactly with that from Ref. 3, for the normalized moments of the universal distribution. Thus, we can assume that this expression for the moments as well as the restrictions imposed on the third normalized moment is, to say generally, common for all such models.

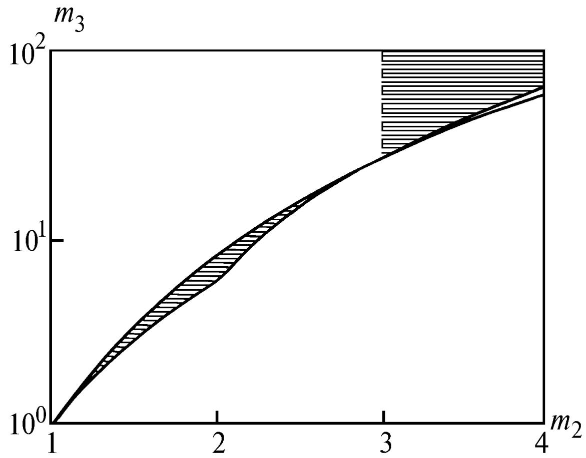

FIG. 1. Applicability ranges of the Beckman distribution.

Analyzing experimental data, for $m_2 < 2$, that is under weak turbulence, we have calculated parameters for a number of sets of the Beckman distribution and inserted them into Eq. (3). The results are shown in Figs. 2–4 together with the model distributions. The measurements were performed with the instrumentation and by the methods described in Refs. 6 and 7. As seen, distribution (3) essentially differs from the experimental one, especially in the deep fading regime.

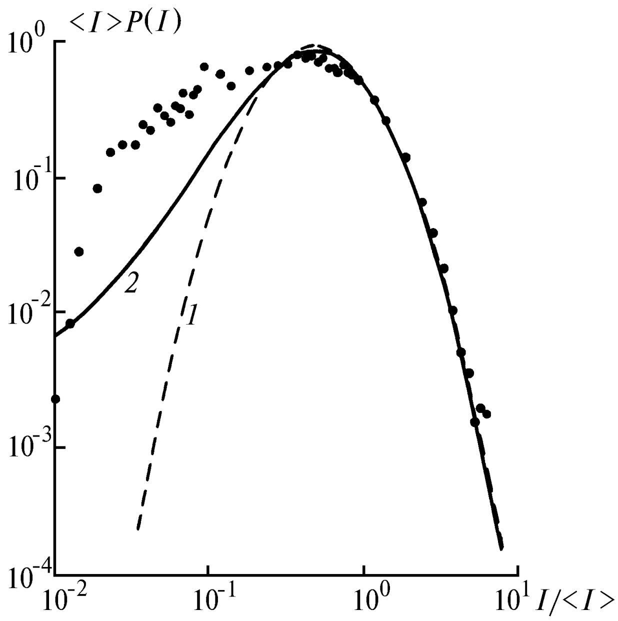

FIG. 2. Comparison of the histogram of normalized intensity with lognormal distribution (curve 1) and with the Beckman distribution (curve 2) at β = 0.74, *for a narrow beam reflected from a plane mirror in rain.*

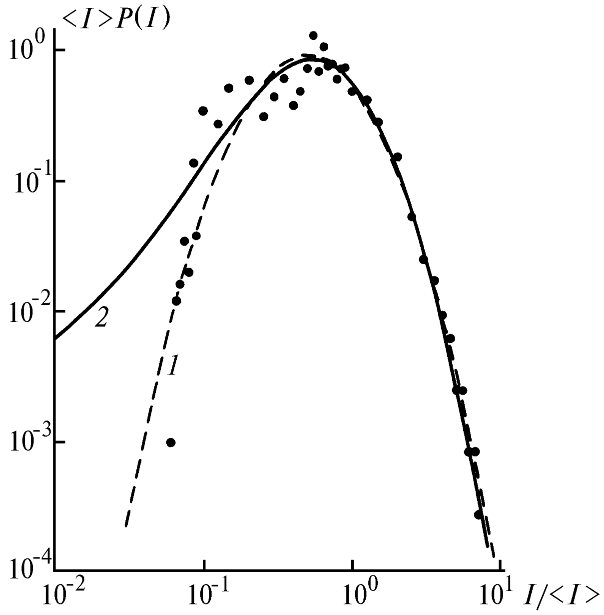

FIG. 3. The same as in Fig. 2 but at β = 0.76, *for a quasiplanar wave reflected from a mirror disk.*

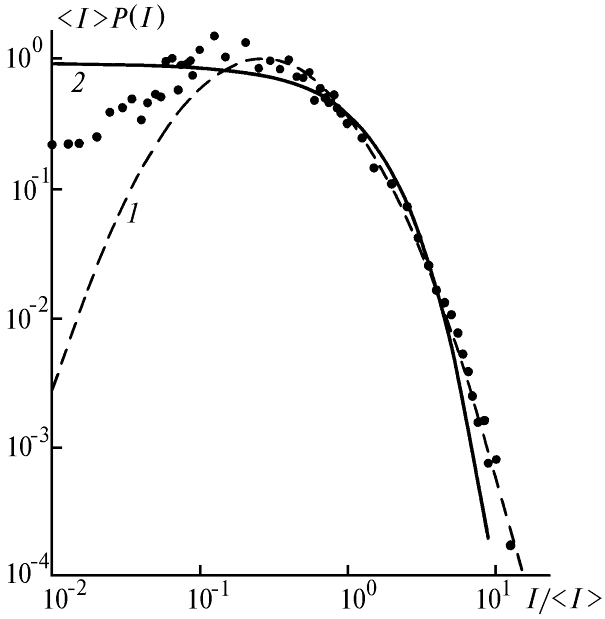

FIG. 4. The same as in Fig. 2 but at β = 0.99, *for a quasiplanar wave reflected from a mirror disk.*

For $m_2 > 2$, none of the realizations has moments falling within the region shown in Fig. 1. This, in our opinion, is attributed to the limited dynamic range of the instrumentation employed, resulting in the systematically low high–order moments in the experiment. Also, doubtful is the use of the Rayleigh distribution to describe the amplitude of the second component in Eq. (1); this could be justified if the overall field were the superposition of multiply scattered waves, statistically independent and abundant in number. This, however, is not generally the case in the real atmosphere, which, while comprising inhomogeneities of a wide range of scales, is, in fact, dominated by radiation fields scattered from large inhomogeneities, thus, being partially correlated. The overall radiation field is therefore not rigorously Rayleigh and, hence, the intensity distributions are not exponential. In this regard, it would be most reasonable to describe the amplitude of the second component in Eq. (2) with the K–distribution which fits experimental data[8] much better than the exponential distribution. Use of such a model, in which the second field component is described with the K–distribution rather than with the Rayleigh one, will make a subject for a separate paper. As to the distribution (3), its narrow applicability range and poor fit of experimental data, meeting the application criteria, makes its use to describe probability densities over a wide range of atmospheric turbulence hardly possible.

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