CORRELATION FUNCTION OF THE LONG–WAVE RADIATION IN BROKEN CLOUDS: THE CALCULATED RESULTS

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The IR–radiation transfer through the broken clouds is considered. The investigation concerned the dependence of variance and correlation function of thermal radiation on the parameters of cloud field and observation conditions. The effect of light scattering on the second moments of the brightness field is estimated.

The investigation of the relationship between the statistical parameters of cloudiness and radiation fields is urgently needed for solving the wide range of scientific and applied problems. The dependences of the mean value of the long–wave radiation intensity on the parameters of the cloud field and on the observation conditions have been studied with an account of multiple scattering effects[1] and without them.[2] The equations for second moment of long–wave radiation intensity have been obtained and solved in Ref. 3. It has been shown in Ref. 1 that if the viewing angle $\xi < 60–70°$ while the optical depth of cumulus clouds $\tau > 15–20$ then in estimating the mean value of the intensity one can neglect scattering and treat the cumulus clouds as absolutely black emitters. In what follows we will call the clouds, in which the scattering is neglected, black clouds.

The purpose of this paper is to investigate the dependence of the variance and correlation function of the long–wave radiation intensity on the optical–geometric parameters of the cloud field and the observation conditions as well as to determine the limits of applicability of the black–cloud approximation in calculating the second moments of the brightness fields.

The model and solution technique. The statistically uniform and anisotropic model of the cloud field $\kappa(\mathbf{r})$ is based on the Poisson point fluxes on the straight lines.[4] The cloud amount N, thickness H and the cloud horizontal size D, which determines the correlation function of cloud fields, are the input parameters of the model.

We use the notation $D_\varphi(\mathbf{x})$, $D_i(\mathbf{x})$, $K_\varphi(\mathbf{x}_1, \mathbf{x}_2)$, and $K_i(\mathbf{x}_1, \mathbf{x}_2)$ for variances and normalized correlation functions of the intensity of direct $\varphi(\mathbf{x})$ and diffuse $i(\mathbf{x})$ long–wave radiation,[3] while the notation $D_0(\mathbf{x})$ and $K_0(\mathbf{x}_1, \mathbf{x}_2)$ – for the statistical characteristics of the radiation intensity of the black clouds ($\lambda = 0$), where $\mathbf{x} = (\mathbf{r}, \boldsymbol{\omega})$ is the point in the phase space X of the coordinates and directions $\boldsymbol{\omega} = (a, b, c)$ and λ is the quantum survival probability. We denote by $V_{\varphi, i}(\mathbf{x}_1, \mathbf{x}_2)$ and $V_{i, \varphi}(\mathbf{x}_1, \mathbf{x}_2)$ the cross–correlation functions of the intensity of direct and diffuse radiation.

To estimate the values $D_i(\mathbf{x})$, $K_i(\mathbf{x}_1, \mathbf{x}_2)$, and $V_{i, \varphi}(\mathbf{x}_1, \mathbf{x}_2)$, the algorithms for the statistical simulation are developed in Ref. 3, while for the calculation of the values $D_\varphi(\mathbf{x})$, $D_0(\mathbf{x})$, $K_\varphi(\mathbf{x}_1, \mathbf{x}_2)$, $K_0(\mathbf{x}_1, \mathbf{x}_2)$, and $V_{\varphi, i}(\mathbf{x}_1, \mathbf{x}_2)$ the formulas have been obtained

$$D_\varphi(\mathbf{x}) = \big(I_z(\boldsymbol{\omega}) - (1-\lambda)B_c\big)^2 D_j(\mathbf{x}) ; \tag{1}$$

$$D_0(\mathbf{x}) = \big(I_z(\boldsymbol{\omega}) - B_c\big)^2 D_j(\mathbf{x}) ; \tag{2}$$

$$V_{\varphi, i}(\mathbf{x}_1, \mathbf{x}_2) = \big(I_z(\boldsymbol{\omega}) - (1-\lambda)B_c\big) V_{j, i}(\mathbf{x}_1, \mathbf{x}_2) ; \tag{3}$$

$$K_\varphi(\mathbf{x}_1, \mathbf{x}_2) = K_0(\mathbf{x}_1, \mathbf{x}_2) = K_j(\mathbf{x}_1, \mathbf{x}_2) , \tag{4}$$

where $I_z(\boldsymbol{\omega})$ is the intensity of the radiation from the external sources located at the cloud field boundaries, $B_c = B(T_c)$ is the Planck function at the temperature of the isothermal clouds T_c, $D_j(\mathbf{x})$ and $K_j(\mathbf{x}_1, \mathbf{x}_2)$ are the statistical characteristics of the function $j(\mathbf{x})$ which have been studied in detail in Ref. 4, $V_{j, i}(\mathbf{x}_1, \mathbf{x}_2)$ is the cross–correlation function for $j(\mathbf{x})$ and $i(\mathbf{x})$ (Ref. 5). The function $j(\mathbf{x}) = \exp\left(-\dfrac{\sigma}{|c|}\right)\displaystyle\int_{E_z} \mathrm{k}(\mathbf{r}')\, d\mathrm{h}$ may be treated as the random intensity of the direct radiation at the point $\mathbf{r}$ provided that the unidirectional source of unit power radiating in the direction $\boldsymbol{\omega}$, is located at the point $\mathbf{r}^{(0)} = (x^{(0)}, y^{(0)}, \xi) = \mathbf{r} - \dfrac{z-\xi}{c}\boldsymbol{\omega}$, where $\xi = 0$ at $c > 0$ and $\xi = H$ at $c < 0$, σ is the extinction coefficient, and $E_z = (0, z)$ at $c > 0$ and $E_z = (H, z)$ at $c < 0$.

The variance $D_I(\mathbf{x})$ and the normalized correlation function $K_I(\mathbf{x}_1, \mathbf{x}_2)$ of the total radiation intensity $I(\mathbf{x}) = \varphi(\mathbf{x}) + i(\mathbf{x})$ have the forms

$$D_I(\mathbf{x}) = D_\varphi(\mathbf{x}) + D_i(\mathbf{x}) + V_{i, \varphi}(\mathbf{x}) + V_{\varphi, i}(\mathbf{x}) , \tag{5}$$

$$K_I(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{D_I(\mathbf{x})}\Big\{D_\varphi(\mathbf{x})\, K_\varphi(\mathbf{x}_1, \mathbf{x}_2) + D_i(\mathbf{x})\, K_i(\mathbf{x}_1, \mathbf{x}_2) + V_{i, \varphi}(\mathbf{x}_1, \mathbf{x}_2) + V_{\varphi, i}(\mathbf{x}_1, \mathbf{x}_2)\Big\} . \tag{6}$$

Hereafter for simplicity we will omit the argument of variances and correlation functions.

If the receivers are oriented in the zenith direction, i.e., $\boldsymbol{\omega}_1 = \boldsymbol{\omega}_2 = \boldsymbol{\omega}_\perp = (0, 0, 1)$ or in the nadir direction ($\boldsymbol{\omega}_\perp = (0, 0, -1)$) Eqs. (1)–(6) are substantially simplified. In this particular case[4, 5]

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$$D_j = N(1 - N)\left(1 - e^{-\tau}\right)^2 , \qquad K_j = e^{-A(\Delta x + \Delta y)} , \tag{7}$$

$$V_{j,\,i} = (N - 1) < i(\tilde{z}, \boldsymbol{\omega}_\perp) > \left(1 - e^{-\tau}\right) e^{-A(\Delta x + \Delta y)} , \tag{8}$$

where $A \sim 1/D$, $\Delta x = |x_1 - x_2|$, $\Delta y = |y_1 - y_2|$, τ is the cloud optical depth, and $\tilde{z} = H$ at $c > 0$ and $\tilde{z} = 0$ at $c < 0$.

We now consider the statistical characteristics of diffuse radiation brightness D_i, K_i, and $V_{i,\,\varphi}$. It is obvious from formulas (33) and (34) (see Ref. 3) that at $\boldsymbol{\omega}_1 = \boldsymbol{\omega}_2 = \boldsymbol{\omega}_\perp$

$$|x_0 - x_2| = \Delta x , \; |y_0 - y_2| = \Delta y , \tag{9}$$

where $\mathbf{r}_0 = (x_0, y_0, z_0)$ is the point of a first collision. According to Ref. 5 at $A(\boldsymbol{\omega}_\perp) = A(|a| + |b|) = 0$ we have

$$<i(\tilde{z}, \boldsymbol{\omega}_\perp)> = Nu(\tilde{z}, \boldsymbol{\omega}_\perp) = <\chi(\mathbf{r})\, i(\mathbf{r}, \boldsymbol{\omega}_\perp)> . \tag{10}$$

Accounting for Eqs. (9) and (10), the sought–after correlations may be represented as follows:

$$D_i K_i = \left((1 - N) <i(\tilde{z}, \boldsymbol{\omega}_\perp)>^2/N\right) e^{-A(\Delta x + \Delta y)} , \tag{11}$$

$$V_{i,\,\varphi} = (1 - N)\left((1 - \lambda)\, B_c - I_z(\boldsymbol{\omega}_\perp)\right) \times$$

$$\times <i(\tilde{z}, \boldsymbol{\omega}_\perp)> \left(1 - e^{-\tau}\right) e^{-A(\Delta x + \Delta y)} . \tag{12}$$

It follows from formulas (3), (4), and (7)–(12) that the normalized correlation functions K_φ, K_i, $V_{i,\,\varphi}$, and $V_{\varphi,\,i}$ coincide with the correlation function of the probability that the zenith (nadir) direction is covered with clouds

$$\tilde{K} = e^{-A(\Delta x + \Delta y)} , \tag{13}$$

while the variances of brightness have the form:

$$D_0 = N(1 - N)\left(I_z(\boldsymbol{\omega}_\perp) - B_c\right)^2\left(1 - e^{-\tau}\right)^2 , \tag{14}$$

$$D_\varphi = N(1 - N)\left(I_z(\boldsymbol{\omega}_\perp) - (1 - \lambda)B_c\right)^2\left(1 - e^{-\tau}\right)^2 , \tag{15}$$

$$D_i = (1 - N) <i(\tilde{z}, \boldsymbol{\omega}_\perp)>^2/N , \tag{16}$$

$$V_{i,\,\varphi} = V_{\varphi,\,i} = (1 - N)\left((1 - \lambda)\, B_c - I_z(\boldsymbol{\omega}_\perp)\right) \times$$

$$\times <i(\tilde{z}, \boldsymbol{\omega}_\perp)> \left(1 - e^{-\tau}\right) . \tag{17}$$

$$D_I = N(1 - N)\Big(I_z(\boldsymbol{\omega}_\perp) -$$

$$- \left(\alpha<i(\tilde{z}, \boldsymbol{\omega}_\perp)> + (1 - \lambda)B_c\right)\Big)^2 \left(1 - e^{-\tau}\right)^2 , \tag{18}$$

where $\alpha = 1/(N\,(1 - e^{-\tau}))$.

In Refs. 4 and 5 it has been shown that in the asymptotic case as $\gamma = H/D \to 0$ the correlation functions tends toward unity

$$D_j = N(1 - N)\left(1 - e^{-\tau/|c|}\right)^2 , \tag{19}$$

$$<i(\tilde{z}, \boldsymbol{\omega})> = N i_{0,s}(\tilde{z}, \boldsymbol{\omega}) , \tag{20}$$

where $i_{0,\,s}(z, \boldsymbol{\omega})$ is the intensity of scattered radiation in a continuous cloud layer. It is not difficult to show using Eqs. (19) and (20) that formulas (14)–(18) may be employed for calculation of the variances of brightness of thermal radiation, modulated by the stratified clouds ($\gamma \ll 1$), partially covering the sky, for various viewing angles by substituting $\tau/|c|$ for τ and $\boldsymbol{\omega}$ for $\boldsymbol{\omega}_\perp$.

The calculated results. The absorption by the atmospheric gases in the atmospheric transparency window 8–13 μm is well known to be very weak,[6] and for this reason we will account for the interaction only between the radiation and cloud matter within the cloud layer in this spectral range. If scattering in the atmosphere above and below the cloud is neglected, then the absorption and emission of radiation by the aerosol and atmospheric gases may be easily taken into account with the boundary conditions (on the function $I_z(\boldsymbol{\omega})$). It is obvious from the above assumptions that aerosols and gases have no effect on the qualitative dependence of statistical characteristics of long–wave radiation intensity on the parameters of the problem, therefore for simplicity we will neglect the influence of the atmosphere beyond the cloud on the thermal radiation transfer. The underlying surface is absolutely black emitter at the temperature $T_s = 290$ K. It follows from the definition of the function $I_z(\boldsymbol{\omega})$ that it equals to B_s at the lower boundary of a cloud layer at $c > 0$, while it equals to zero at the upper boundary at $c < 0$, where $B_s = B(T_s)$. The isothermal clouds at the temperature $T_c = 255$ K have the thickness $H = 1$ km and the horizontal size $D = 0.5$ km. These values of T_s and T_c are typical of mean temperature of the underlying surface and of the middle clouds in summer over the territory of the USSR (see Ref. 7). The scattering phase function and the single–scattering albedo were calculated based on the Mie theory at a wavelength of 10 μm and for C1 cloud.[8] It should be noted that at these temperatures the Planck function of the underlying surface differs almost by a factor of two from that of clouds. The azimuth angle counted off from the *XOZ* plane was taken to be zero, the other parameters are given in figure captions.

The effects of the cloud amount N and of the zenith viewing angle ξ on the variance of intensity of the upwelling ($\uparrow$) and downwelling ($\downarrow$) radiation D_I and its components (see Eq. (5)) are illustrated in Fig. 1. Here and in Figs. 2 and 3 the half–sum $(V_{i,\,\varphi} + V_{\varphi,\,i})/2$, denoted for convenience by $\tilde{V}_{i,\,\varphi}$, is shown.

Let us consider the statistical characteristics of brightness in the directions of zenith and nadir. According to Eqs. (14) and (15) D_0 and D_φ are independent of the parameter γ, they are symmetric about $N = 0.5$, and for cumulus clouds they weakly depend on τ. Without temperature inversion and at $\lambda \neq 0$ it follows from Eqs. (8) and (9) that $D_0^\uparrow < D_\varphi^\uparrow$ while $D_0^\downarrow > D_\varphi^\downarrow$, therefore the inequality $D_0^\downarrow/D_0^\downarrow < D_\varphi^\uparrow/D_\varphi^\downarrow$ is valid in any case. For fixed temperatures of clouds and underlying surface and at $\lambda = 0.638$ the ratio $D_0^\uparrow/D_0^\downarrow$ is equal approximately to unity while the ratio $D_\varphi^\uparrow/D_\varphi^\downarrow$ is of the order of 20 (Fig. 1).

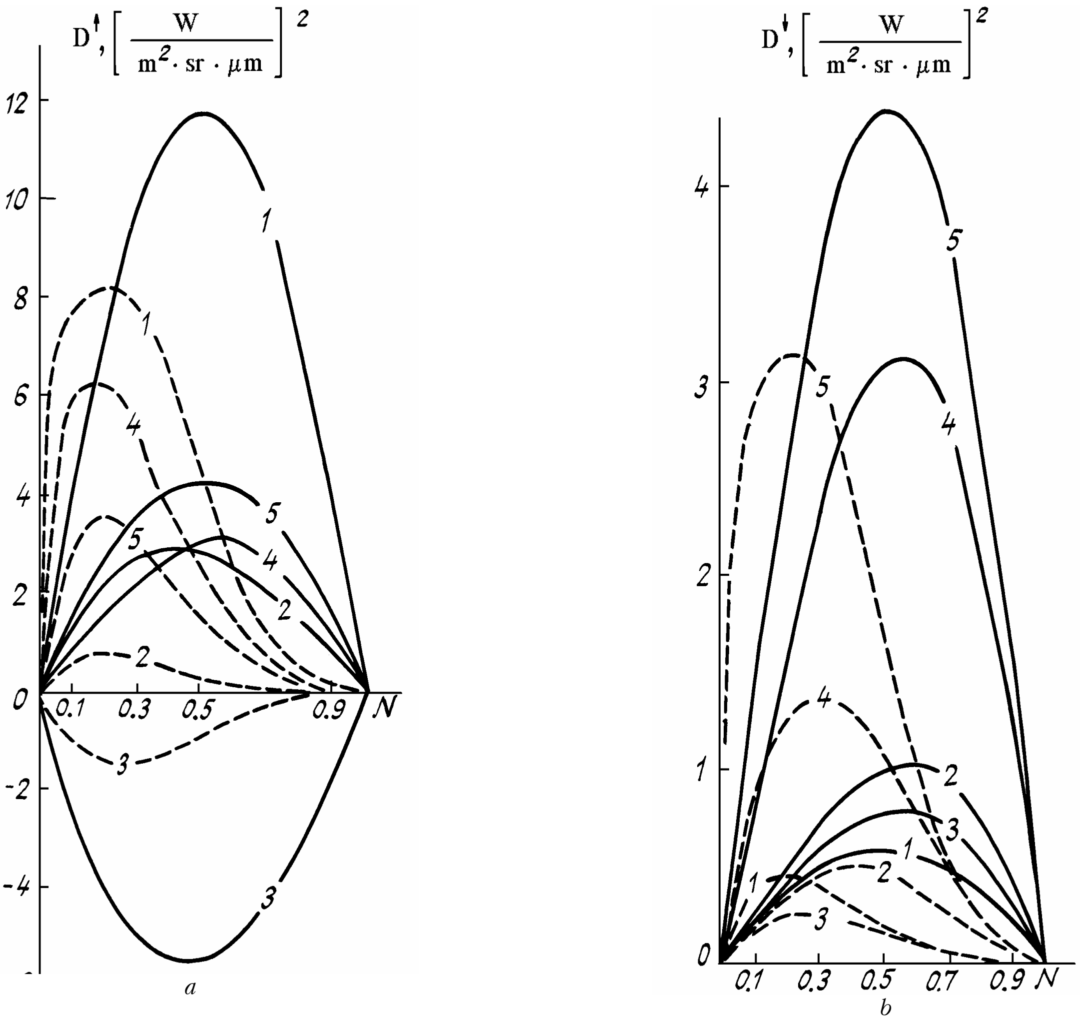
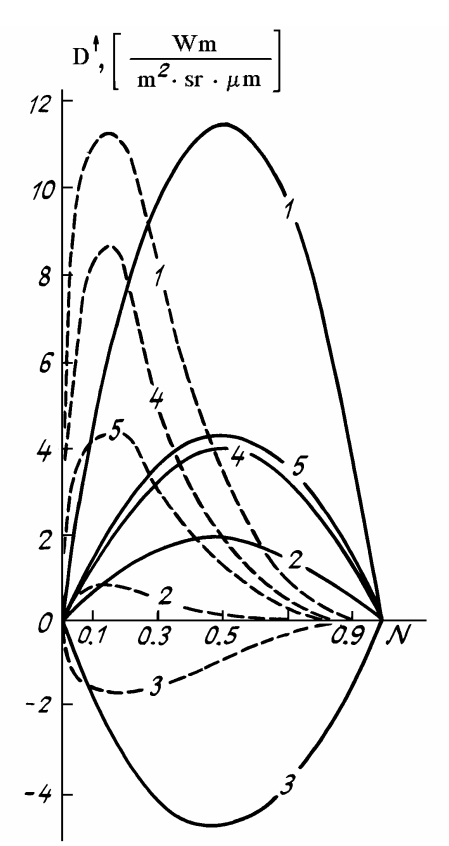


FIG. 1. Variances of the upwelling (a) and downwelling (b) radiations as functions of the zenith viewing angle and of the cloud amount for $\sigma = 5$ km^{-1}. *Here and in Fig. 2 the solid lines refer to* $\xi = 0°$ *and the dashed lines refer to* $\xi = 60°$. *1)* D_{φ}*; 2)* D_i*; 3)* $\tilde{V}_{i,j}$*; 4)* D_I*; and 5)* D_0.

The dependence of variances D_i, $V_{i,\varphi}$, and D_I on the parameters of cloud field is more complicated because these statistical characteristics are the explicit functions of N and τ (see Eqs. (16)–(18) and depend implicitly on N, τ, and the parameter γ via the mean intensity $\langle i(\tilde{z}, \omega_{\perp})\rangle$. The linearity of the transfer equation makes it possible to represent $\langle i(\tilde{z}, \omega_{\perp})\rangle$ as the sum $\langle i(\tilde{z}, \omega_{\perp})\rangle = i_s(\tilde{z}, \omega_{\perp}) + i_c(\tilde{z}, \omega_{\perp})$, where i_s and i_c correspond to the mean intensity of diffuse radiation from clouds and from the underlying surface. The statistical uniformity of cloud field, the constancy of optical characteristics, and the isothermality of clouds result in the equality of $i_c^{\uparrow}(H, \omega_{\perp}) = i_c^{\downarrow}(0, \omega_{\perp})$, therefore the differences between the variances of upwelling and downwelling radiations are caused by warm underlying surface contributing differently to $i_s^{\downarrow}(H, \omega_{\perp})$ and $i_s^{\downarrow}(0, \omega_{\perp})$.

The radiation of the underlying surface exiting through the cloud top via a process of multiple scattering vanishes as $N \to 0$ while for optically thick clouds it happens even as $N \to 1$. This explains the nonmonotonic character of the dependence of $i_s^{\uparrow}(H, \omega_{\perp})$ on the cloud amount. With increase of N the radiation emitted by the underlying surface and then reflected from the cloud layer also increases, and as a result, $i_s^{\downarrow}(0, \omega_{\perp})$ is the monotonically increasing function of N. The fact that the qualitative character of the dependence of $i_s^{\uparrow}(H, \omega_{\perp})$ on N differs from that of $i_s^{\downarrow}(0, \omega_{\perp})$ results in the shift of the maximum values of variance D_i and cross–correlation function $\tilde{V}_{i,\varphi}$ for upwelling and downwelling radiations (Figs. 1 and 2). For small and intermediate N and fixed parameters of the problem the strong elongation of the scattering phase function in the forward direction results in the fact that $i_s^{\uparrow}(H, \omega_{\perp})$ is larger than $i_s^{\downarrow}(0, \omega_{\perp})$, and hence, $D_i^{\uparrow} > D_i^{\downarrow}$ and $|\tilde{V}_{i,\varphi}^{\uparrow}| > \tilde{V}_{i,\varphi}^{\downarrow}$. With increase of the cloud optical depth $i_s^{\uparrow}(0, \omega_{\perp})$ increases, as a result, $D_i^{\downarrow}$ and $\tilde{V}_{i,\varphi}^{\downarrow}$ increase too, while for the upwelling radiation the dependence is inverse.



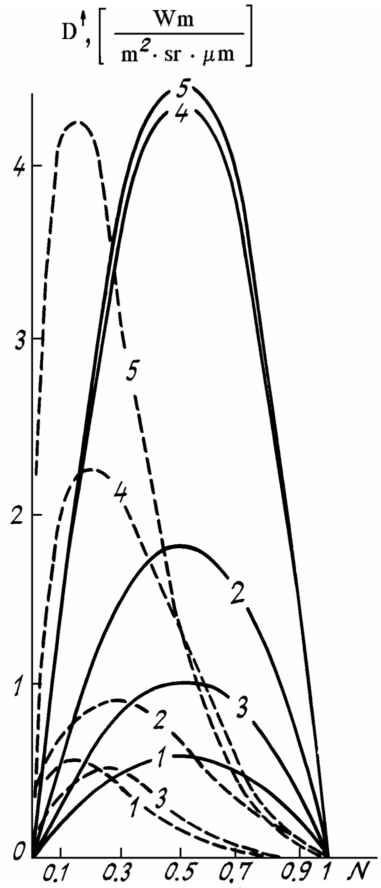


FIG. 2. The effects of the zenith observation angle and of the cloud amount on the variance of the upwelling (a) and downwelling (b) radiation for $\sigma = 40\ \mathrm{km}^{-1}$.

The radiation coming from the warm underlying surface can entry and exit not only through the cloud bottom (top) but also through the cloud sides. The strong anisotropy of the scattering phase function and the presence of absorption cause the sides to play an important role in the formation of $\tilde{i}_i^{\uparrow(\downarrow)}(z, \omega_{\perp})$ because the radiation exiting and entering through them undergoes, on the average, fewer scattering events than the radiation exiting or entering through its bottom (top). This is the reason for $i_s^{\uparrow}(H, \omega_{\perp})$ and therefore for $D_i^{\uparrow}$ to increase, while for $i_s^{\downarrow}(0, \omega_{\perp})$ and $D_i^{\downarrow}$ to decrease with increase of the parameter γ (for instance, H is fixed and D decreases). According to Eq. (17) and prescribed boundary conditions, $\tilde{V}_{i,\varphi}^{\uparrow} < 0$ for the upwelling radiation and $\tilde{V}_{i,\varphi}^{\uparrow} > 0$ for the downwelling one. The negative values of $\tilde{V}_{i,\varphi}^{\downarrow}$ are the result of the fact that with an increase of N, the intensity of the direct radiation $\varphi^{\uparrow}$, on the average, decreases, while the intensity of the diffuse radiation $i^{\uparrow}$, on the average, increases for fixed parameters of the problem.

For the slant viewing angles ξ the effect of finite horizontal cloud size on the formation of the brightness fields of direct and diffuse radiations and their statistical characteristics intensities. With increase of ξ the probability for the viewing direction to be covered with cloud sides increases and results in the shift of maxima of variances toward smaller values of N and the decrease of variances D_{φ} and D_0 for both upwelling and downwelling radiations.

At $\xi > 0°$ in every individual realization of cloud field the receiver records the radiation coming not only from the cloud bottom (top), but also from the cloud sides. The mean optical depth of such a viewing direction, on the average, increases and the amplitude of fluctuations of scattered radiation probably decreases, as a result of multiple scattering. This may be the reason of the decrease of D_i and $V_{i,\varphi}$.

From finished the qualitative analysis of dependences of variances we proceed to some quantitative estimates. The results of calculations show that for the upwelling radiation the $D_i^{\uparrow}$ value is 4 times less than $D_{\varphi}^{\uparrow}$ at $\xi = 0°$ and is 10 times less at $\xi = 60°$ (Fig. 1). It means that $D_{\varphi}^{\uparrow}$ and cross–correlation $\tilde{V}_{i,\varphi}^{\uparrow}$ mainly contribute to the variance of the intensity of the total radiation $D_I^{\uparrow}$. For the downwelling radiation at $\xi = 0°$ $D_i^{\downarrow}$ is approximately 2 times greater than $D_{\varphi}^{\downarrow}$ while at $\xi = 60°$ $D_i^{\downarrow} \approx D_{\varphi}^{\downarrow}$. With increase of optical depth τ from 5 to 40, $D_i^{\uparrow}$ decreases while $D_i^{\downarrow}$ increases approximately by a factor of 1.5–2.

The investigation of the ratio $\delta_1 = (D_I - D_0)/D_0 \cdot 100\%$, which determines the error of ignoring long–wave radiation scattering by clouds in estimating the variance, is of particular interest. In calculating the mean intensity in Ref. 1 the accuracy of the black–cloud approximation was estimated from the condition

$$\Delta T = \left| T_0 - T \right| \leq 1\ K\ , \tag{21}$$

where T_0 and T_1 are the values of brightness temperatures, corresponding to the mean intensities of "black" $\langle I_0(\mathbf{x})\rangle$ and cumulus (with an account for scattering) clouds.

For the stratified clouds ($\gamma \ll 1$) inequality (21) holds at $\xi < 60$–$70°$ and $\tau > 10$ (Ref. 1). It follows from the calculations of variances D_0 and D_I that for the above–indicated values of γ, ξ, and τ and for an arbitrary N the highest δ_1 does not exceed 5%. With increase of the parameter γ and the zenith viewing angle ξ and with decrease of the cloud optical depth τ the differences between D_I and D_0 increase. In the case of the cumulus cloudiness ($0.5 \le \gamma \le 2$) the error in determining the temperature due to the ignorance of scattering does not exceed 1 K at $\xi < 60$–$70°$ and $\tau > 15$–20 (see Ref. 1). The results of calculations show that for the cumulus clouds with $\tau > 15$–20 for an arbitrary N the differences between the variance values of the zenith (nadir) brightness D_I and D_0 do not exceed 5%. At large values of the zenith observation angles δ_1 substantially increases and strongly depends on the cloud amount. For instance, $\delta_1 \sim 100\%$ at $\xi = 60°$, $\gamma = 2$, and $\tau = 40$ for $N = 0.2$ and $\delta_1 \sim 10\%$ for $N = 0.5$.

We will now consider the correlation functions of the long–wave radiation intensity. The characteristic horizontal cloud size is the main parameter, which determines the correlation distance L. When D increases and the other parameters of the problem remain unchanged the average area of the region, in which the intensities are close enough in values, became larger and, therefore, the statistic interrelation between the intensities at the two different points separated at a fixed distance gets stronger and results in the increase of L. With increase of H or ξ the correlation distance also increases though the amount of this increase is less in comparison with an increase of D. For the zenith (nadir) direction L reaches its maximum value at $N = 0.5$ and further decreases when the cloud amount increases or decreases. For slant viewing angles $\xi \ge 30°$ the correlation distance decreases with increase of N. The above dependences of correlation distance L on D, H, ξ, and N are valid for correlation functions of both direct (K_φ) and diffuse (K_i, $V_{i,\varphi}$) radiation.

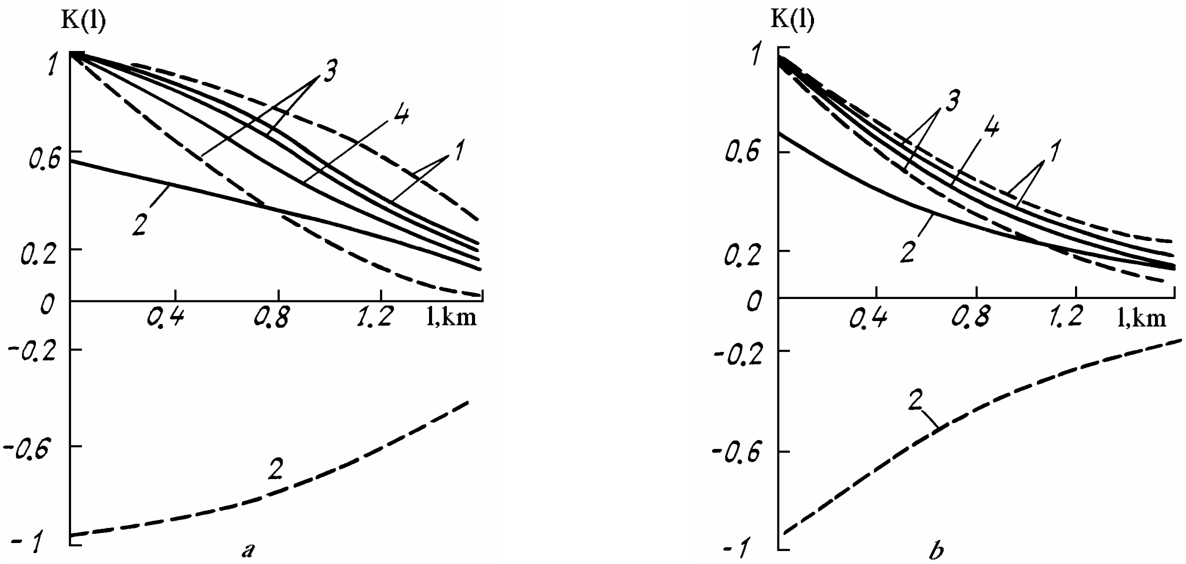


FIG. 3. The dependence of the correlation functions of the intensity on the extinction coefficient at $N = 0.5$, $\xi = 60°$, and $\sigma = 5$, (a) and 40 (b) km^{-1}: 1) $K_i(l)$, 2) $V_{i,\varphi}(l)$, 3) $K_I(l)$, 4) $K_0(l)$. Here $l = |x_1 - x_2|$, solid curves indicate the downwelling radiation, and dashed curves indicate the upwelling radiation.

We introduce the notation L_i, $L_{i,\varphi}$, L_I, and L_0 for the correlation distances defined at e^{-1} level and corresponding to the functions K_i, $\tilde{V}_{i,\varphi}$, K_I, and K_0. According to Eq. (4), K_φ and K_0 coincide with the correlation function K_j, which has been studied in detail in Ref. 4. The calculated results show that for $0° \le \xi \le 60°$, $5 \le \tau \le 40$, and $0 \le \gamma \le 2$ the correlation distance $L_i^{\uparrow(\downarrow)} > L_0$. The anisotropy of scattering results in the differences not only between the variances, but also between the correlation functions of the upwelling and downwelling diffuse radiations, i.e., $L_i^{\uparrow} > L_i^{\downarrow}$ and $|L_{i,\varphi}^{\uparrow}| > L_{i,\varphi}^{\downarrow}$. It follows from Eq. (6) that the relative contribution of the correlations K_φ, K_i, and $V_{i,\varphi}$ to the total correlation function K_I is determined by the contribution of the variances D_φ, D_i, and $\tilde{V}_{i,\varphi}^{\uparrow}$ to the variance D_I. At $N = 0.5$, $\xi = 60°$, $\tau = 5$, $H = 1$ km, and $\gamma = 2$ the variance of the downwelling diffuse radiation is almost twice as large as $D_\varphi^{\downarrow}$ and $V_{i,\varphi}^{\downarrow}$, therefore $K_I^{\downarrow} \approx K_i^{\downarrow}$, and, hence, it follows that $K_I^{\downarrow} > K_0$ (Fig. 3), while the variance of the upwelling radiation $D_I^{\uparrow}$ is determined mainly by $D_\varphi^{\uparrow}$ and $\tilde{V}_{i,\varphi}$, therefore, the correlations $K_\varphi^{\uparrow}$ and $\tilde{V}_{i,\varphi}^{\downarrow}$ mainly contribute to $K_I^{\uparrow}$. Since $\tilde{V}_{i,\varphi}^{\downarrow} < 0$ the inequality $K_I^{\uparrow} < K_0$ holds (Fig. 3).

At the slant observation angles ξ with increase of cloud optical depth τ (other parameters remains unchanged) the correlation distances L_i, $L_{i,\varphi}^{\downarrow}$, $|L_{i,\varphi}^{\uparrow}|$, and L_0 decrease. The correlation distances L_i and $L_{i,\varphi}$ depend on τ stronger than L_0 (see Fig. 3). For instance, at $N = 0.5$, $\xi = 60°$, $H = 1$ km, and $\gamma = 2$ with increase of τ from 5 to 40, L_0 decreases approximately by 10% while L_i and $L_{i,\varphi}$ – by approximately 20–30%. With increase of cloud optical depth both the correlation distances and the differences between L_I and L_0 decrease. For cumulus clouds with $\tau > 20$ at the observation angle $\xi = 60°$ the sings in inequalities $L_I^{\downarrow} < L_0$ and $L_I^{\uparrow} > L_0$ remain unchanged while the error in determining the correlation distance due to ignorance of scattering is approximately 10%.

Conclusion. It has been shown in Ref. 1 that in determining the mean intensity, scattering may be neglected (the error in determining the corresponding brightness temperature does not exceed 1 K) if the observation angle $\xi < 60$–$70°$ while the optical depth of cumulus clouds $\tau > 15$–20. For stratified cloudiness this approximation is valid at $\xi < 60$–$70°$ and $\tau > 10$. We call the clouds, in which scattering can be neglected, as the black clouds.

The calculated results, analyzed in the paper enables us to draw the following conclusions.

1. For stratified cloudiness ($\gamma < 1$) the differences between the variances of the scattering and black clouds do not exceed 5% at $\tau > 10$ and $\xi \leq 60°$, the correlation functions are independent of τ, and ξ tends to unity.

2. For cumulus clouds ($0.5 \leq \gamma \leq 2$) with $\tau > 20$ at viewing angles near zenith (nadir) the ignorance of scattering in calculating the variances results in the error of approximately 5%. The correlation functions coincide with the correlation function of the probability that the viewing zenith (nadir) direction is covered with clouds. With increase of the observation angle the difference between the variances of the scattering and black clouds increases and for large zenith viewing angles reaches its maximum of approximately 100% at small N, while the error in determining the correlation functions is approximately 10 %.

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